Crack Modelling by a Coupled Method of Scaled Boundary Finite Element - Finite Element Methods

Graduate School of Science & Engineering, Ehime University ○ Santosh Shrestha (Student Member, JSCE)
Department of Civil & Environmental Engineering, Ehime University Mitao Ohga (Member, JSCE)
Department of Civil & Environmental Engineering, Ehime University Kazuhiro Taniwaki (Member, JSCE)

Introduction

The finite element method (FEM) is widely used numerical method in fracture mechanics to determine the stress field near a given discontinuity (material or geometric). However, in order to capture the stress singularity occurring near the crack-tip, it is necessary the use of extremely fine meshes around the crack-tip, which reduces its efficiency, or the use of specialized elements of the singular type. In this particular, when studying crack growth phenomena, working with the FEM can become cumbersome, because a precise representation and discretization of the discontinuity must be preformed for each configuration of the discontinuity during crack growth. To address these deficiencies, the scaled boundary finite element method (SBFEM) recently developed by Wolf and Song, which tries to combine the advantages of FEM and boundary element method (BEM) with unique properties of its own, is emerging as an efficient alternative numerical method for crack analysis. Authors [1,2] and other researcher have presented the versatility of SBFEM to compute the fracture parameters: stress intensity factors (SIFs), *T*-stress and higher order terms of the crack tip stress field and crack propagation simulation.

However, SBFEM has certain limitations such as requirement the scaling of material variations with relative to the so-called scaling center, difficult to deal patch load within the domain, and considering linear elastic material behaviors for elastostatics problems. These SBFEM's weaknesses can be FEM's strengths. Therefore, it is often desirable to couple the SBFEM with FEM to overcome these limitations of both methods. To our knowledge, none of the previous studies have addressed the fracture analysis coupling SBFEM-FEM. It is often advantageous to use SBFEM only in the sub-domains closed to cracks, where their capabilities can be exploited to the greatest benefit and FEM in areas away from cracks. In this paper, a coupling of SBFE-FEM is proposed for evaluating the fracture parameters- SIFs and T-stress and then the validity of the proposed coupling method is examined by a crack problem comparing the results with available numerical and analytical solutions in literature.

Coupling the SBFE -FEM

Consider the problem domain Ω is divided into two non-overlapping domains – Ω_{SBFEM}, SBFEM domain and Ω_{FEM}, FEM domain, i.e. $\Omega = \Omega_{SBFEM} \cup \Omega_{FEM}$, with interface boundary, Γ_I as shown in Fig. 1. In Fig. 1, *e* (white circle), *s* (gray circle), and *i* (black circle) present the nodes on FEM region, SBFEM region and interface boundary, respectively. The coupling formulation is as follows

In the FEM, the approximation of the displacements $\{u(x,y)\}$ can be written as

$$\{u(x,y)\} = \sum_{i=1}^{n} N_i(x,y)u_i = [N(x,y)]\{\hat{u}\} \tag{1}$$

where $N(x, y)$ is the shape functions and $\hat{u}$ is the nodal displacement. The equilibrium equation

$$[K]\{\hat{u}\} = \{P\} \tag{2}$$

can be written in matrix form as

$$\begin{bmatrix} K_{ee}^{f} & K_{ei}^{f} \\ K_{ie}^{f} & K_{ii}^{f} \end{bmatrix} \begin{Bmatrix} u_e \\ u_i \end{Bmatrix} = \begin{Bmatrix} P_e \\ P_i \end{Bmatrix} \tag{3}$$

where, the subscripts *i* and *e* designate interaction and non-interaction nodes as shown in Fig.1, respectively and superscript, *f* designate for the FEM.

Similarly, in SBFEM formulation, the approximation of the displacements $\{u(x,y)\}$ can be written as

$$\{u(\eta)\} = \sum_{i=1}^{n} N_i(\eta)u_i = [N(\eta)]\{u\} \tag{4}$$

where $N(\eta)$ is the shape functions and the equilibrium equation (Eq. 2) can be written as in matrix form as

$$\begin{bmatrix} K_{ss}^{s} & K_{si}^{s} \\ K_{is}^{s} & K_{ii}^{s} \end{bmatrix} \begin{Bmatrix} u_s \\ u_i \end{Bmatrix} = \begin{Bmatrix} P_s \\ P_i \end{Bmatrix} \tag{5}$$

where, the subscripts, *i* and *s* designate interaction and non-interaction nodes for the SBFEM region as shown in Fig.1, respectively and superscript, *s* designate for the SBFEM.

