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REDUCED STIFFNESS BUCKLING STRENGTH ANALYSIS OF AXIALLY LOADED CYLINDRICAL SANDWICH SHELLS

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1.0 : Introduction

A reduced stiffness method is presented for the estimation of lower bounds to the imperfection sensitive general buckling of axially loaded elastic sandwich cylindrical shells. Careful analysis of the energy changes during the buckling process allows definition of a reduced stiffness theoretical model that provide compact, explicit, analytical expressions that could be proved practically suited to design. Isotropic sandwich shells with core carrying shear stresses only and with equal faces are considered for the analysis.

2.0 : Classical buckling analysis

A convenient way of examining the various possible equilibrium paths described by the stationarity of the total potential energy is to first define the fundamental path emerging from the unloaded and undeformed state. For the present problem this is approximated by the axisymmetric membrane solution $(N_x^F, N_s^F, N_{xs}^F) = (-2\sigma t_f, 0, 0)$ and $(M_x^F, M_x^F, M_{xs}^F) = (0,0,0)$ with corresponding strains $(E_x^F, E_s^F, E_{xs}^F) = (-\sigma/E, \mu\sigma/E, 0)$, where, σ - externally applied end stress, E – the modulus of elasticity and μ - the Poison's ratio.

Depending on the incremental membrane strains that are linear $(\varepsilon_x', \varepsilon_s', \varepsilon_{xs}')$ and quadratic $(\varepsilon_x'', \varepsilon_s'', \varepsilon_{xs}'')$, the total potential energy of an axially loaded sandwich cylinder of length L, mean radius a, face thickness h_f and core thickness h_c can be broken down into components as below.

$$\Pi = \Pi_0 + \Pi_1 + \Pi_2 + \Pi_3 + \ldots\ldots \quad (1)$$

Here, Π_0, Π_1, Π_2 are independent, linear and quadratic contributions to the total potential energy. Of the present concern are the quadratic components, Π_2 of the total potential energy, for it is that control the stability of the fundamental path and from which the Eigenvalue problem yielding the critically stable state is derived. Assuming that the displacements (u, v, w) in the directions of (x, s, ξ) and rotations (β_x, β_s) about the s and x axes are sufficiently small, the quadratic component Π_2 can be expanded into its main energy components as

$$\Pi_2 = U_M + U_B + U_S + V \quad (2)$$

where, $U_M = U_M^x + U_M^s + U_M^{sx}$: the axial, circumferential and twist strain energy terms, $U_B = U_B^x + U_B^s + U_B^{sx}$: corresponding bending strain energy terms and $U_S = U_S^{x\xi} + U_S^{s\xi}$: the shear strain energy terms in the superscripted planes. The last term $V = V^x + V^s$, depends upon the quadratic membrane displacement relations and should accordingly be seen as part of the non-linear membrane strain energy. The classical buckling strength q_c is derived from the following equation.

$$U_M + U_B + U_S + q_c(\overline{V}^x + \overline{V}^s) = 0, \quad k = q_c / 2E_f h_f \quad (3)$$

For a shell with classically simply supported ends, making use of linear stress and moment strain relations and quadratic stress strain relations, stationarity of the total potential energy with respect to kinematically admissible displacements $(u, v, w, \beta_x, \beta_s)$ results in a linear eigenvalue problem. The classical buckling load, q_c and modes of sandwich cylindrical shell are obtained as eigenvalues of the Eigen vector. This q_c is identical with the value given by equation (3).

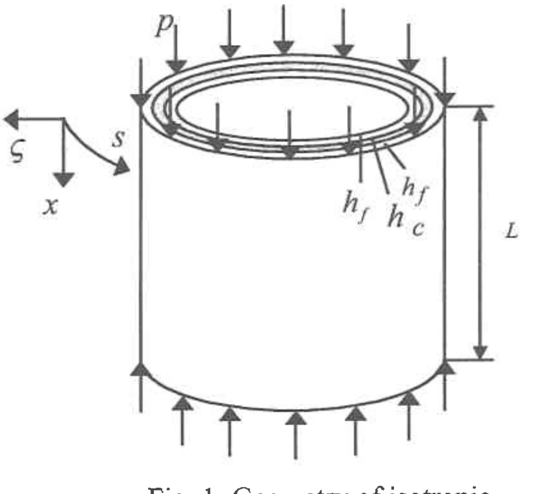

Fig. 1: Geometry of isotropic cylindrical sandwich shell

3.0: RS buckling analysis

By identifying the energy components that may be eroded during buckling, it is possible to define the RS buckling strength of the sandwich cylinder by eliminating the appropriate energy components. The non-linear circumferential membrane energy term, V^s is a consequence of the fundamental state Poisson expansion and the non-linear membrane stress resultant. It is this component of non-linear circumferential stress that, as pointed out by Donnell as early as 1934, will be eroded in the subsequent interactive post-buckling behavior. At small deflection, a mode coupling will result in the loss of the initial stabilization provided by the non-linear membrane strain energy V^s and the linear axial strain energy U_M^x. This leads to the simple idea that a lower limit to the sandwich cylindrical shells post critical loss of stiffness, would be provided by the reduced total potential energy which then finally leads to

