Ⅱ-19 A Particle Level Set Approach to Incompressible Two-Phase Flows

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1. Introduction

Numerical simulation results are presented for incompressible two-phase (gas-liquid) flows using level set methods[1)]. A particular emphasis is placed on: 1. how to satisfy jump condition at the gas-liquid interface accurately, and 2. how to conserve fluid mass numerically. A boundary-condition-capturing method[2)] is applied to satisfy the jump condition while a particle level set method[3)] is introduced to improve mass conservation.

2. Boundary-Condition-Capturing Method (Ghost Fluid Method)

Generally, densities and viscosities are discontinuous across the interface. Both the discontinuous viscosity and surface tension force cause the pressure to be discontinuous across the interface as well. In addition, the discontinuous viscosity leads to kinks in the velocity field across the interface. In the traditional level set method[4)], discontinuities of density and viscosity across the interface are removed by using a δ-function formulation to smear out the numerical solution near the interface and surface tension effects are introduced by the CSF method[5)], in which a surface force is formulated to model numerically surface tension effects having finite thickness. In this study we solve governing equations without smearing out these discontinuities by applying a boundary-condition-capturing method (ghost fluid method)[2)]. The CSF method is also unnecessary in this approach since the pressure jump is modeled directly. See 2) for details.

Figure 1 shows simulation results of a steady state air bubble in water, where gravity is set to zero. It is seen that the ghost fluid method can capture a jump in the pressure distribution accurately while it is smeared out in the solution of the traditional level set (delta function) method.

3. Particle Level Set Method

The advantages of level set methods are that the numerical implementation is easy and one can accurately compute the local curvature of an interface using the level-set representation for the interface. It is known, however, that level set methods are prone to lose fluid mass. While many methods have been proposed to improve mass conservation in level set methods[1)], a particle level set method[3)] is applied in this study and the mass conservation is handled well. In the particle level set method, massless Lagrangian particles are placed in a band around the zero level set (as shown in Figure 2) and used to correct the level set evolution. See 3) for details.

Simulation results of the Zalesak's disk rotation problem[3)] are shown in Figure 3. It indicates that particles correct the zero level set well, especially in the sharp corners.

4. Examples

Figures 4 and 5 show simulation results for the rise of an air bubble in water by using the ghost fluid method without and with particle correction, respectively. The particle level set solution does not depend on the number of grid points used as strongly as does the level set only solution. This indicates that the particle level set method can produce more reliable results than the traditional level set method, even in computations where the spatial resolution is not as high.

Acknowledgments

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References

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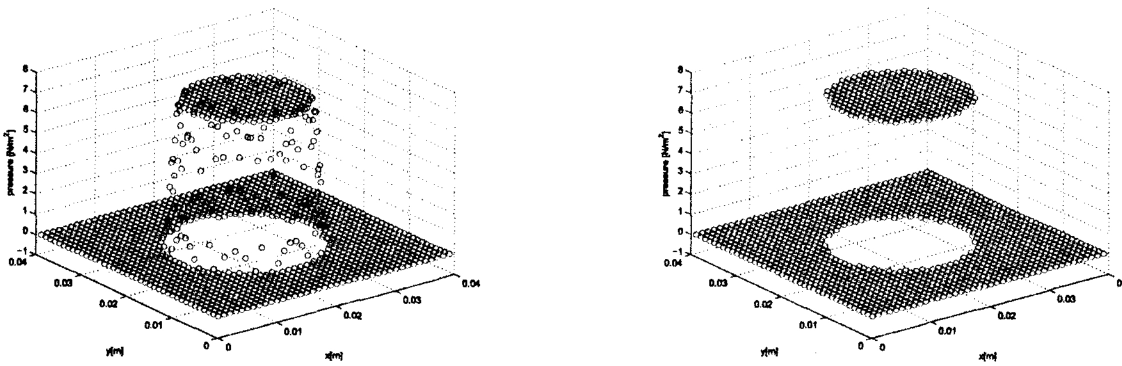


(a) delta function method (b) ghost fluid method

Figure 1: Comparison of calculated pressure field around a steady state air bubble.

