Modeling of Transport Phenomena of Two - Phase Flow

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ABSTRACT

A theoretical approach of two-phase flow and its three-dimensional numerical realization are presented. Water-, air and vapor migration through a porous media, under special consideration of an additional thermal gradient of a second driving force, serves as the practical background.

Introduction

The study of multiphase flow through geological material has been major importance during the past decade, often connected with a wide variety of application, namely simulation of oil-resovouirs or contermination. One area of such interest is the question, how to use geological formation as depositories for nuclear waste, as figure1.

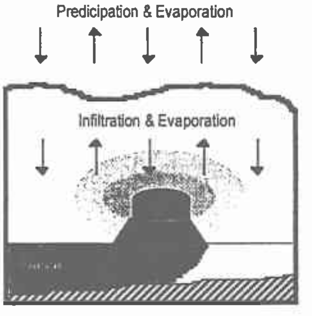

Figure 1 Principle of nuclear deposit

Model description

Subsurface can be characterized using the multiphase/multitracer concept. A phase is defined as to have different properties from the contiguous material and to have a bounding surface. (Fredlund, et.al. 1993).

Due to the long-term character of such facilities, after a certain period of time a thermal gradient can be observed. The flow conditions around the deposit become partially unsaturated. In addition to it, water may pass the contractile water-air skin and appear as a third vapor phase.

Because energy and mass migrate simultaneously, the system is described by two independent state variables, pressure and temperature. While for pressure, the governing equation have to be formulated for all three mobile phases water, air and vapor, for temperature a fourth phase have to be regarded, the solid matrix.

In the current model stage elastic, plastic or thermoelastic behavior of the solid matrix is not regarded. In addition to it, the non wetting mobile phases (air and vapor) follow the Dalton law while the wetting phase (water) is defined as to be a Boussineq - fluid. The mobil phases have to satisfy the Richard equation and diffusion of the gaseous phases based on the theory of dilute gases (Hirschfelder 1954). The radiation of heat is neglected and the energy transport based only on conduction and convection. The phases are assumed to be in a local thermal equilibrium.

Mass flow equations

Bear (1990) gave for the adequate conservation equation for each phase the following description. Without regarding external sources and sinks, and under the assumption, that the porosity of the solid matrix is not a function of the state variables, the equation can be written as:

$$\frac{\partial \Theta_l^V \rho_l}{\partial t} = -\nabla(\rho_l q_l^M) - f_{l\to v}^l \qquad (1)$$

$$\frac{\partial \Theta_g^V \rho_v}{\partial t} = -\nabla(\rho_v q_v^M) - f_{v\to l}^l \qquad (2)$$

$$\frac{\partial \Theta_g^V \rho_a}{\partial t} = -\nabla(\rho_a q_a^M) \qquad (3)$$

Θ_l	*volumetric moisture content*
Θ_g	$(\Phi-\Theta_l)$
Φ	*porosity*
ρ_l	*partial pressure liquid phase*
ρ_v	*partial pressure vapor phase*
ρ_a	*partial pressure air phase*
q_l^M	*mass flux vector liquid phase*
q_v^M	*mass flux vector vapor phase*
q_a^M	*mass flux vector air phase*
$f_{l\to v}'$	*volumetric rate of vaporation*
$f_{v\to l}'$	*volumetric rate of condensation*

According to Peaceman (1977) the motion force of the liquid phase can be formulated as follows:

$$q_l^m = \frac{K_{ij} K_l}{\mu_l}\nabla(p_l + \rho_l g z) \qquad (4)$$

Hirschfelder (1954) showed, that for the momentum equation of a dilute gas is identical to that its components in some cases if the gas velocity is replaced by the mass average velocity of the mixture.

$$v = \frac{m\, n_1 v_1 + m_2 n_2 v_2}{m\, n_1 + m_2 n_2} \qquad (5)$$

Introducing in such a way the impact of the Knudsen diffusion (second term), the Binary diffusion (third term) and the Thermo-diffusion, it can be written:

$$
\begin{aligned}
q_v^{m} = {} & \frac{K_{ij}K_v}{\mu_v}\nabla(p_v+\rho_v g z) \\
& -(\Phi-\Theta)\rho_v \frac{D_{Ka}D_{Kv}}{\rho_g D_B+\rho_v D_{Ka}+\rho_a D_{Kv}}\nabla(p_g+\rho_g g z) \\
& -(\Phi-\Theta)\rho_v \frac{\frac{\rho_g}{\rho_v}D_B D_{Kv}}{\rho_g D_B+\rho_v D_{Ka}+\rho_a D_{Kv}}\nabla(p_v+\rho_g g z) \\
& -(\Phi-\Theta)\rho_g \frac{R_a D_{Ka}D_T}{\rho_g D_B+\rho_v D_{Ka}+\rho_a D_{Kv}}\nabla T
\end{aligned} \qquad (6)
$$

