I −23

# EFFECT OF LATERAL PRESSURE ON THE LOWER BOUND STRENGTH OF AXIALLY LOADED SANDWICH CYLINDRICAL SHELL

Department of Civil and Environmental Engineering, Ehime University Mitao Ohga (Member)
Graduate School of Science and Engineering, Ehime University ○ A.S. Wijenayaka (Student Member)
Daio Paper Corporation, Shikoku Chuuou City Kenji Manabe (Student Member)

## Introduction

Examination of buckling of the sandwich cylindrical shell allows the identification of its energy components that are lost in the non-linear imperfection sensitive mode coupling[1]. This has lead to the reduced stiffness lower bound buckling strength of both sandwich cylindrical shells under pure axial and lateral loading[2]. Further to that, the effect of uniform constant external lateral pressure on the Reduced Stiffness (RS) lower bound buckling strength of axially loaded sandwich cylindrical shell is examined in this paper.

## RS lower bound buckling strength

The pre-buckling membrane fundamental state of a simply supported axially loaded sandwich cylinder of length $L$, mean radius $a$, face thickness $h_f$ and core thickness $h_c$, (Fig 1) under constant uniform lateral pressure and can be defined as

$$(N_x^F, N_s^F, N_{xs}^F) = (-2\sigma h_f, -qa, 0) \quad (1)$$

$$(M_x^F, M_x^F, M_{xs}^F) = (0, 0, 0) \quad (2)$$

$$(\mathrm{E}_x^F, \mathrm{E}_s^F, \mathrm{E}_{xs}^F) = \left(-\frac{\sigma}{E_f}, \left(\frac{q}{E_f} + \frac{\upsilon_f \sigma}{E_f}\right), 0\right) \quad (3)$$

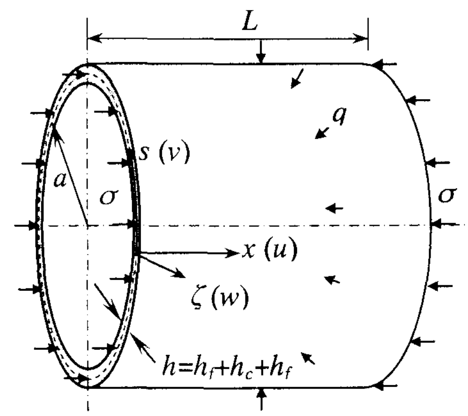



Fig. 1. Sandwich cylindrical shell

Here, $\sigma$ and $q$ denotes the applied axial stress and constant uniform external lateral pressure respectively. The axial load is supported by only face sheets while the core is assumed to support only transverse shear. $v_f$ and $E_f$ denote the Poison parameter and Young's modulus of face respectively. $N_x^F$, $N_s^F$ and $N_{xs}^F$ denote the membrane fundamental stresses in the $x$ and $s$ directions and $xs$ plane (Fig. 1) respectively. $M_x^F$, $M_s^F$ and $M_{xs}^F$ are the relevant fundamental state moments. $E_x^F$, $E_s^F$ and $E_{xs}^F$ denotes the relevant strains. Depending on the incremental membrane strains that are linear and quadratic at the point of buckling, the total potential energy of the sandwich cylindrical shell can be divided into its constant, linear, and quadratic components.

$$\Pi = \Pi_0 + \Pi_1 + \Pi_2 + \ldots \quad (4)$$

$\Pi_2$ controls the stability of the shell and can be effectively approximated by[3, 4]

$$\Pi_2 = \frac{1}{2}\iiint[\sigma_x'\varepsilon_x' + \sigma_s'\varepsilon_s' + \tau_{xs}'\gamma_{xs}' + \tau_{x\xi}'\gamma_{x\xi}' + \tau_{s\xi}'\gamma_{s\xi}']dsdxd\zeta + \iint(N_x^F\varepsilon_x'' + N_s^F\varepsilon_s'')dsdx = U + V_M^x + V_M^s \quad (5)$$

Here, $U$ is the sum of strain energies. $V_M^x$ and $V_M^s$ denote the axial and circumferential nonlinear membrane energy components. Application of the variational principles to Eq. (5) results in a set of homogeneous equations. The classical bucking strength $(\sigma_{alcl})$ can be obtained by further manipulation of those equations.

$$\sigma_{alcl} = ((C_{31}A_1' + C_{32}A_2' + C_{33} + C_{34}A_4' + C_{35}A_5') - \lambda_2 q)/2h_f\lambda_1 \quad (6)$$

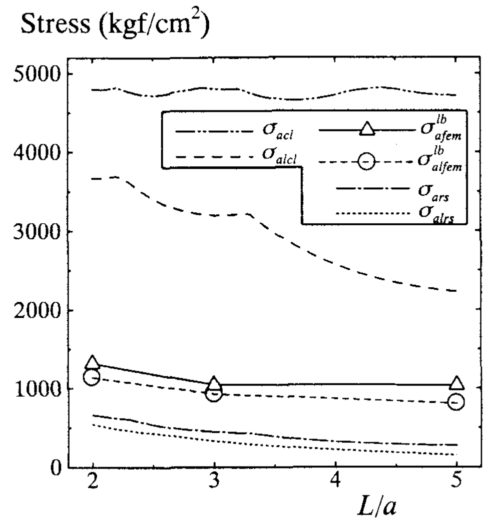
The critical stresses and the relevant mode shapes are obtained as a solution to Eq. (6) by computational iteration. Here, $\lambda_1 = (m\pi/L)^2$, $\lambda_2 = a(n/a)^2$. $m$ and $n$ denote the axial half and circumferential full wave numbers. $A_i'$ depends on amplitude of displacement functions and coefficients, $C_{ij}$ of the homogeneous equations. From Eq. (5),

$$U_M + U_B + U_S + \sigma_{alcl}(\overline{V}_M^x + \overline{V}_M^{s1}) + q\overline{V}_M^{s2} = 0 \quad (7)$$