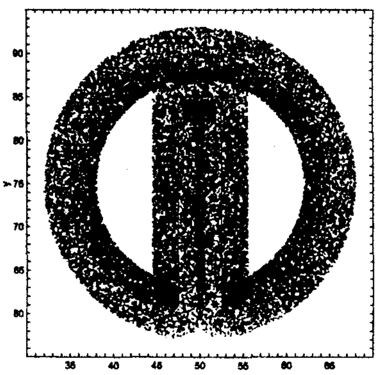
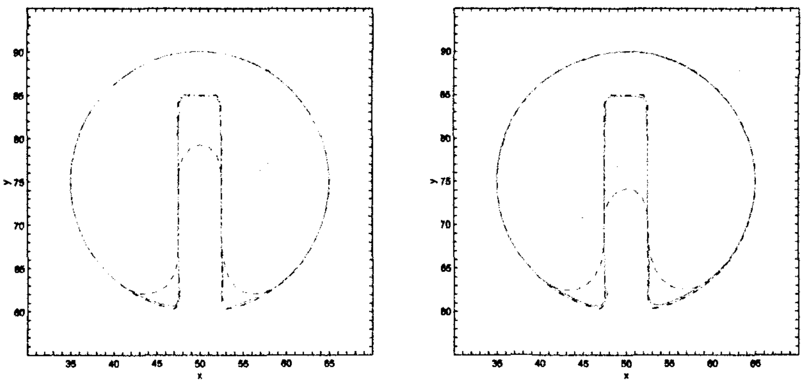
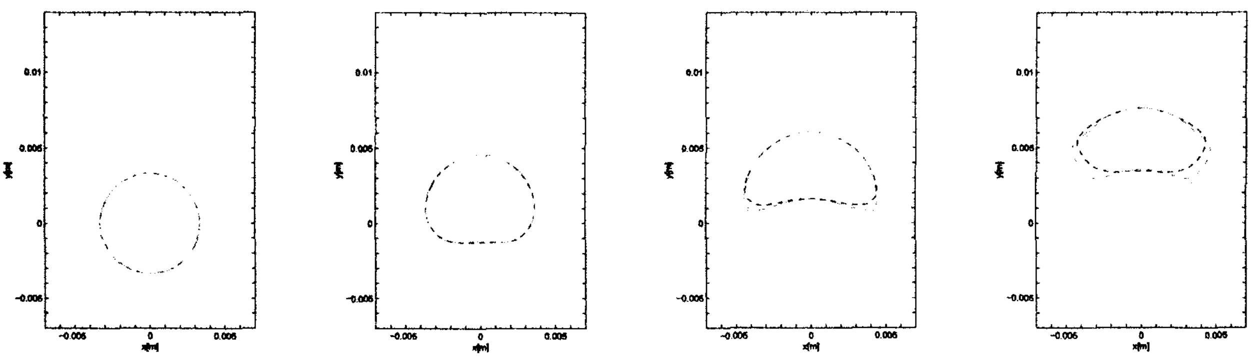


Figure 2: Initial zero level set and particles.



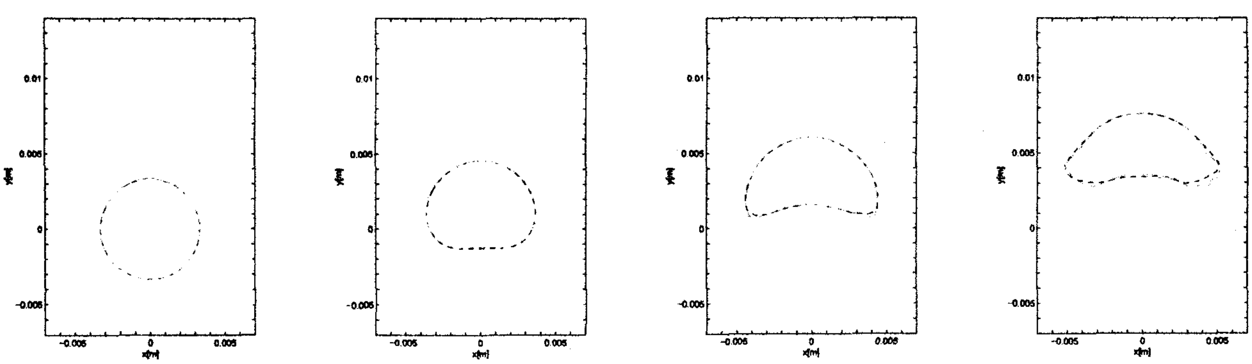
(a) after one revolution (b) after two revolutions

Figure 3: Comparison of the level set solution(- - -), particle level set solution(——) and theory(-·-·-·).



(a) $t = 0.0$[sec] (b) $t = 0.02$[sec] (c) $t = 0.035$[sec] (d) $t = 0.05$[sec]

Figure 4: Comparison of the level set solutions: - - -, 40×60; -·-·-, 80×120; ——, 160×240 grid points.



(a) $t = 0.0$[sec] (b) $t = 0.02$[sec] (c) $t = 0.035$[sec] (d) $t = 0.05$[sec]

Figure 5: Comparison of the particle level set solutions: - - -, 40×60; -·-·-, 80×120; ——, 160×240 grid points.