A FUNDAMENTAL STUDY ON OPTIMAL DESIGN OF PRESTRESSED CONCRETE CONTINUOUS BEAMS

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1. INTRODUCTION

In the conventional design procedure of prestressed concrete structures, the flexural design considering both serviceability limit state and ultimate limit state may become a fairly tedious task that involves several iterations to obtain a feasible design. Furthermore, this requires extensive computations and there is no guarantee that the solution is optimal. In this fundamental research, an optimum design method for prestressed concrete structures, which can determine the optimum prestressing force, tendon layout, height and width of uniformed cross section considering the stress constraints in serviceability limit state by ACI code, are studied and the efficiency of the method is illustrated by the numerical examples.

2. OPTIMUM DESIGN PROBLEM

In this fundamental study, the cross section is assumed to be rectangular. As the design variables, prestressing force P and tendon eccentricities e from the center of the cross section, height H and width B of a uniformed cross section in structures depicted in Fig. 1, are taken into account. The cross-sectional area of tendon A_{ps} is determined by P/f_{pe} where f_{pe} is the permissible tensile stress of prestressing tendon. The following governing design constraints $g_j (j=1,\dots,q)$ of stress limitations in serviceability limit state are considered as design constraints and the primary optimal design problem is formulated as to find the P, e, H and B which minimize the total cost of the structure W.

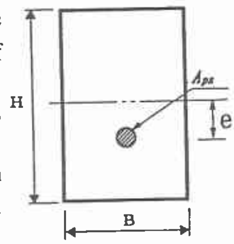

Fig.1 Design Variables

For the case at transfer

for tension $g_{\sigma j} = |\sigma_j| - 3\sqrt{f'_{ci}} \leq 0$ (1) for compression $g_{\sigma j} = |\sigma_j| - 0.6\,f'_{ci} \leq 0$ (2)

For the case at service

for tension $g_{\sigma j} = |\sigma_j| - 6\sqrt{f'_{c}} \leq 0$ (3) for compression $g_{\sigma j} = |\sigma_j| - 0.45\,f'_{c} \leq 0$ (4)

where $|\sigma_j|$ is the stress at top or bottom fiber in the section. f'_{ci} and f'_c are, respectively, the concrete strength developed at the time of transfer of prestressing force into the concrete and the compressive strength of concrete(28 days).

At the transfer, the dead loads and initial prestressing force are considered as external loads. At the service, the dead loads, live loads and effective prestressing force are considered. The maximum and minimum bending moments due to live loads are calculated by applying a uniformly distributed live load to each span per once and summing up all positive or negative bending moment separately. The secondary bending moment due to prestressing force is obtained by considering the primary prestressing bending moment as the equivalent loads for each element. In the analysis of structures, section properties such as cross-sectional area and moment of inertia are calculated by taking the mean values of the properties at both-end nodes of each member element.

3. OPTIMUM DESIGN ALGORITHM

Utilizing the convex and linear approximation concept, the primary optimal design problem can be approximated as the following convex and separable subproblem by using the first-order partial derivatives with respect to the design variables and the direct and reciprocal design variables.

Find P, e, H, B which

minimize $$\Delta W(P,e,H,B) = \sum_{i=1}^{m} \left(\omega_{pi} P + \sum_{k=1}^{n} \omega_{eki} e_k + \omega_{Hi} H + \omega_{Bi} B \right) \quad (5)$$

subject to

$$g_j(P,e,H,B) = a_{j_{(+)}}P - a_{j_{(-)}}(P^0)^2\frac{1}{P} + \sum_{k=1}^{n}[y_{jk_{(+)}}e_k - y_{jk_{(-)}}(e_k)^0\frac{1}{e_k}] + h_{j_{(+)}}H - h_{j_{(-)}}(H^0)_2\frac{1}{H}$$

$$+ b_{j_{(+)}}B - b_{j_{(-)}}(B^0)^2\frac{1}{B} + \overline{U}_j \le 0 \qquad (1,\dots,q) \qquad (6)$$

$$P^l \le P \le P^u,\quad e_k^{\,l} \le e_k \le e_k^{\,u}\ (k=1,\dots,n),\quad H^l \le H \le H^u,\quad B^l \le B \le B^u \qquad (7)$$