Finally the compatibility requirement along the interface is introduced as

$$\{u(x,y)\} = [N(x,y)]\{\hat{u}\} = [N(\eta)]\{u\} \quad \text{over } \Gamma_I \tag{6}$$

As this equation can be satisfied exactly over the entire interface as the SBFEM region has some number of degrees of freedom controlling the approximation. Due to the compatibility of the displacements and equilibrium of the forces at the interaction nodes between FEM and SBFEM regions, the stiffness matrices of FEM can be added to the stiffness matrix of SBFEM as

$$\begin{bmatrix} K_{ee}^{f} & K_{ei}^{f} & 0 \\ K_{ie}^{f} & K_{ii}^{f} + K_{ii}^{s} & K_{is}^{s} \\ 0 & K_{si}^{s} & K_{ss}^{s} \end{bmatrix} \begin{Bmatrix} u_e \\ u_i \\ u_s \end{Bmatrix} = \begin{Bmatrix} P_e \\ P_i \\ P_s \end{Bmatrix} \tag{7}$$

Eq. (14) is full system equation of the coupling method. The stiffness matrix can be assembled to form the system stiffness matrix without any difficulty. In fact, the assembly procedure is the same as usually used in the FEM.

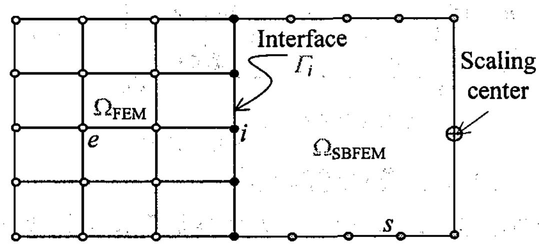

Fig 1. Domains contains SBFEM and FEM regions

SBFEM formulation for fracture parameters

To compute the fracture parameters i.e. SIFs and T-stress and higher order terms of the crack-tip stress fields, authors have presented a simple and direct formulation by comparing the classical linear elastic field solution (Williams' eigenfunction series) in the vicinity of a crack-tip with the SBFEM the stress and displacement fields along the radial direction emanating from the crack tip in [1]. The necessary condition of the formulation is that the scaling center is chosen at the crack-tip that leads to only the boundary, but not the straight crack faces and faces passing through the crack tip is discretized. The presented formulation for SIFs and T-stress are as follows.

$$K_I = c(\hat{\sigma}_y)\sqrt{2\pi\hat{r}} \tag{8}$$

$$T = c(\hat{\sigma}_x) \tag{9}$$

where, $\hat{\sigma}_y$ and $\hat{\sigma}_x$ are the stress components along perpendicular and parallel to crack surface, respectively; $\hat{r}$ is the radial distances of the boundary nodes from scaling center, and c is the integration constant. In this paper only the basic equations of the proposed formulation are presented. For a more detailed description we refer to [1].

Validation of SBFE-FEM

A computer coding in MATLAB was developed on the base of above formulation and a three-point bending beam with single edge crack at the middle was analysed to demonstrate the effectiveness of the coupled method. The main objective of the analysis is to compute the fracture parameters - SIF and T-stress. Near crack region was model from SBFEM and other regions were model from FEM. The discretization employed in this study consisted of three-node iso-parametric quadratic line elements on the boundary for SBFEM region and eight-node iso-parametric quadratic elements for FEM regions. The scaling center of SBFEM was placed at the crack-tip in SBFEM mesh.

