Free-Rocking Experiment test for verifying The 2DOF Model of Flat-Bottom Cylindrical Shell Tank

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1. Introduction

Evaluation of rocking motion of cylindrical tanks subjected to strong earthquake motions is a fundamental topic in the context of the seismic vulnerability assessment of this particular kind of structures. Based on mechanical analogy of rocking motion of between the 2DOF model consisting of a spring-mass system and a lower mass attached to its base and cylindrical tanks, the equations of motion for the tank bulging motion, that for the tank rocking motion, the base shear and reaction are derived in previous study [1]. In this study, Free-rocking experiment test was carried out to verify the 2DOF model.

2. Nomenclature

k: Spring constant
m_1: Mass of the Spring-Mass System (SMS)
m_2: Mass of the mass attached to the base of the SMS
R_1: Length between origin O and gravity center of m_1
R_2: Length between origin O and gravity center of m_2
x_1: Displacement of m_1
$\ddot{z}_H$: Horizontal ground acceleration
α_1: Angle between vertical line y and R_1
α_2: Angle between vertical line y and R_2
θ: Rotational angle of 2DOF model

3. Equation of Motion for 2DOF Model

Consider a 2DOF system consisting of a Spring-Mass (SM) system and a lower mass attached to its base (hereafter 2DOF model). Figure 1 gives its geometry. The SM system begins to vibrate at the initiation of the ground motion, while the 2DOF model commences to rock when the Overturning Moment (OM) induced by the SM system of the 2DOF model overcomes the Restoring Moment (RM) inherent in the 2DOF model. The Lagrangian of 2DOF model is given as;

$$
\begin{aligned}
L = &\frac{1}{2}m_1\left\{\dot{x}_1^2 + x_1^2\dot{\theta}^2 + 2x_1R_1\dot{\theta}^2\sin\alpha_1 - 2\dot{x}_1R_1\dot{\theta}\cos\alpha_1\right\} \\
&+\frac{1}{2}\left\{I_1 + m_1R_1^2\right\}\dot{\theta}^2 \\
&-m_1g\left\{R_1\cos\left(\alpha_1-\theta\right)+\sin\theta x_1 - R_1\cos\alpha_1\right\} \\
&+\frac{1}{2}\left\{I_2 + m_2R_2^2\right\}\dot{\theta}^2 \\
&-m_2g\left\{R_2\cos\left(\alpha_2-\theta\right)-R_2\cos\alpha_2\right\}-\frac{1}{2}kx_1^2
\end{aligned}
\tag{1}
$$

The equation of motion for translational motion of the vibrant mass of the SM system of the 2DOF model is derived as;

$$
\begin{aligned}
&m_1\ddot{x}_1 - \lambda m_1R_1\ddot{\theta}\cos\alpha_1 + \lambda m_1g\sin\theta - m_1\left(x_1 + \lambda R_1\sin\alpha_1\right)\dot{\theta}^2 \\
&+kx_1 + m_1\ddot{z}_H\cos\theta = 0
\end{aligned}
\tag{2}
$$

Here, to simplify notation of Eq. (2), the damping term of the SM system of the 2DOF model is not explicitly shown. However, this study assumes that the damping effects on the response of the SM system of the 2DOF model are naturally included by giving the inertia force of the tank bulging system based on the spectral response acceleration given later. The equation of motion for the rocking motion of the 2DOF model is also derived as;

$$
\begin{aligned}
&\left\{m_1\left(x_1^2 + 2\lambda x_1R_1\sin\alpha_1 + R_1^2\right) + I_1 + I_2 + m_2R_2^2\right\}\ddot{\theta} - \lambda m_1\ddot{x}_1R_1\cos\alpha_1 \\
&+2m_1\dot{x}_1\dot{\theta}\left(x_1 + \lambda R_1\sin\alpha_1\right) + m_1g\left\{R_1\sin\left(\alpha_1-\theta\right)+\lambda x_1\cos\theta\right\} \\
&+m_2gR_2\sin\left(\alpha_2-\theta\right) \\
&-\left[m_1\left\{\lambda R_1\cos\left(\alpha_1-\theta\right)+x_1\sin\theta\right\}+\lambda m_2R_2\cos\left(\alpha_2-\theta\right)\right]\ddot{z}_H = 0
\end{aligned}
\tag{3}
$$

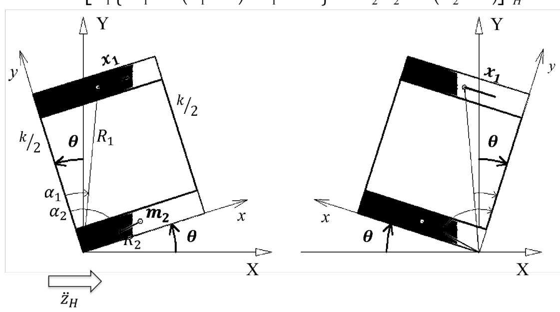

Figure 1. 2DOF model in displaced and rotated position

The condition to initiate the rocking motion of the 2DOF model is;

$$\mathrm{RM} < \mathrm{OM} \tag{3}$$

$$RM = (m_1R_1\sin\alpha_1 + m_2R_2\sin\alpha_2)g$$

$$OM = m_1R_1\cos\alpha_1\left(\ddot{x}_1 + \ddot{z}_H\right) + m_2R_2\cos\alpha_2\ddot{z}_H$$

