# Non-Linear Kinematic Hardening Model for Cyclic Elasto-Plastic Analysis

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## 1. Introduction

The available non-linear kinematic hardening models for multaxial cyclic plasticity still produce little bit wrong predictions to their behavior when it reaches to high values of total strains or effective plastic strains, (Navarro et al, 2005). This paper presents a new non-linear kinematic hardening elasto-plastic model to describe the material behaviour more realistically in any strain range when it is subjected to multiaxial cyclic loading.

## 2. Proposed Hardening Model

The proposed model consists of new hardening rule, which gives proper idea to the non-linear plastic flow than the previous hardening rules.

In plasticity theory, (Lubliner et al, 1993), for a material exhibiting the kinemetic hardening, the von Misses yield criteria is defined as,

$$\phi = \left(\sigma'_{ij} - \alpha'_{ij}\right)\left(\sigma'_{ij} - \alpha'_{ij}\right) - \frac{2}{3}\sigma_0^2 = 0 \tag{1}$$

where $\phi$ = the yield function; $\sigma'_{ij}$ = the deviatoric stress tensor ; $\alpha'_{ij}$ = the deviatoric component of back stress tensor ; and $\sigma_0$ = the initial yield stress in uniaxial tension.

When the plastic flow is assumed to be associated, the incremental plastic strain is found from the Drucker's normality condition and it is mathematically described as,

$$d\varepsilon_{ij}^{p} = d\lambda\left(\frac{\partial\phi}{\partial\sigma'_{ij}}\right) \tag{2}$$

where $d\lambda$ = the plastic multiplier; and $d\varepsilon_{ij}^{p}$ = the incremental plastic strain.

For loading increment in the plastic region the effective plastic strain is defined as,

$$d\bar{\varepsilon}^{p} = \sqrt{2/3\, d\varepsilon_{ij}^{p} d\varepsilon_{ij}^{p}} \tag{3}$$

where $d\bar{\varepsilon}^{p}$ = the incremental effective plastic strain; and $d\varepsilon_{ij}^{p}$ = the incremental plastic strain tensor.

The new hardening rule is proposed as bellow by making a little extension of previous Armstrong-Fredrick rule.

$$d\alpha_{ij} = \frac{2}{3}c\,d\varepsilon_{ij}^{p} - \gamma\alpha_{ij}d\bar{\varepsilon}^{p} + \beta d\sigma_{ij} \tag{4}$$

where $c$, $\gamma$, $\beta$ = plastic constants , which can be determined by uniaxial experimental results of particular material; and $d\alpha_{ij}$ = the incremental back stress tensor which describes the manner of yield surface translation in stress space.

The new term $\beta d\sigma_{ij}$ is increasing with the plastic loading and finally it contributes to increase the effect of non-linear behaviour more especially in large strain range.

## 3. Elasto-Plastic Constitutive Relation for FEM

Finite Element (FE) formulation of the elasto-plastic stiffness matrix for incremental loading is described as bellow.

The function $\phi$ is such that the material is elastic for $\phi \le 0$ and $(\partial\phi/\partial\sigma'_{ij})d\sigma'_{ij} < 0$ . In this case material is described by,

$$d\sigma_{ij} = D_e d\varepsilon_{ij}^{e} \tag{5}$$

where $D_e$= material elastic stiffness matrix; and $d\varepsilon_{ij}^{e}$= the incremental elastic component of total strain.

If $\phi = 0$ and $(\partial\phi/\partial\sigma'_{ij})d\sigma'_{ij} \ge 0$ then the process is said to be plastic and incremental stress is decomposed as,

$$d\sigma_{ij} = D_e\left(d\varepsilon_{ij}^{e} - d\lambda\left(\frac{\partial\phi}{\partial\sigma'_{ij}}\right)\right) \tag{6}$$

The consistency condition ($d\phi = 0$) is used together with definitions and proposed hardening rule to obtain the material elasto-plastic stiffness tensor as follows.

