Effect of skewness and standard deviation on porosity in sediment with wide particle size distribution

Hiroshima University Student Member ○Jiang Yicheng
Hiroshima University Regular Member Tatsuhiko Uchida

1. INTRODUCTION

According to the increase of heavy rain fall caused by climate change, increasing in sediment supply into rivers including the discharge from the upstream sabo dam often exceeds the sediment capacity. This can be a main cause of the sediment-flood disaster, which is a big threat to the people living downstream. As a result, the study of sediment transport is an important topic in hydraulics engineering.

One challenging topic in this field is to clarify the variation of porosity in sediment mixture. For example, if the porosity of sediment changes in a sabo dam due to the variation in grain size distribution, the volume will change as well, leading to variation in the storage capacity of the dam and fine sediment discharge into the downstream river. So it's an important topic to evaluate sediment volume.

The bed elevation calculation models have been developed based on the concept of 'active layer' proposed by Hirano (1971). However, little research has paid attention to the problem that fine particles will be transported by the water flow with the variation in porosity and sediment volume. Uchida et al. (2020) developed an Eulerian sediment deposition model, which has advantages compared with conventional models. For example, it does not need to employ an artificial layer and calculate porosity, while it can be obtained from the volume fraction. For the porosity prediction, Francisco et al. (2016) found a connection between standard deviation and minimum porosity. However, the skewness of the three-dimensional moment for the grain size distribution has not been well-researched yet. To find a more reliable method to estimate porosity, this study investigated the effects of skewness and standard deviation of grain size distribution on the porosity in mixed size sediment.

2. METHOD OF NUMERICAL ANALYSIS

The basic equations of this method were expanded from binary packing formula to sediment mixture by Uchida et al. (2020). We set the sediment class i in the order of diameter as i=1,2,3,… from the biggest one to the smallest. Then the deposition process is calculated in this order and obtain the sediment mixture formula as

$$\lambda = 1 - \sum_{i=1}^{k} P_i \tag{1}$$

$$\lambda_i = 1 - \sum_{j=1}^{i-1} \alpha_{ij} P_j \tag{2}$$

$$\alpha_{ij} = 1 + \frac{\lambda_0}{1-\lambda_0} \gamma_{ij}^{C_1} \left(1 - \beta_{ij}\right)^{C_2} \tag{3}$$

$$\beta_{ij} = \left(1 - \gamma_{ij}\right) \left\{1 - \frac{P_j}{(1-\lambda_0)\lambda_j}\right\} \tag{4}$$

$$\gamma_{ij} = min\left(C_3 \frac{d_i}{d_j}, 1\right) \tag{5}$$

Keywords: Porosity, Sediment mixture, Skewness, Standard deviation
Contact address:1-3-2 Kagamiyama, Higashi Hiroshima city, Hiroshima, 739-8511, Japan

where λ_0 is the porosity of the uniform diameter sediment; $C_1C_2C_3$ is the coefficient decided by experiment; λ_i is the available porosity of particle *i*; P_i is the portion of particle *i*.

As for the numerical calculation, we have introduced analytical layers to calculate the porosity. The thickness of the layers are decided by the particle diameter. We assume that the sediment mixture is deposited little by little in small amounts in the order of diameter from the biggest one to the smallest one. Then the analytical layers are divided into unsaturated surface layer and saturated deposition layer. The packing process in each layer is changed from unsaturated to saturated. In an unsaturated situation, the deposition height will not increase when sediment deposits; but in saturated situation, the deposition height will increase. As a result, the variation of deposition height and surface fraction are calculated by

$$\frac{\partial z_{bi}}{\partial t} = \frac{D_i}{\lambda_i(1-\lambda_0)}\ ,\ \frac{\partial P_i}{\partial t} = 0 \quad (saturated\ P_i = \lambda_i(1-\lambda_0)) \tag{6}$$

$$\frac{\partial z_{bi}}{\partial t} = 0\ ,\ \frac{\partial P_i}{\partial t} = \frac{D_i}{d_i} \quad (unsaturated\ P_i < \lambda_i(1-\lambda_0)) \tag{7}$$

where D_i is deposition volume rate of particle *i* for unit area; z_{bi} is the deposition height on the deposition layer.