where

$$\overline{U}_j = g_j(P^0,e^0,H^0,B^o) - P^0[a_{j_{(+)}}-a_{j_{(-)}}] - \sum_{k=1}^{n} e^0_k[y_{jk_{(+)}}-y_{jk_{(-)}}] - H^0[h_{j_{(+)}}-h_{j_{(-)}}] - B^0[b_{j_{(+)}}-b_{j_{(-)}}]$$

$$\omega_P = \frac{\partial W}{\partial P},\ \omega_{ek} = \frac{\partial W}{\partial e_k}\ \omega_H = \frac{\partial W}{\partial H},\ \omega_B = \frac{\partial W}{\partial B},\ a_j = \frac{\partial g_j}{\partial P},\ y_{jk} = \frac{\partial g_j}{\partial e_k},\ h_j = \frac{\partial g_j}{\partial H},\ b_j = \frac{\partial g_j}{\partial B}$$

m is the number of elements. The sensitivities of W and g_j with respect to P, $\boldsymbol{e}$, H and B are calculated by using the forward-difference approximation.

The above approximated subproblem is solved by a dual method where the separable Lagrangian function is minimized with respect to the design variables and maximized with respect to Lagrange multipliers (dual variables). At the minimization process, the design variables are improved by simple expressions derived from stationary conditions of separable Lagrangian function. Then at the maximization process, the dual variables are improved by a Newton-type algorithm. The optimum solution is obtained by iterating the above min.-max. process.

4. NUMERICAL DESIGN EXAMPLES

The above method has been applied to various minimum cost designs of prestressed concrete structures. In this study, the numerical results for the two-span continuous prestressed concrete beam shown in Fig. 2 are discussed.

In the numerical design example, f'_c, f'_{ci} and f'_{pe} are, respectively, set at 41 Mpa, 31 Mpa and 1,103 Mpa. The unit costs of pre-stressing tendon and concrete are 6916800 /m^3 and 24000 /m^3, respectively. The layout of tendon is idealized to be parabolic at each span, but the tendon in a member element is assumed to be straight for the calculation of tendon length l_i. The move limit on $\boldsymbol{e}$, H and B is set as 20%. The structure is divided into 16 member elements in order to obtain the accurate results.

Fig. 3 shows the optimum tendon layout and cross section. The optimum solution is obtained after 6 iterations efficiently. As seen in Fig. 3, the optimum design variables obtained are quite reasonable and the width B is determined by the lower limit. The initial prestressing force 901 KN is reduced to 576 KN. By starting the algorithm with different initial variables, the quite similar solutions are obtained within 10 iterations.

From other numerical results, it is clear that the proposed optimum design method can determine prestressing force P, tendon eccentricities $\boldsymbol{e}$ from the center of the cross section, height H and width B of a uniformed cross section for prestressed concrete structures quite efficiently.

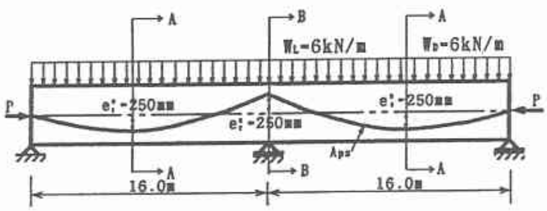

Fig. 2 The geometory and loading condition of 2-span continuous prestressed concrete beam

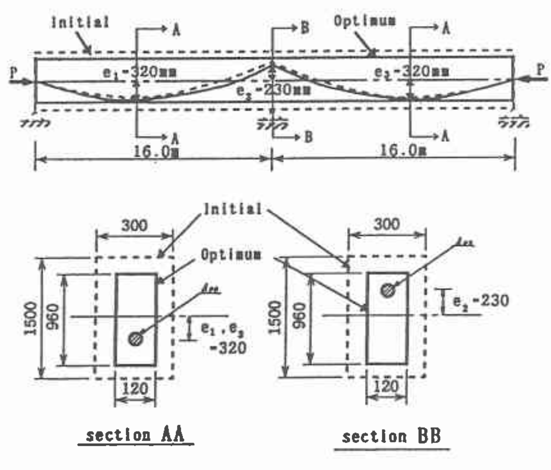

Fig. 3 The initial, the optimum tendor layouts and cross section of the beam