The schematic diagram is as shown in Fig.2, where L and D are the span and depth of beam respectively and a is the crack length. The applied point load per unit thickness was P = 1 unit at middle. The analyses were carried out using plane strain condition with Young's modulus E = 1.0 and Poisson's ratio υ = 0.3. Unit thickness was assumed for all the specimens. All units are consistent with that of E. Only a half of the specimen (hatched portion in schematic diagrams) was modeled by virtue of symmetry. The problem was analysed with span to depth ratio L/D = 4 and the relative crack length, a/D = 0.2. A typical FE-SBFEM model with L_S/L_F = 0.25 used for analysis is given in Fig. 3 (a), where L_S and L_F are length of SBFEM region and FEM region, respectively.

The computed results of the normalized SIFs, $K_I/\sigma_0(\pi a)^{1/2}$ and the normalized T-stress, T/σ_0, for three different length ratios, L_S/L_F are presented in Table 1,

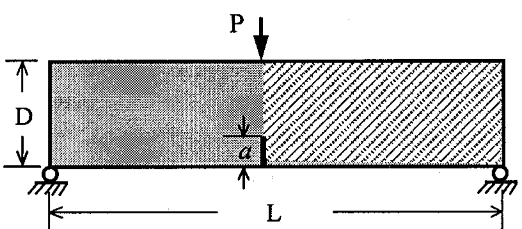

Fig. 2. Schematic diagram

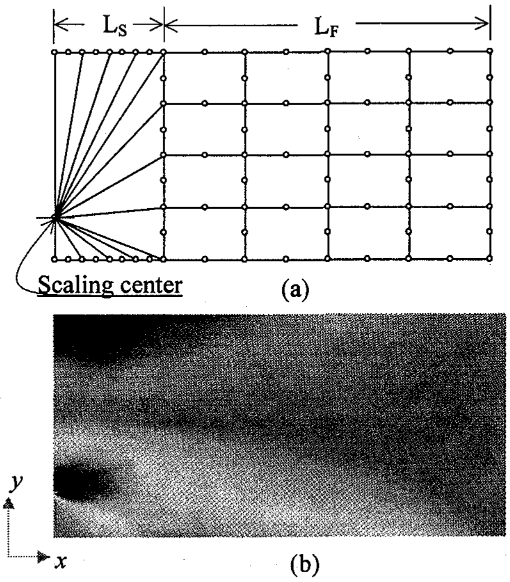

Fig. 3 (a) Analysis model and (b) stress, σ_x contour

Table 1. Computed results

L_S/L_F	Normalized SIFs		Normalized T-stress	
	Present	Ratio	Present	Ratio
0.25	0.3101	0.99	0.2353	1.00
0.50	0.3090	0.99	0.2361	1.00
0.75	0.3083	0.99	0.2367	1.01

* Ratio with the values (0.312 & 0.2355) from [3]

where, $\sigma_0 = 3PL/2D^2$ and the 'ratio' is the comparison of the computed results with respect to the references results. These SBFE-FEM results of the normalized SIFs are compared with the results obtained by Guinea et al. (1998), while the computed normalized T –stress are compared with the results obtained by hybrid crack element method from [3]. The comparison shows that SBFE-FEM results are in an excellent agreement with the references values. Fig.3 (b) is presented the stress, σ_x contour of the problem.

Conclusion

In this paper, a coupling of the recently developed SBFEM and the traditional FEM was applied for linear-elastic fracture analysis. The computed results obtained by the proposed method are in remarkable agreement with the corresponding ones in the literature. Based on this study, it can be confined that a significant saving in the computation cost can be achieved due to the proposed new coupling method comparing with the traditional FEM.

References

1. Shrestha, S. and Ohga, M (2005): An efficient method for computation of singular and higher order terms of a crack-tip stress field, *Journal of Applied Mechanics* JSCE, 8, pp. 171-178.
2. Shrestha, S., Ohga, M and Taniwaki, K. (2006), On the crack propagation simulation by scaled boundary finite element method, *Proceeding of Tenth East Asia Pacific Conference on Structural Engineering & Construction (EASEC-10)*, Bangkok, Thailand, pp.413-418.
3. Karihaloo, B. L. and Xias, Q. Z., (2001): Higher order terms of the crack tip asymptotic field for a notched three-point bend beam. *In. J. of Fracture*, 112, pp 111-128.