Keywords 2DOF Model, Free-Rocking Experiment, Rocking-Bulging Interaction

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The transition from the liftoff around an edge to the liftoff around another one in accompanied by an impact. The associated loss of energy is taken into account by reducing the angular velocity of the system after the impact. In particular, it can be expressed as follow:

$$\dot{\theta}(t^{+}) = e\dot{\theta}(t^{-}) \qquad 0 \le e \le 1 \tag{4}$$

where e is the coefficient of restitution; t^{+} is the time just after the impact; t^{-} is the tome just before the impact. Changes in angular velocity are considered to occur instantaneously.

4. Experimental Test Results and Comparison with Analytical Simulation

In order to validate the accuracy of the 2DOF model and its adequacy in simulating the complex motion described by the coexistence of horizontal displacement and rotation, both experimental tests and analytical simulation have been conducted.

A high-speed camera has been used to record the free rocking motion of a 2-story model composed of two rigid masses connected by two flexible columns (see Fig. (2)). The materials utilized are steel for masses and junctions, aluminum for columns. The upper mass m1 is 3.33kg, the lower mass m_2 is 2.34kg (these values include also mass of steel junctions). The distance R_1 between the pivot of rotation and the gravity center of m_1 is 299mm while for the lower mass R_2 is 101mm. The angles between the axis y and respectively R_1 and R_2 are 0.341 and 1.446rad. The natural frequency of the vibrant mass m1 is 3.9 Hz. In order to avoid sliding motion, the two contact surfaces have been coated with sandpaper.

An initial uplift angle of 0.125 rad (12.3 degree) is enforced to the model and consequently also an initial displacement of the upper mass occurs due to the inclined component of the weight force. The motion is recorded in high-speed photography whose time interval is 1/3000 second and then the responses of interest are measured with an image processor. On the other hand, solving the equation of motion presented in the previous paragraph, the time history of the response in terms of displacement x_1 and rotation θ are calculated.

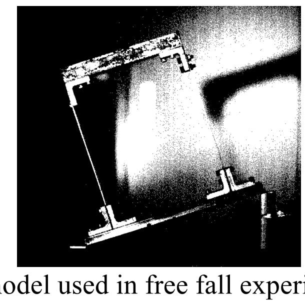

Figure 2. Steel model used in free fall experimental test.

Fig.3 compares the time history of the rotational angle θ obtained from the experimental test with that obtained from the analytical resolution. Fig.4 shows the same comparison in terms of relative displacement x_1.

As figures show, the experimental result is well simulated by the analytical model. In case of the rotational angle θ, the two curves match well both in terms of maxima values and bouncing times. Therefore, the equations of motion for the 2DOF model are verified.

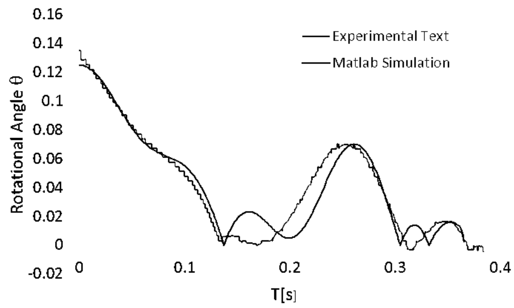

Figure 3. Comparison of analytical and experimental results in term of rotational angle θ

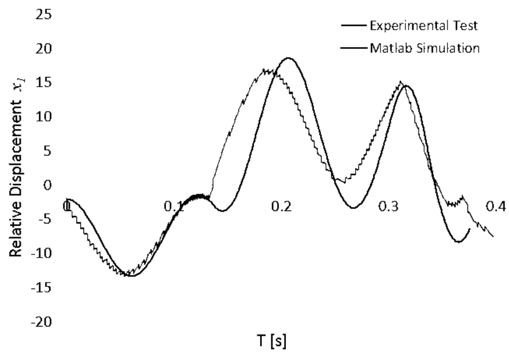

Figure 4. Comparison of analytical and experimental results in term of relative displacement x_1

5. Conclusion

In this study, Free-rocking experiment test was carried out to verify the equation of motion of the 2DOF model that was obtained in previous study. In the experiment test, a 2-story model composed of two rigid masses connected by two flexible columns is used and its motions are recorded in high-speed camera. The experimental result is well simulated by the analytical model. In case of the rotational angle θ, the two curves match well both in terms of maxima values and bouncing times. Therefore, the equations of motion for the 2DOF model are verified.

6. Acknowledgements

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7. References

[1] Taniguchi, T., Okui, D., 2014, "A Case Study of Evaluation of Tank Rock Motion with Simplified Analysis Procedure," Proc.Seismic Engineering, ASME Paper No.PVP2014-28635.