In the analytical layers, the portion of particle *i* in arbitrary layer *n* is

$$d_{max} \cdot (\delta P_i)_n = \{(P_i)_{n-1} - (P_i)_n\} \cdot \Delta z_b + (\Delta t \cdot D_i)_n \tag{8}$$

where $(P_i)_n$ is the portion of particle *i* in *n* layer; $(\Delta t \cdot D_i)_n$ is the deposition volume per unit area of particle *i* in n layer. We can get the porosity for each grain size distribution from this equation.

In this study, average grain size, standard deviation σ and skewness *S* are defined based on the logarithmic distribution of grain diameter as

$$\log_{10}\bar{d} = \frac{\sum_{i=1}^{k} V_i \log_{10} d_i}{V_d} \tag{9}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{k} V_i\left(\log_{10} d_i - \log_{10}\bar{d}\right)^2}{V_d}} \tag{10}$$

$$S = \frac{\frac{1}{V_d}\sum_{i=1}^{k}\left\{V_i\left(\log_{10} d_i - \log_{10}\bar{d}\right)^3\right\}}{\left[\frac{1}{V_d}\sum_{i=1}^{k}\left\{V_i\left(\log_{10} d_i - \log_{10}\bar{d}\right)^2\right\}\right]^{\frac{3}{2}}} \tag{11}$$

where V_d is the total volume of dried sediment; V_i, d_i is the volume and diameter of each particle size. The skewness *S* is a three-dimensional moment to show the range of size distribution. When the portion of the finer particles is more than the coarser particles, $S > 0$; when the portion of the coarse particle is more than the finer particles, $S < 0$.

3. RESULT

With data of Roy et al. (2011) measured in the Line River, data measured in the Noro River, Hataka River, Seno River, Ooyaoo River and Yano River were used for analysis to calculate the skewness, standard deviation and porosity of each sample point.

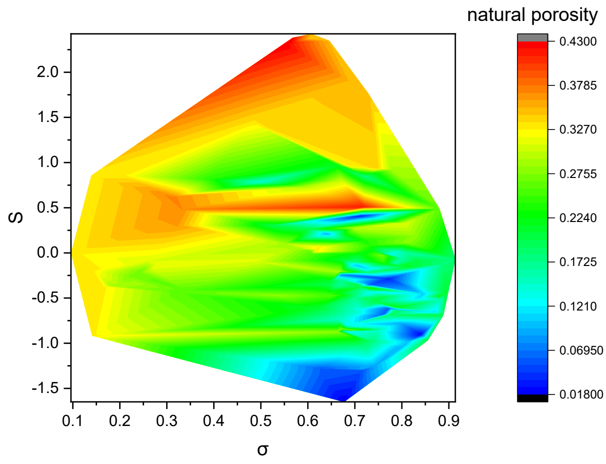

Fig.1 contour graph between natural deposition porosity and skewness and standard deviation

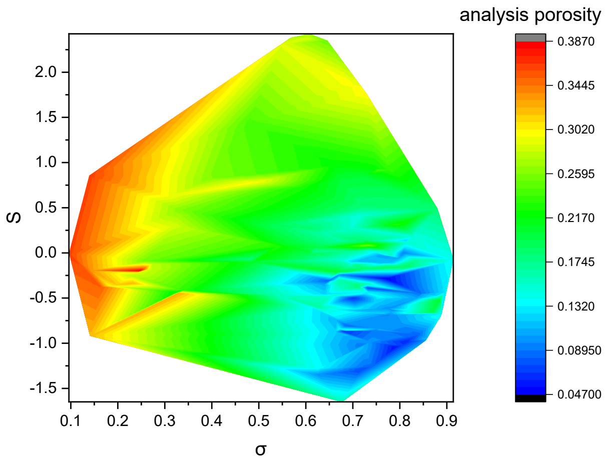

Fig.2 contour graph between analysis porosity and skewness and standard deviation

