I−14 BUCKLING OF AXIALLY LOADED SANDWICH CYLINDRICAL SHELLS DUE TO LATERAL PRESSURE

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Introduction

The previously postulated reduced stiffness and classical buckling methods for the purely axially loaded sandwich cylindrical shell is revised for when the shell is subjected to a constant lateral pressure. The changes in the classical and reduced stiffness buckling coefficients of axially loaded sandwich cylindrical shell due to the constant lateral pressure are presented. In addition, these behaviors are compared to that of purely axially loaded sandwich cylindrical shells to understand how its behavior might change when lateral pressure is acted upon.

Classical buckling

A convenient way of examining the various possible equilibrium paths described by the stationarity of the total potential energy of a sandwich cylinder of length L, mean radius a, face thickness h_f and core thickness h_c, (Fig 1) is to first define the membrane fundamental state. For the present problem this is defined as given below;

$$(N_x^F, N_s^F, N_{xs}^F) = (-2\sigma h_f, -qa, 0) \quad (1)$$

$$(M_x^F, M_x^F, M_{xs}^F) = (0, 0, 0) \quad (2)$$

Here, N^F and M^F are the fundamental stresses and moments in subscripted planes and directions. The corresponding strains can be written as

$$(\mathrm{E}_x^F, \mathrm{E}_s^F, \mathrm{E}_{xs}^F) = (-\frac{\sigma}{E_f}, -\left(\frac{q}{E_f} - \frac{\upsilon_f \sigma}{E_f}\right), 0) \quad (3)$$

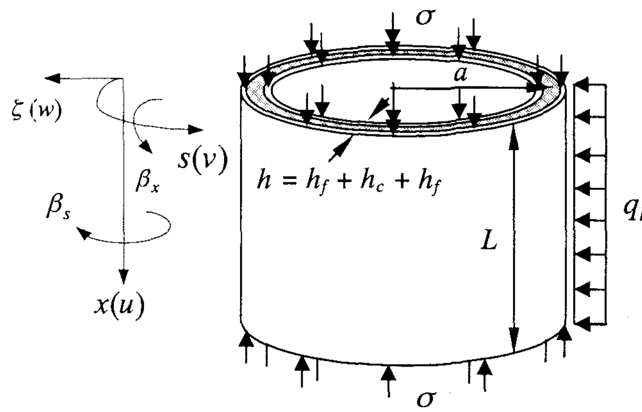

Fig 1. Sign convention, geometry and applied loads

Where, σ is the axial load, E_f, the modulus of elasticity of face and ν_f, the Poison's ratio of face. q_l is the constant lateral pressure acting all around the sandwich cylindrical shell. The quadratic component (Π_2) of the total potential energy is in concern as controls the stability of the problem.

$$\Pi_2 = U + V_M^x + V_M^s \quad (4)$$

Where, U is the sum of strain energies. And, V_M^x and V_M^s are the nonlinear membrane energy terms in axial and circumferential direction respectively. Assuming that the shell is simply supported, application of the variational principles to the above equation results in an eigenvalue problem. The critical stresses and the respective mode shapes are obtained as a solution to this linear eigenvalue problem. When the axially loaded sandwich cylindrical shell is subjected to constant lateral pressure (q_l), the classical axial buckling coefficient is obtained as follows;

$$k_c = \frac{1}{2h_f E_f \lambda_1}\left((C_{31}A_1' + C_{32}A_2' + C_{33} + C_{34}A_4' + C_{35}A_5') - \lambda_2 q_l\right) \quad (5)$$

Here, $\lambda_1 = \{h_f + D_{M1}(1-\nu_f^2)/2E_f\}\rho^2$, $\lambda_2 = \{(D_{M1}/2E_f) + a\}\alpha^2$ α and ρ are the axial half and circumferential full wave number respectively. While, $D_{M1} = 2E_f h_f/(1-\upsilon_f^2)$. A_i' is a function of the amplitude of displacement functions and coefficients, C_{ij} of the eigenmatrix. As it can be seen in Fig 2, when L/a<0.5, the classical buckling coefficient (k_c) due to the lateral pressure is exactly similar to that of the pure axial loading of the sandwich cylindrical shell (k_{ac}). However, when L/a>0.5, the buckling coefficient reduces due to lateral pressure. Also, as it is evident from Fig 3, k_c reduces as the lateral pressure increases. Here, k_{ac} is the classical buckling coefficient due to pure axial loading. q_c is the classical buckling pressure due to pure lateral pressure loading. Lateral pressure equals to q_c reduces k_c by almost 50% as it is evident from Fig 3.

