I -26 Buckling Analysis of Laterally Loaded Sandwich Cylindrical Shells

Graduate School of science and Engineering, Ehime University ○ Wijenayaka A. S. (Student Member, JSCE)
Department of Civil and Environmental Engineering, Ehime University Mitao Ohga (Member, JSCE)

1. Introduction

The laterally loaded sandwich cylindrical shell is finding many applications in civil, mechanical, and other fields of engineering. Ever expanding applications of the laterally pressure loaded sandwich cylindrical shell have lead to many imperfectly understood buckling problems. Therefore, a study into the buckling of laterally pressure loaded sandwich cylindrical shell is made here. Firstly, by using energy methods and variational principals, a classical buckling method was proposed. Derivation of the classical buckling method allowed the careful analysis of the energy changes during the bucking process. After that, a reduced stiffness lower bound for the bucking of laterally loaded sandwich cylindrical shell was proposed. The buckling of the geometrically imperfect laterally loaded sandwich cylindrical shell was accomplished by using a FEM code. In this way, the validity of the proposed reduced stiffness method was ascertained.

2. Classical buckling method

A convenient way of examining the various possible equilibrium paths described by the stationarity of the total potential energy is to first define the fundamental state. For the present problem this is approximated as given bellow:

$$N_s^F = -qa,\ N_x^F = N_{xs}^F = M_x^F = M_s^F = M_{xs}^F = 0 \quad (2.1)$$

Where, N^F and M^F are the fundamental external loads and moments. And, q is the applied lateral pressure as given in Fig. 1. The model consists of a core of thickness h_c sandwiched between two equal thickness (h_f) face sheets.

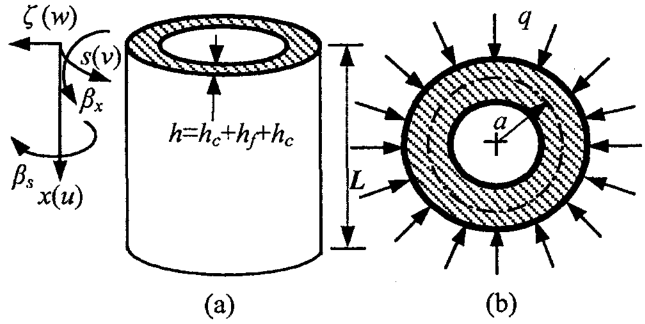

Fig. 1. Geometry and applied stresses

The mean radius is taken as a, while the length of the cylinder is designated as L. Depending on the incremental membrane strains that are linear $(\varepsilon_x', \varepsilon_s', \varepsilon_{xs}')$ and quadratic $(\varepsilon_x'', \varepsilon_s'', \varepsilon_{xs}'')$, the total potential energy of a laterally loaded sandwich cylindrical shell can be broke down into its components as below:

$$\Pi = \Pi_0 + \Pi_1 + \Pi_2 + \ldots \quad (2.2)$$

Here, Π_0, Π_1, Π_2 are independent, linear and quadratic contributions to the total potential energy. Of the present concern are the quadratic components, Π_2 of the total potential energy, for it is that control the stability of the fundamental path and from which the Eigenvalue problem yielding the critically stable state is derived. Assuming that the displacements (u, v, w) in the directions of (x, s, ξ) and rotations (β_x, β_s) about the s and x axes (Fig. 1) are sufficiently small, the quadratic component Π_2 can be written as:

$$\Pi_2 = U_M + U_B + U_S + V \quad (2.3)$$

Where, U_M, U_B, and U_S are the membrane, bending and shear energies respectively. And, V ($=qU_E$) depends upon the quadratic membrane displacement relations and should accordingly be seen as part of the non-linear membrane strain energy. The classical buckling strength q_c is given by the following equation.

$$U_M + U_B + U_S - q_c U_E = 0 \quad (2.4)$$

For a classically simply supported shell, making use of the linear stress and moment strain relations and quadratic stress strain relations, stationarity of the total potential energy with respect to kinematically admissible displacements (u, v, w, β_x, β_s) (Fig. 1) results in a linear Eigen value problem. The classical critical mode shape is obtained as a solution to this Eigen value problem.

3. Reduced stiffness buckling method

The reduced stiffness lower bound is derived by eliminating the membrane strain energy component as could disappear as the deformation develop into the critical mode shape.

$$U_B + U_S - q_{rs} U_E = 0 \quad (2.5)$$

Where, q_{rs} is the reduced stiffness buckling coefficient.