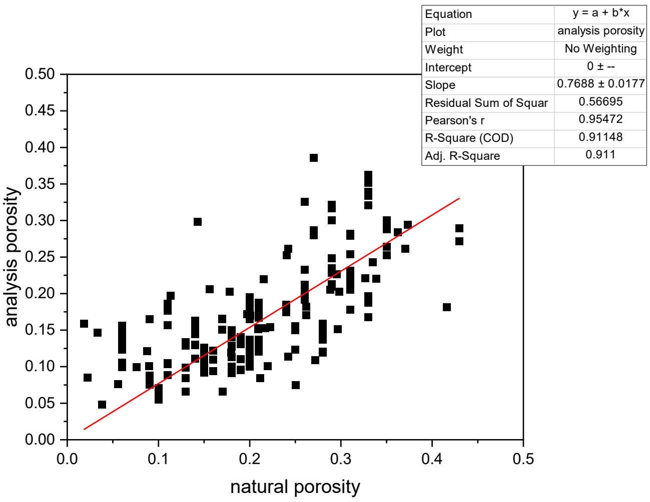

Fig.3 comparation between analysis porosity and natural porosity

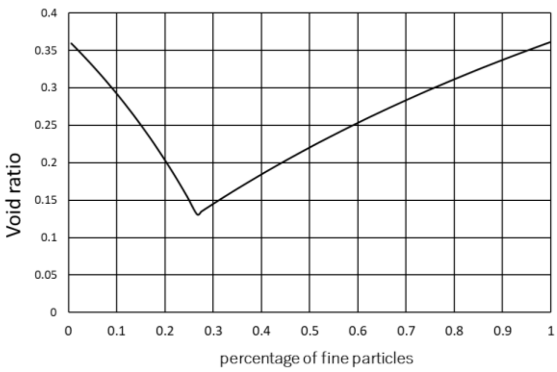

Fig.4 porosity variation in binary packing

Fig.1 shows that the porosity has a decreasing tendency with decreasing in skewness. Most part of the porosity is decreasing from low standard deviation to high deviation as well. However, there is an extremely high porosity area at around 0.5 skewness and 0.4 to 0.7 standard deviation. The low porosity part is found around -1.0 to -1.5 skewness and around 0.67 standard deviation; and the high part of porosity is found around 2.0 skewness and 0.4 to 0.6 standard deviation. Most part of Fig.2 shows the same tendency as Fig.1. In addition, it has a more significant characteristic that the porosity is decreasing from the low standard deviation to high standard deviation. The low porosity part is found around -0.25 to -1.25 skewness and 0.7 to 0.8 standard deviation; and high porosity part is found around -0.5 to 1.0 skewness and 0.15 standard deviation.

The correlation between analysis porosity and natural porosity is compared in Fig.3. The linear fitting result shows that analysis porosity is about 0.7688 times of natural porosity in the used data.

Fig.4 appears that the porosity has a decreasing and then increasing tendency in binary packing when the ratio of finer particles changes. The *x*-axis represents the portion of the finer partials, and skewness also varies when the portion of the finer particles changes. There may be some similar points between these two parameters on the influence of porosity.

The binary packing figure shows a decreasing and then increasing tendency when the portion of finer particles increases. As the skewness also represents the percentage of fine particles, there may be also the same tendency in *y*-axis of Fig.1 and Fig.2, beyond the minimum value of *S*. As the collected data is limited, if we can obtain lower skewness data, it may have similar tendency in *y*-axis.

There is some horizonal part in Fig.1 and Fig.2, in which the porosity changes rapidly between them. The explanation of this phenomena in natural porosity is that the natural porosity is not only decided by two parameters of sediment distribution. This may indicate that the shape of sediment distribution is not regular, which means that standard deviation and skewness can not express the porosity well. In addition, the natural deposition condition is also an important influence that the analysis porosity did not consider. So finding other parameters to represent porosity is a challenging task of this topic in the future.

CONCLUSION

This study found the variation rule of the porosity on the influence of skewness and standard deviation of the grain size distribution. The porosity decreases when the skewness becomes smaller and the standard deviation becomes bigger. A conjecture of the connection between skewness and portion of finer particles in binary packing is aroused as well – maybe the porosity will turn to increase with smaller skewness value beyond the range of this study. Although the future discussion is needed, such a small negative value of the skewness to satisfy the coarse packing conditions in binary packing problem is considered to be barely induced in the deposition process in natural rivers.

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