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An Accurate Technique for Crack Propagation Simulation using SBFEM

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Introduction

Crack propagation is an important failure mechanism requiring accurate numerical modeling to implement simulation essential for reliable prognoses of the safety and the durability of engineering structures. Generally, experimental studies and analytical solutions for crack propagation in all the structures are not attainable. Thus, the realistic numerical simulation of the crack propagation is a prerequisite for reliable prognoses of the safety and the durability of structures. But, crack propagation simulation is a complex and difficult task since it must be constantly redefined and updated the crack shape and tip location, which may be time-consuming.

For the crack propagation simulation, the most widely used numerical methods include finite element method (FEM), boundary element method (BEM) and meshless method. However, various studies have revealed that all these methods have certain limitations and are inefficient for modeling crack problems. The main problem is the difficulty in meshing and remeshing to accurately represent singularities and discontinuities near a crack-tip and computationally costly. The review of these methods has presented in Ref [1]. Recently authors have presented a simple, efficient and more flexible quasi-automatic crack propagation simulation technique using recently developed *scaled boundary finite element method* (SBFEM) [2].

However, most of the previous studies have considered the single fracture parameter, stress intensity factors (SIFs) for the crack propagation simulation even though many studies have highlighted the effects of T-stress in crack growth angles. To our knowledge, none of the previous studies have addressed the effects of T-stress in crack propagation simulation except authors try to attempt the T-stress effects in crack propagation simulation using SBFEM in [1]. Thus the main objective of this paper is to demonstrate the T-stress effect to predict crack growth trajectories.

Crack propagation Criteria

In the numerical crack propagation simulation, the crack growth direction plays an important role. Several criteria have been proposed to predict local direction of crack propagation. Among them, one of the most commonly used is based on maximum hoop or principal stress at the crack tip. In this study, the maximum principal stress criterion, which predicts the direction of crack growth from the stress state prior to the crack extension, is considered for crack propagation simulation. In this criterion, it is considered that the crack will propagate from its tip in the direction along which the maximum hoop stress $\sigma_{\theta\theta}$ occurs. The hoop (circumferential) stress in the direction of crack propagation is a principal stress. Therefore, the critical angle, θ_0, defining the radial direction of propagation can be determined by setting the shear stress $\sigma_{r\theta}$ to zero.

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta_0} = 0 \tag{1}$$

Considering both the singular (SIFs) and constant (T-stress) terms of the stress field near crack-tip on maximum circumferential stress criteria, the direction of crack propagation, θ_0, is computed by solving the following equation.

$$K_I \sin\theta_0 + K_{II}(3\cos\theta_0 - 1) - \frac{16}{3} T\sqrt{2\pi r_c}\,\sin\frac{\theta_0}{2}\cos\theta_0 = 0 \tag{3}$$

where K_I and K_{II} are mixed mode SIFs, and T is T-stress for any instance during the crack-growth. r_c is an additional length scale representing the fracture process zone size. When the values of K_I , K_{II} and T are known, θ_0 can be easily solved by means of Eq. (3). Since r_c is generally assumed to be very small, the first two terms (SIFs effects) of Eq. (3) are considered for crack propagation simulation.

SBFEM crack propagation simulation procedure

Computation of fracture parameters

To compute the fracture parameters i.e. SIFs and T-stress and higher order terms of the crack-tip stress fields, authors have presented a simple and direct formulation by comparing the classical linear elastic field solution (Williams' eigenfunction series) in the vicinity of a crack-tip with the SBFEM the stress and displacement fields along the radial direction emanating from the crack tip in [1]. The necessary condition of the formulation is that the scaling center is chosen at the crack-tip that leads to only the boundary, but not the straight crack faces and faces

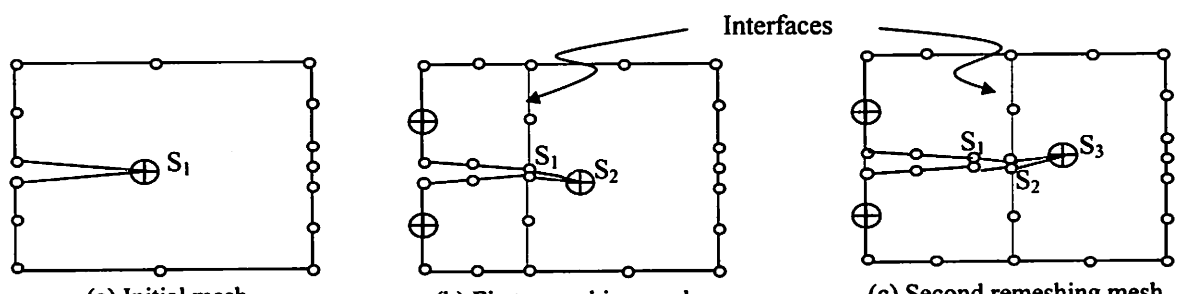

Fig. 3. The proposed SBFEM's remeshing procedure
(S_i & S_{i+1} are the old and new crack-tip)

passing through the crack tip is discretized. The presented formulation for SIFs and T-stress are as follows.

$$K_I = c(\hat{\sigma}_{yy})\sqrt{2\pi\hat{r}} \qquad (4)$$

$$K_I = c(\hat{\sigma}_{xy})\sqrt{2\pi\hat{r}} \qquad (5)$$

$$T = c(\hat{\sigma}_{xx}) \qquad (6)$$

where, $\hat{\sigma}_y$ and $\hat{\sigma}_x$ are the stress components along perpendicular and parallel to crack surface, respectively; $\hat{r}$ is the radial distances of the boundary nodes from scaling center, and c is the integration constant. In this paper only the basic equations of the proposed formulation are presented. For a more detailed description we refer to [1].