$$D_p = D_e - \frac{(1-\beta)D_e\left(\frac{\partial\phi}{\partial\sigma'_{ij}}\right)\left(\frac{\partial\phi}{\partial\sigma'_{kl}}\right)D_e}{\frac{2}{3}c\left(\frac{\partial\phi}{\partial\sigma'_{ij}}\right)\left(\frac{\partial\phi}{\partial\sigma'_{kl}}\right) - \frac{4}{3}\gamma\sigma_0\left(\frac{\partial\phi}{\partial\sigma'_{ij}}\right)\alpha_{kl} + (1-\beta)\left(\frac{\partial\phi}{\partial\sigma'_{ij}}\right)D_e\left(\frac{\partial\phi}{\partial\sigma'_{kl}}\right)} \tag{7}$$

Finally the obtained $D_p$ is used to formulate global elasto-plastic stiffness matrix for whole structure in FEM program code.

## 4. Verification of the Proposed Model

In favor of the verification, the model behaviors were compared with previously proposed models and the experimental behaviors of 316L stainless steel material, (Rashid, 2004) for both tensile and cyclic loading.

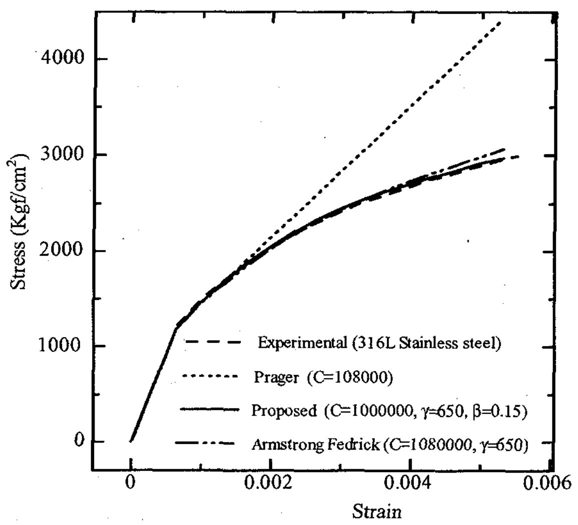


Figure 1. Comparison of uniaxial tensile loading behavior

The modified in-house FEM code is used to simulate the uniaxial tensile loading and the mean stress zero cyclic loading behaviors as shown in Figure 1& 2 respectively.

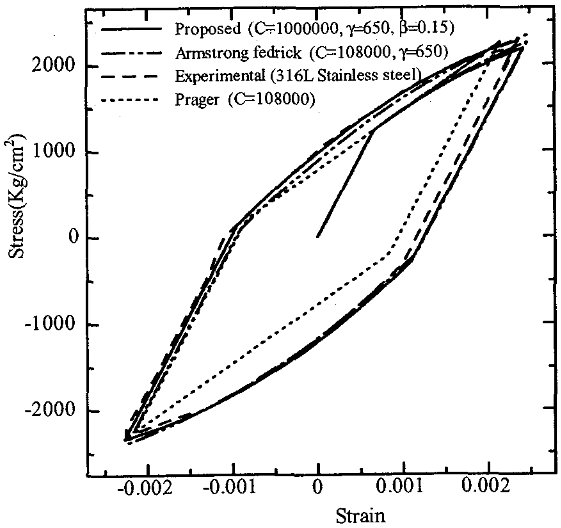


Figure 2. Comparison of cyclic behavior (∓2300 Kgf/cm$^2$)

The results reveals that the model prediction has better agreement with real behavior of the material than previous models and its exhibits more non- linearity than others.

## 5. Case Study

In the case study, two different applications of cyclic elasto-plastic analysis have been mentioned.

### 5.1 Ratcheting failure life estimation

Figure 3 illustrates the stress-strain curves of the stainless steel 316L bar under uniaxial stress cycling with non-zero mean stress with a cycle number 5. During the plastic loading process the accumulated effective plastic strain reaches to the threshold value of the material, the ratcheting life of the component is assumed to be over, (Taher et al. 1993). Hence fatigue failure life is obtained and compared with the previous model predicted life as shown in Table 1.