4. Buckling analysis of geometrically imperfect sandwich cylindrical shells

The development of appropriate methods for the analysis of shell structures is increasingly in demand to ensure the integrity of structural designs. Analytical solutions to shell structures are limited to scope and in general are not applicable to arbitrary shapes, load conditions, irregular stiffening and support conditions, cutouts and many other aspects of practical design. The finite element method has consequently become prominent in the analysis of such shells in view of the ease with which such complexities can be dealt with. Therefore, a non-linear analysis method has been developed into a finite-element code to allow investigation of the behavior of laterally loaded, geometrically imperfect sandwich cylindrical shells.

The FEM program developed for this purpose uses so called 9-node Isoparametric shell element with independent rotational and displacement degrees of freedom, in which the three dimensional stress and strain conditions are degenerated to shell behavior. A layered approach is employed in order to allow different material properties through the thickness of the shell. The critical mode shape from the classical buckling method was introduced as the initial imperfection of the shell in the FEM analysis.

Buckling analysis of the geometrically imperfect shell was carried out by varying the amplitude of the initial imperfection. The resulting plot of equilibrium paths for the case of *L/a* equals two is given in Fig. 2. Here, v_f–the Poison ratio, E_f – Young's module of face material, and V [$=E_f h_f/\{4aG_c(1-v_f)\}$] where, G_c is the core material shear strength. w is the lateral deformation of the shell, while q_{fem} is the stress obtained from the finite element analysis. As it can be seen in Fig. 2, as the initial imperfection increases, the respective maximum stress on the equilibrium path reduces. When the maximum stress ratio (q^{max}_{fem}/ q_c) on each equilibrium path is plotted against the respective initial imperfection (w_0/h), the imperfection sensitivity plot results. The same for *L/a* equals two is given in Fig. 3. As it is evident from that figure, the maximum stress ratio reach from and above the reduced stiffness lower bound as the initial imperfection increases. Therefore, the reduced stiffness method can be expected to

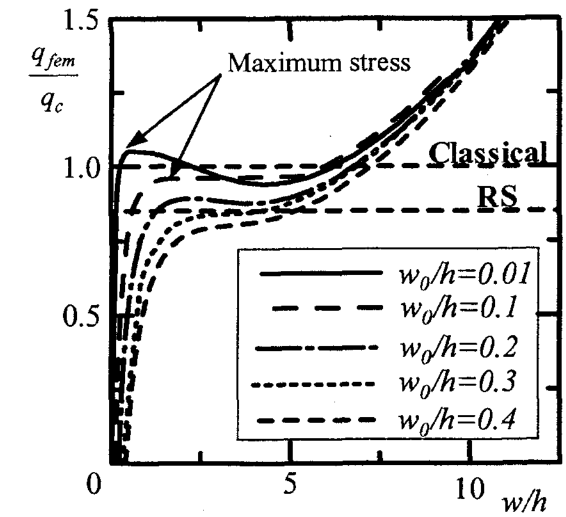

Fig. 2. Buckling analysis of geometrically imperfect sandwich cylindrical shell with L/a=2.0, v_f=0.3, E_f=2.06x10^5 Mpa, V=0.01, a/h_f=500, a=100 cm, h_c/h_f=6

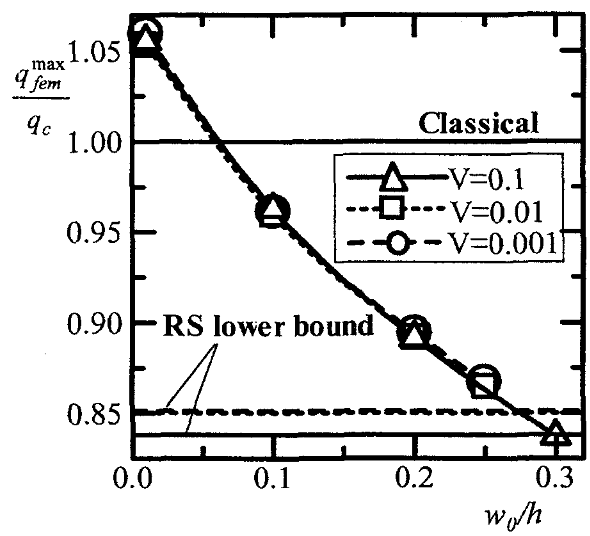

Fig. 3. Imperfection sensitivity plot (*L/a*=2.0)

provide close lower bounds for the buckling of laterally pressure loaded sandwich cylindrical shells.

5. Conclusions

The reduced stiffness method provides close lower bounds for the buckling of laterally loaded sandwich cylindrical shells.

6. Reference

Plantema, F. J. (1966). *Sandwich Construction: The bending and buckling of sandwich beams, plates and shells*, John Wiley & Sons Inc., New York, London, Sydney.