Crack propagation simulation

The simulation of crack propagation involves a number of successive analyses. Each analysis consists of the following steps.

i. A SBFEM analysis of a crack structure is performed placing a scaling center at crack-tip as shown in Fig. 1 (a). Then the mixed mode SIFs and T-stress are computed using Eqs. (4) to (6) respectively.
ii. The direction of crack propagation is calculated from Eq. (4).
iii. A virtual increment of crack length (Δa) is defined according to the user's specifications and the location of new crack-tip is determined.
iv. By adding two nodes locating on the opposite sides of the crack in the old crack tip, the sub-domain that includes crack-tip is further sub-divided into three sub-domains as shown in Fig.1(b), and then discretization of the boundaries and interfaces are updated according to the requirement for accurate computation of stress singularity. Since SBFEM has a unique property that certain fixed and free boundaries passing through scaling centers need not be discretized, the scaling centers are placed in such as way that discretization should be minimized.
v. Step (i) to step (iii) are repeated to locate new crack-tip. Then the interfaces of sub-domains near the crack are shifted to previous crack-tip, as shown in Fig.1(c), and upgrade the discretization for analysis.
vi. Then step (i) to step (v) are repeated for further simulation

In this procedure, users should define the incremental crack length to locate the crack-tip in every increment. Therefore, the proposed procedure is quasi-automatic.

Effectiveness of proposed technique

To demonstrate the effectiveness of the proposed technique, a L-shaped concrete beam with inclined crack at the corner was analysed. The schematic diagram with loading conditions and dimensions of the problem used for the analysis are as shown in Fig. 2a. The analyses were carried out using plane strain condition with thickness =100 mm, Young's modulus E = 20,000 N/mm^2 and Poisson's ratio υ = 0.18. The initial crack at the corner, a = 30 mm was considered

The problem was first analyzed using the single domain locating the so-called scaling center at the crack-tip. and then remeshing was performed by dividing into three sub-domains as shown in Fig. 1 (b). The 21st remeshing

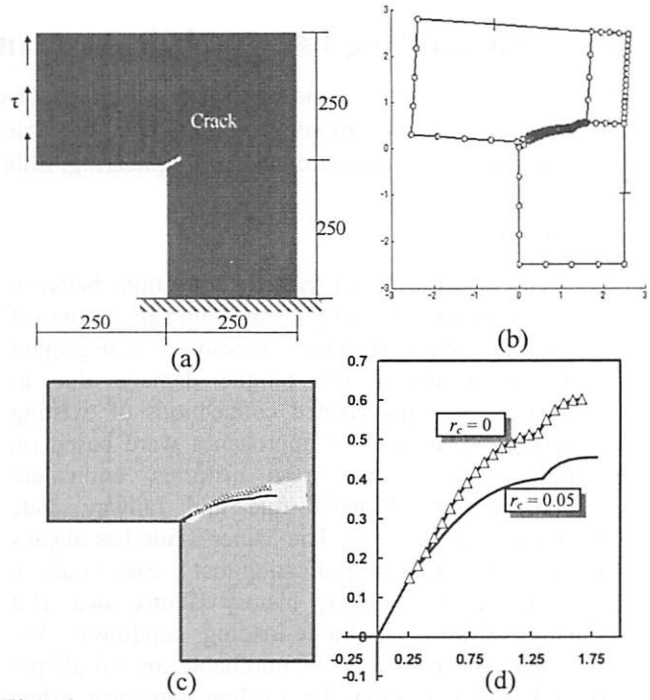

Fig. 2: L-shaped beam (a) schematic diagram (b) deflected shape at 21st remeshing (c) comparing of computed crack growth paths (d) computed crack growth paths

model with corresponding deflections was as shown in the Fig. 2 (b), where '+' signs are so-called scaling centers. In remeshing models, two scaling centers were located in the top and right hand boundaries that lead to minimize the discretization.

A constant crack extension length, Δa = 7.5 mm, (i.e. 25% of initial crack length) has been chosen for the crack propagation simulation. At each increment, the crack propagation direction was estimated using Eq. (3) by considering r_c= 0 (i.e. only SIFs) and r_c= 0.05 (i.e. SIFs and T-stress). The computed crack trajectories are as shown in Fig. 2(d) and are compared with the experimental results from the Ref [3]. The comparison shows in Fig. 2 (c) that the computed crack growth paths using the proposed algorithm are within the range of the experimental data. It can be also seen that the computed paths considering T-stress effects are more accurate than the computed results considering only SIFs effects.

Conclusion

In this paper, a simple, efficient and more flexible quasi-automatic crack propagation simulation technique using recently developed *scaled boundary finite element method* was presented and the effect of T-stress to predict crack growth paths has been demonstrated by analyzing a L-shaped concrete beam. Based on this study, it can be confined that the proposed crack propagation simulation technique can be achieved more accurate results with simple and efficient remeshing models.

References

1. Shrestha, S. (2007): Scaled boundary finite element method for linear elastic fracture analysis, PhD Thesis, Ehime University.
2. Shrestha, S., and Ohga, M and Taniwaki, K. (2007), Computation of SIFs of the branched crack by scaled boundary finite element method, *J. of structural Engineering*, 52A,pp. 133-140.
3. Dumstorff, P. and Meschke, G., Crack propagation criteria in the framework of X-FEM-based structural analyses, *Int. J. for Num. Analy Meth. in Geomech.* 31, pp. 239-259, 2007.