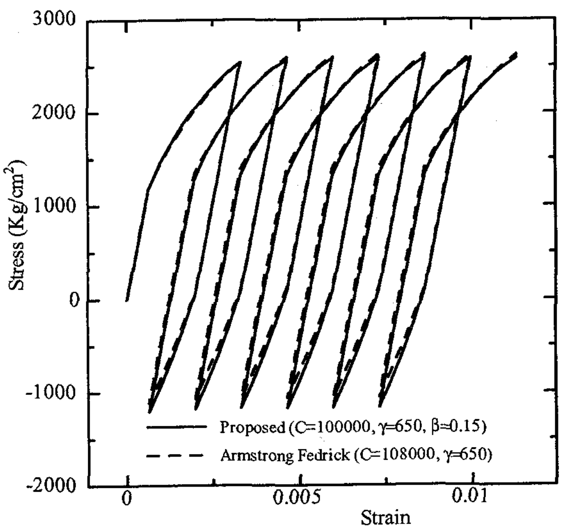


Figure 3. Ratcheting stress/strain behavior comparison with previous model

The predicted failure lives indicate the proposed model prediction extends the life up about 3 % more than the pervious estimation.

Table 1: Comparison of ratcheting failure lives

| Hardening model | $\bar{\varepsilon}^p$ *10$^{-4}$ | Fatigue life (cycles) |
|---|---|---|
| Armstrong Fredrick | 5.809 | 689 |
| Proposed model | 5.653 | 708 |

### 5.2 Low cycle fatigue life estimation

Stainless steel 316L riveted plate member which is subjected to constant amplitude cyclic load in the range of ∓9 Tons applied on the far end from the rivets, is considered for the low cycle fatigue life evaluation. The geometry consists of 120*10 mm plate cross section with two symmetrically located rivet holes apart from 60 mm with 20 mm diameter. By considering the symmetricity of the problem, half of the member with one rivet hole was analyzed using the modified in-house FEM code. The FE mesh consists nine node isoperimetric shell element as shown in Figure 4 and fully bonding restraint condition simulates the unilateral contacts between rivet and plate.

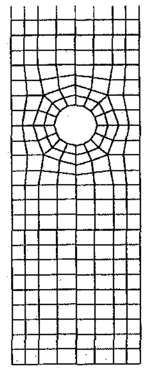

Figure 4: FE model

In view of same failure criteria, mentioned in sub chapter 5.1, the fatigue lives are calculated as illustrated in Table 2.

Table 2: Comparison of fatigue failure lives

| Hardening model | $\bar{\varepsilon}^p$ *10$^{-3}$ | Fatigue life (cycles) |
|---|---|---|
| Armstrong Fredrick | 3.585 | 111 |
| Proposed model | 3.307 | 121 |

The predicted fatigue lives indicate the proposed model prediction extends the life up about 10% more than the pervious estimation.

## 6. Conclusions

The results illustrate that the present model represents real behavior of kinematic hardening material with higher accuracy than previous model in vicinity of cyclic elasto-plastic analysis. Case study reveals that contribution of proposed model provides more optimized prediction to the real application of cyclic elasto-plastic analysis such as low cycle fatigue evaluation, ratcheting failure analysis, etc.

Since this paper presents usage of only one material for the comparison purposes, further verification with few other materials and more case studies with different loading conditions are recommended for future studies.

## References

Lubliner, J. Taylor, R.L. & Auricchio, F.1993, A model of Generalized Plasticity and Its Numerical Implementation, *Journal of Solid and Structures*, Vol.30: pp. 3171-3184

Navarro,A. Giraldez, J.M. Vallellano, C. 2005, A constitutive model for elastoplastic deformation under variable amplitude, *International Journal of Fatigue, Elsevier*, Vol.27: pp 838-846.

Rashid Kamel Abu Al-Rub. 2004, Material Length Scales in Gradient-Dependent Plasticity/Damage and Size Effects: Theory and Computation, *PhD thesis, Louisiana State University*.

Taher, M.A. Voyiadjis, G.Z. 1993, Plastic Damage Model for Concrete Under Cyclic Multiaxial Loading, *Journal of Engineering Mechanics*, Vol.19: pp. 1465-1484