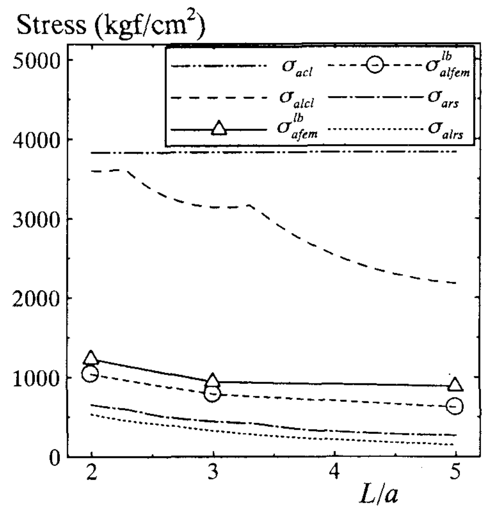
Here, $U_M = U_M^x + U_M^s + U_M^{sx}$ : the axial, circumferential and twist membrane energies, $U_B = U_B^x + U_B^s + U_B^{sx}$ : corresponding bending strain energies and $U_S = U_S^{x\xi} + U_S^{s\xi}$ : the shear strain energies in the superscripted planes. The nonlinear membrane energy components, $\overline{V}_M^{x1}$ and $\overline{V}_M^{s1}$ associate with applied axial stress. Whereas, $\overline{V}_M^{s2}$ associates with the lateral pressure. In the case of the axially loaded sandwich cylindrical shell, the mode coupling can result in the loss of initial stabilization provided by the non-linear circumferential strain energy $V_M^s$ and $U_M^x$ [1~3]. Therefore, elimination of these two energy components from the total potential energy leads to the reduced stiffness strength ($\sigma_{alrs}$), which can be obtained as a function of the classical buckling strength.

$$\frac{\sigma_{alrs}}{\sigma_{alcl}} = \left( \frac{U_M^s + U_M^{xs} + U_B + U_S + q\overline{V}_M^{s2}}{U + q\overline{V}_M^{s2}} \right) \left( \frac{\overline{V}_M^{x1} + \overline{V}_M^{s1}}{\overline{V}_M^{x1}} \right) \tag{8}$$

The validity of the proposed reduced stiffness strength is ascertained by comparing it with that of purely axially loaded sandwich cylindrical shell ($\sigma_{ars}$) and the imperfection sensitive Finite Element Method (FEM) lower bound ($\sigma_{alfem}^{lb}$) obtained with numerical examples[1]. Variation of the FEM and reduced stiffness lower bound and classical buckling strengths are shown in Fig. 2(a) [$V$=0.0] and (b) [$V$=0.1]. Where, $V$ $(= E_f h_f / 4aG_c(1-\nu_f^2))$ is the transverse shear flexibility parameter. Here, $G_c$ denotes the core shear stiffness. $\sigma_{afem}^{lb}$ denotes the FEM lower bound of purely axially loaded shell. $q$ equals 0.02 kgf/cm$^2$. And, it causes the readily available classical buckling strength ($\sigma_{alcl}$) to deviate considerably from that of purely axially loaded shell ($\sigma_{acl}$). This deviation increase as $L/a$ increases. Though small, the slight reduction in FEM and reduced stiffness lower bound buckling strengths are corresponding. Thus, the proposed reduced stiffness lower bound buckling strength avoids the uncertainty of consequences associated with both initial geometric imperfections and external uniform lateral pressure of strong ($V$=0.0) as well as weak ($V$=0.1) core axially loaded sandwich cylindrical shells.




(a) $V$ = 0.0




(b) $V$ = 0.1

Fig 2. Variation of RS and FEM lower bounds ($L/a$=2.0, $E_f$=2.06×10$^5$ MPa, $a/h_f$ = 5000, $h_c/h_f$=10, $a$=1.0 m, $\nu_f$ =0.3)

## Conclusions

An additional uniform external lateral pressure applied on axially loaded sandwich cylindrical shell reduces its reduced stiffness lower bound buckling strength. However, the proposed reduced stiffness lower bound buckling strength avoids the uncertainty of consequences associated with both initial geometric imperfections and external uniform lateral pressure.

## References


1. Ohga, M and Wijenayaka, A. S. (2003). Explicit Lower Bounds for Imperfection Sensitive Buckling of Axially Loaded Sandwich Cylindrical Shells, *International Symposium on New Perspectives of Shell and Spatial Structures*, Taipei, Taiwan, pp 156.1-8.
2. Ohga, Mitao and Umakoshi, M. 2001. RS buckling strength of cylindrical sandwich shells under axial pressures, *Journal of Structural Engineering*, 47A, pp 27-34.
3. Croll, J. G. A. & Batista, R.C. 1981. Explicit lower bounds for the buckling of axially loaded cylinders, *Int. J. Mech. Sci.*, 23(6), pp 331-343.
4. Yamada, S. & Croll, J. G. A. 1993. Buckling and post buckling characteristics of pressure-loaded cylinders, *Journal of Applied Mechanics*, ASME, 60, pp 290-299.