$$
\begin{aligned}
q_a^{m} = {} & \frac{K_{ij}K_g}{\mu_a}\nabla(p_a+\rho_a g z) \\
& -(\Phi-\Theta)\rho_a \frac{D_{Ka}D_{Kv}}{\rho_g D_B+\rho_v D_{Ka}+\rho_a D_{Kv}}\nabla(p_g+\rho_g g z) \\
& -(\Phi-\Theta)\rho_a \frac{\frac{\rho_g}{\rho_a}D_B D_{Kv}}{\rho_g D_B+\rho_v D_{Ka}+\rho_a D_{Kv}}\nabla(p_a+\rho_g g z) \\
& -(\Phi-\Theta)\rho_g \frac{P_a D_{Ka}D_T}{\rho_g D_B+\rho_v D_{Ka}+\rho_a D_{Kv}}\nabla T
\end{aligned} \qquad (7)
$$

p_l	*partial pressure liquid phase*
p_v	*partial pressure vapor phase*
p_a	*partial pressure air phase*
p_g	*partial pressure gas phase*
K_{ij}	*intrinsic permeability*
k_l	*permeability(liquid)*
k_g	*permeability(gas)*
μ_l	*viscosity(liquid)*
μ_l	*viscosity(vapor)*
μ_l	*viscosity(air)*
D_B	*Coefficient for binary Diffusion*
D_B	*Coefficient for thermal Diffusion*
D_{Kv}	*Knudsen Diffusion Coefficient(vapor)*
D_{Ka}	*Knudsen Diffusion Coefficient(air)*
R_v	*Ideal gas Constant(vapor)*
R_v	*Ideal gas Constant(air)*
T	*local Temperature*
C_l^v	*heat capacity(liquid phase)*
C_v^v	*heat capacity(vapor phase)*
C_a^v	*heat capacity(air phase)*
C_s^v	*heat capacity(solid matrix)*
Λ	*thermal conductivity tensor*

After adding up all four phases of the energy migration:

$$\langle aHC\rangle_{hw} = -(C_l j_l + C_v j_v + C_a j_a)\nabla T + (\Lambda\nabla T) - F_v^v + Q \qquad (8)$$

with

$$\langle aHC\rangle_{hw} = (1-\Phi)\rho_s C_s + \Theta\rho_l C_l + (\Phi-\Theta)(\rho_v C_v + \rho_a C_a) \qquad (9)$$

· Verification

The model was realized as a three-dimensional Finite Element Program using the Galerkin-approach for the spatial discretization and Crank-Nicholson the time. Mass and energy equation are interconnected by following four interactions:

$\mu = f(T),\ \ \rho = f(P,T),\ \ \Theta = f(P)$ and $v = f(P)$ (10a), (10b), (10c), (10d),

The diffusions coefficients were obtain using the following formula:

$$D_{Kv} = \frac{2}{3}\frac{r_{Pore}}{\tau}\sqrt{\frac{8R_vT}{\pi}}, \qquad D_{Ka} = \frac{2}{3}\frac{r_{Pore}}{\tau}\sqrt{\frac{8R_aT}{\pi}} \qquad (11a), (11b)$$

$$D_B = \frac{.000044\,T^{2.334}}{(p_v+p_a)\tau} \qquad (12)$$

$$D_T = \frac{\lambda}{\left[(1-\Phi)\rho_s c_s + \Theta\rho_l c_l + (\Phi-\Theta)(\rho_v c_v + \rho_a c_a)\right]} \qquad (13)$$

For the verification, the well known results of Touma and Vachaud (1984) experiment were used (Figure 2). The simulation results fit in a quite good way with the experimental results of Touma, but on top and bottom exist certain inconsistencies. The reason therefore consist in some differences in the initial conditions.

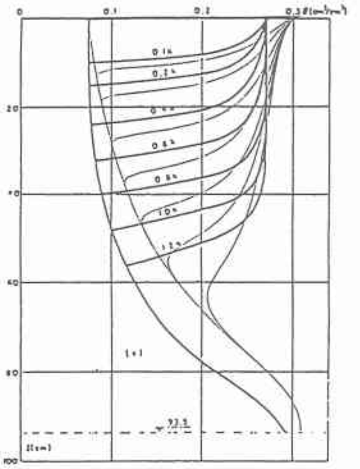

Figure 2 Comparasion simulation and experiment

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