$$k^* = \left(\frac{U_M^s + U_M^{sx} + U_B + U_S}{U_M + U_B + U_S} \right) \left(\frac{\overline{V}^x + \overline{V}^s}{\overline{V}^x} \right) k \quad (4)$$

Where k^* is the RS buckling stress. The variation of the k and k^* with the length of the cylinder is given in Fig. 2, where $V (= E_f t_f / 4aG_c(1 - v_f^2))$ is the representative parameter of core material shear strength. As can be seen from this graph, the RS buckling stress constantly reduces as the length of the cylinder increases. But, the classical buckling stress is almost constant from L/a=0.3 to 10 and then reduces as the length of the cylinder increases.

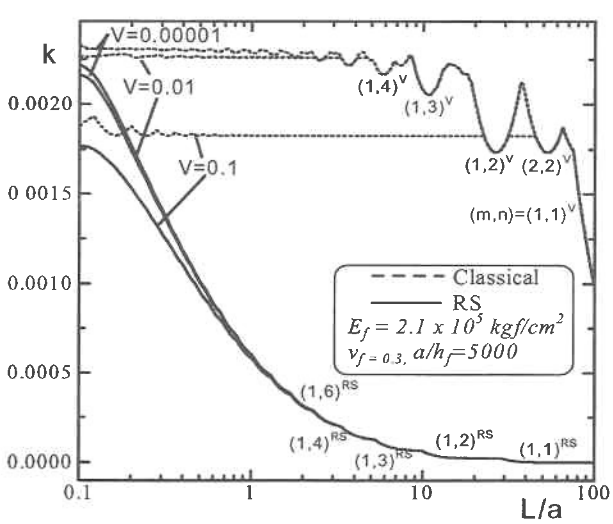

Fig. 2: Variation of classical and RS stress with V and L/a
Sandwich shell with h_c/h_f=10, L/a=2

4.0: Post buckling analysis

An elastic and geometrically nonlinear analysis method has been developed to allow investigation of the post buckling behavior of axially compressed sandwich cylindrical shells. The FEM program developed for this purpose uses so called 9-node Isoparametric shell element with independent rotational and displacement degrees of freedom, in which the three dimensional stress and strain conditions are degenerated to shell behavior. In the post buckling analysis, the mode shape at the critical buckling load from the RS analysis was introduced as the initial imperfection of the shell. It is evident from Fig. 2 that for a wide range of V and L/a ratio the critical classical and RS axial wave number equals to one. As can be seen in Fig. 3, analysis were continued with increasing initial displacements. For small initial displacements, the equilibrium paths show a peaking followed by a saddle point, which then increase as the applied load increases. In contrast, the curves E and F do not show such a trend. In curve D, the peak point (Point 1) and the saddle point (Point 2) are almost equal to each other, indicating that the respective stress at Point 1 is obviously the minimum buckling stress. As can be seen from Fig. 3, point 1 is much closer to RS critical buckling stress than the classical stress giving a lower limit to the imperfect sensitive analysis.

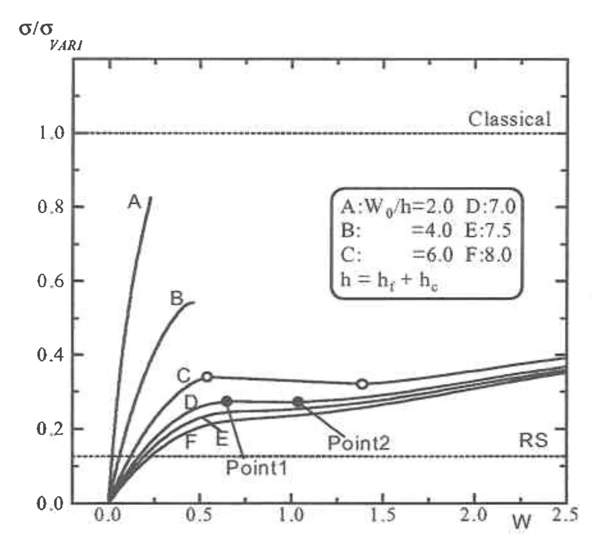

Fig. 3: Post buckling analysis
Sandwich shell with V=0.01, L/a=2