IDENTIFICATION OF THE IMPEDANCE MATRIX FOR SOIL-STRUCTURE INTERACTION SYSTEMS

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1.INTRODUCTION In constituting a dynamic model for aseismic analyses of a soil-structure interaction system supported on the surface of soil, one important step is determining impedance matrix that represents the resistance of soil medium to the vibration of the structure. In recent decades, a larger amount of research work[1, 2] has been done for this problem. However, all of the methods and the corresponding results are subject to be verified in the case of a real structure under earthquake as they are based more or less on differently specified assumptions. Thus, an effective method able to produce impedance matrix from real recordings of the responses and excitations will undoubtedly be needed. In this paper, an identification method to produce impedance matrix for a soil-structure interaction system under earthquake is proposed.

2.BASIC EQUATIONS OF MOTION A structure with a rigid foundation supported on the surface of soil under earthquake is shown in Fig.1. To differentiate between the various nodes subscripts are used: *b* for the node of the foundation located on the soil-structure interface, *s* the remaining nodes of the superstructure, and *f* for the ground or free field. The equations of motion in the frequency domain for steady-state responses are formulated as[2]:

$$\begin{bmatrix} S_{ss} & S_{sb} \\ S_{bs} & S_{bb}+I_{bb} \end{bmatrix} \begin{Bmatrix} U_s \\ U_b \end{Bmatrix} = \begin{Bmatrix} 0 \\ I_{bb}U_f \end{Bmatrix} \qquad (1)$$

in which

$$S_{ij} = -\omega^2 M_{ij} + i\omega C_{ij} + K_{ij}; \quad i,j{=}s,b \qquad (2)$$

where *U*'s is the amplitude vector of absolute displacements; *M*'s, *C*'s and *K*'s are mass, damping and stiffness matrix, respectively, of the superstructure with a fixed base. ω is the circular frequency. I_{bb} is the impedance matrix of soil(Fig.2).

$$I_{bb} = K_{bb}(k(a_o) + ia_o c(a_o)) \qquad (3)$$

$$a_o = \omega a / v_s \qquad (4)$$

The matrix K_{bb} contains the static-stiffness coefficients determined by the shear modulus G, Poisson's ratio of soil ν and half width of the base a. The variable a_o represents the dimensionless frequency, in which v_s is the shear-wave velocity of soil. The Matrix $k(a_o)$ and $c(a_o)$ are the dimensionless spring and damping coefficients, respectively, and generally frequency-dependent , i.e. the function of a_o.

Eq.(1) and (3) can be expressed in submatrix form as:

$$S_{ss}U_s + S_{sb}U_b = 0 \qquad (5)$$

$$I_{bb}(U_b - U_f) = (S_{bs}U_s + S_{bb}U_b) \qquad (6)$$

From Eq.(5), matrix S_{ss} and S_{sb} can be identified by general method[3]. Eq.(6) is the input-output equation for soil. Based on the reciprocal theorem of linear structures, $S_{bs}{=}S_{sb}^T$. On the other hand, as the base is rigid matrix S_{bb} only contains the parameters related with mass and geometric quantities alone. So attentions are only devoted on the impedance matrix I_{bb}.

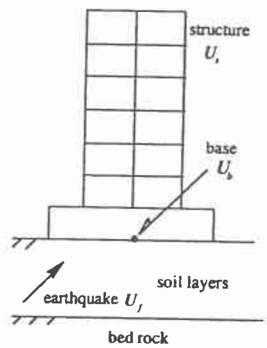


Fig.1. A soil-structure interaction system

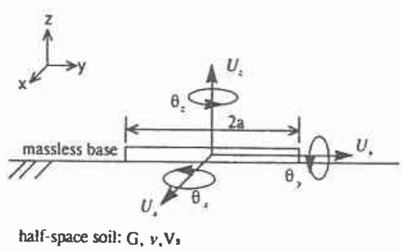


Fig. 2. A substructure of the foundation

3.IDENTIFICATION OF IMPEDANCE MATRIX

Since the common characteristics of I_{bb} are that all the elements of matrix *k* and *c* in Eq.(3) are single-valued function of the dimensionless frequency a_o, they can thus be approximated with polynomials of the following forms:

$$k_{ij} = \sum_{k=0}^{N_{ij}} b_{ij}^k p_k(a_o); \qquad c_{ij} = \sum_{k=0}^{P_{ij}} d_{ij}^k p_k(a_o); \qquad i,j{=}\,x, \theta x, y, \theta y, z, \theta z \qquad (7)$$

in which $p_k(a_o)$ indicates a polynomial function of degree k, b_{ij}^k and d_{ij}^k are the constants of the polynomial and N_{ij} and P_{ij} represent the number of degrees of the polynomials. Subscripts i,j=x, θ_x, y, θ_y, z, θ_z denote the six translational and rotational directions. The criterion for identification of the constants is expressed in the following expression:

$$J(\boldsymbol{b},\boldsymbol{d}) = \sum_{l=1}^{m} \{\hat{U}_b(i\omega_l) - U_b(i\omega_l)\}^2 \rightarrow \min \quad (8\text{-}1)$$

$$\boldsymbol{b} = (b^o_{xx}, b^1_{xx} \cdots b^{N_{xx}}_{xx}, \cdots, b^o_{ij}, b^1_{ij} \cdots b^{N_{ij}}_{ij}, \cdots, b^o_{zz}, b^1_{zz} \cdots b^{N_{zz}}_{zz});\quad i,j = x, \theta_x, y, \theta_y, z, \theta_z \quad (8\text{-}2)$$

$$\boldsymbol{d} = (d^o_{xx}, d^1_{xx} \cdots d^{P_{xx}}_{xx}, \cdots, d^o_{ij}, d^1_{ij} \cdots d^{P_{ij}}_{ij}, \cdots, d^o_{zz}, d^1_{zz} \cdots d^{P_{zz}}_{zz});\quad i,j = x, \theta_x, y, \theta_y, z, \theta_z \quad (8\text{-}3)$$

in which $\hat{U}_b$ is the recorded output and vectors $\boldsymbol{b}$ and $\boldsymbol{d}$ are to be identified.

4.NUMERICAL EXAMPLES FOR A SHEAR-TYPE BUILDING The system parameters are given in table 1. The simplified impedance matrix only containing horizontal and rotational terms given in Ref.[1] is used here to represent the impedance of soil. The form of theimpedance matrix is

$$I_{bb} = \begin{bmatrix} \frac{8Ga}{(2-\nu)}[k_{yy} + ia_o c_{yy}] & 0 \\ 0 & \frac{8Ga^3}{3(1-\nu)}[k_{\theta x\theta x} + ia_o c_{\theta x\theta x}] \end{bmatrix} \quad (9)$$

in which the frequency-dependent dimensionless spring and damping coefficients are approximated by Lengendre polynomial within frequency range 0~20 Hz, where $N_{yy}=N_{\theta x\theta x}=7$ for k_{yy} and $k_{\theta x\theta x}$, $p_{yy}=p_{\theta x\theta x}=2$ for c_{yy} and $c_{\theta x\theta x}$. By Using the normalized variable t, Legendre polynomials are defined by:

$$t = [\omega - (\omega_{max} + \omega_{min})/2]/[(\omega_{max} - \omega_{min})/2] \quad (10\text{-}1)$$

$$p_o(t) = 1 \quad (10\text{-}2)$$

$$p_1(t) = t \quad (10\text{-}3)$$

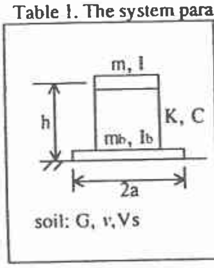
$$(k+1)p_{k+1}(t) = (2k+1)tp_k(t) - kp_{k-1}(t) \quad (k=1,2,.....) \quad (10\text{-}4)$$