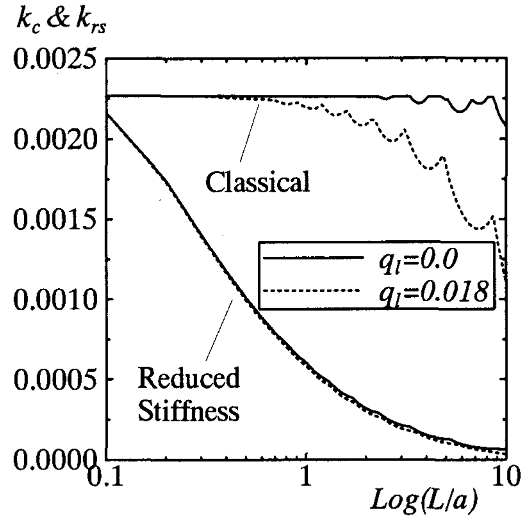

Fig 2. Variation of classical and RS buckling coefficients (q_l = Classical buckling load of laterally pressure loaded sandwich cylindrical shell with L/a=10.0. E_f=2.05x105 MPa, V=0.01, a/h_f=5000, h_c/h_f=10, a=1.0 m)

Reduced stiffness buckling

From Eq. (4),

$$\Pi_2 = U_M + U_B + U_S + \sigma \overline{V}_M^{x1} + \sigma \overline{V}_M^{s1} + q_l \overline{V}_M^{s2} \quad (6)$$

Where, $U_M = U_M^x + U_M^s + U_M^{sx}$: the axial, circumferential and twist membrane energies, $U_B = U_B^x + U_B^s + U_B^{sx}$: corresponding bending strain energies and $U_S = U_S^{x\xi} + U_S^{s\xi}$: the shear strain energies in the superscripted planes. $\overline{V}_M^{x1}$, $\overline{V}_M^{s1}$ and $\overline{V}_M^{s2}$ are the contributions arising from the nonlinear membrane stresses and strains in axial and circumferential directions respectively. Finally, the reduced stiffness buckling coefficient (k_{rs}) can be obtained as a function of classical buckling strength as given bellow;

$$\frac{k_{rs}}{k_c} = \left(\frac{U_M^s + U_M^{xs} + U_B + U_S + q_l \overline{V}_M^{s2}}{U + q_l \overline{V}_M^{s2}} \right) \left(\frac{\overline{V}_M^{x1} + \overline{V}_M^{s1}}{\overline{V}_M^{x1}} \right) \quad (7)$$

As can be seen in Fig. 2, the reduced stiffness buckling coefficient slightly reduces due to the lateral pressure. However, for sandwich cylindrical shells having L/a<0.5 the difference is almost unnoticeable. As the lateral pressure increases, k_{rs} also reduces as it is evident from Fig 3. Here, k_{ars} is the classical buckling coefficient due to pure axial loading. A lateral pressure equals to q_c reduces k_{rs} by almost 35% as it is evident from Fig 3.

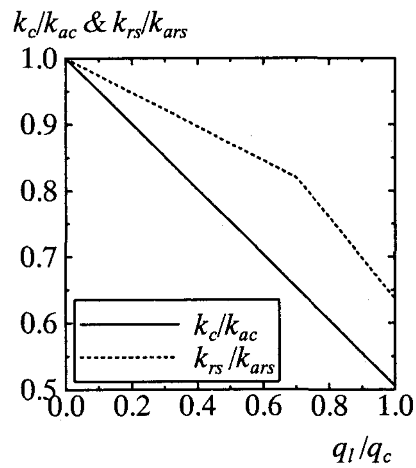

Fig 3. Variation of classical and reduced stiffness bucking coefficients with applied lateral pressure (L/a=4.0. E_f=2.05x105 MPa, V=0.01, a/h_f=5000, h_c/h_f=10, a=1.0 m)

Conclusions

When L/a<0.5, the classical buckling coefficient of axially loaded sandwich cylindrical shell due to lateral pressure is exactly similar to that of the pure axial loading. However, when L/a>0.5, the buckling coefficient reduces due to lateral pressure. The reduced stiffness buckling coefficient slightly reduces due to the applied lateral pressure. Both classical and reduced stiffness buckling coefficients of the axially loaded sandwich cylindrical shell reduces as the lateral pressure increases.

References

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