in which w_{min} and w_{max} indicate the frequency range of interest. The measured responses are generated numerically supposing the system under El Centro earthquake and then Gaussian white noise with frequency bandwidth 0.1~20 Hz and levels of a certain percentage of the root-mean-square of the unpolluted responses are added to the generated responses as well as the free-field acceleration to simulate noise-polluted records. For demonstration, the estimated k_{yy}, $k_{\theta x\theta x}$, c_{yy} and $c_{\theta x\theta x}$ versus the assumed true ones and the residuals for noise levels: 0% (noise-free cases), 2%,3% and 5% are shown in Fig.3 and Table 2, respectively. It is evident that for the noise-free case the estimated k_{yy}, $k_{\theta x\theta x}$, c_{yy} and $c_{\theta x\theta x}$ are completely identical to the true ones and the residuals are relatively small; for the noise-contaminated cases the errors arise and increase as the noise level increases, but the estimated coefficients fluctuate about the true ones and the errors are acceptable for noise levels: 2%~3%.

5.CONCLUDING REMARKS (a)The best functional representation of soil impedance matrix can be inversely produced with considerable accuracy. (b) Although the accuracy is deteriorated by noise contamination, the errors are acceptable for noise level 2%-3% by this method.

6.REFERENCES [1] Veletsos, A. S."Lateral and rocking vibration of footings." J. Soil Mech., ASCE, Vol.97, No.SM9, 1971, pp 1227-1248. [2] Wolf, J. P."Dynamic Soil-structure Interaction." Prentice-Hall, Inc., New Jersey.,1985. [3] Zhao, Q., Sawada, T., Hirao, K. and Nariyuki, Y. "Localized identification of MDOF structures in the frequency domain." Earthquake Eng. Struct. Dyn., Vol.24, 1995, (in print).

Table 1. The system parameters



m = 28300 kg
I = 290000 kg·m²
m_b = 18300 kg
I_b = 5440 kg·m²
h = 3.5 m
a = 2.63 m
K = 4.76x10⁸ kN/m
C = 270000 kN·s/m
G = 6.1x10⁷ kN/m²
ν = 1/3
v_s = 150 m/s

Table 2. Residuals of k_{11}, c_{11}, k_{22} and c_{22}

noise	0%	2%	3%	5%
k_{yy}	1.82E-4	1.42E-1	2.12E-1	3.50E-1
$k_{\theta x\theta x}$	4.58E-5	1.23E-1	2.21E-1	3.77E-1
c_{yy}	1.15E-5	1.55E-1	2.32E-1	3.86E-1
$c_{\theta x\theta x}$	2.61E-7	1.48E-1	1.68E-1	6.18E-1
average	5.99E-5	1.42E-1	2.08E-1	4.32E-1

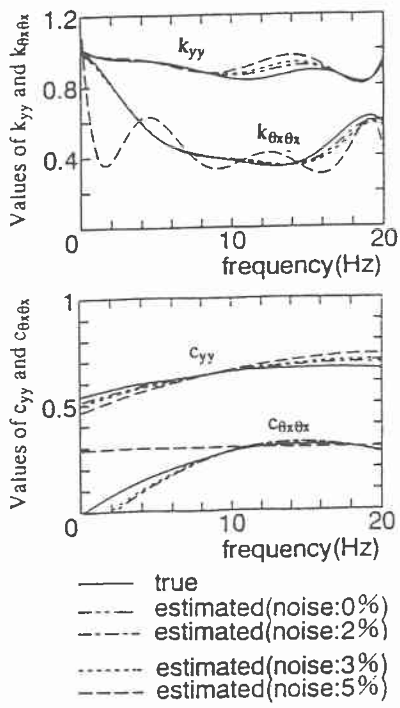


Fig.3 Comparison of the estimated k_{yy}, $k_{\theta x\theta x}$, c_{yy} and $c_{\theta x\theta x}$ with the true ones