3. Automatic Mesh Generation by Voronoi's Theory.

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【Abstract】 It has been problematic to develop simple and practical automatic mesh generation techniques that can minimize the amount of required input data and can generate a lot of nodes. The necessary data include the specification of external boundary of the given domain and an internal node density distribution. In particular,for the application of automatic mesh generation to stress concentration area,simple computer algorithms for managing input and output data are necessitated. In many cases,the difficulty of the finite element analysis lies in the mesh generation which is free from errors in the mesh formation.

【Key Words】 Voronoi,stress concentration,FEM

1. Introduction

The usefulness, Capability and reliability of the finite element method in solving diversified engineering problems have led to the development of computer programs over the past decades. Although these programs are now well established, the input data preparation still remains extensive, time consuming. tedious and error prone. especially in generating a finite element mesh. The need for automatic mesh generation has been realised since last decade, because of the large amount of data involved. The meshes for large areas which are often divided regularly into elements, can be generated by simple and effective mesh generation schemes. A large lock is defined as such region of the structure that is suited for automatic mesh generation. Several large-blocks can then be fabricated to complete the structure, with a relatively few data for individual nodal geometry and element topology to fill in any remaining gaps. Both nodal geometry and element topology can be generated in a manner Suitable for two or three-dimensional structures of complicated shape with irregular boundaries. A number of mesh generation algorithms of varying levels of automation have been proposed. At one extreme is the fully automatic mesh generation methods which demand a minimum amounts of user input and determine regions of high and low element densities. At the other extreme is simple methods which require the user to completely define the element mesh. A brief review of techniques either to developed can be found in the paper by- Thacker. Quite a number of general techniques have been suggested in generating elements within the given domains. (1)(2)

2. A sphere of influence theory

Voronoi Theory is one of the representative mesh generation method. It has the application fields like physics, theory of ecology, Urban Engineering etc. In Voronoi Theory,the territory condition of animals can be seen when their number is increesed in a large scale withitheir habitation area, if the dersity is high,the territor can be arranged without void.(3)The habitation area if we consider as a two dimensinsl plain, the bounddary of to two neighbourdary force-balanced can be expressed by the Vertical line in accordance with theircenter connecting line. In that case,the minimum least multi-angle is called the Voronoi multiangle. This angle is the nearest point from any other terriitory. In figure-1,the animals are organizing territory at point A as the bait is given at point A. Then they are moving to point B when the bait is given at both point A and B. In this theory, we are focussing/concentrating on analogical re-organiization of territory of animals and the re-generation of stress concentration area using Finite Element Analysis.(1)(4) (See Figure 1)

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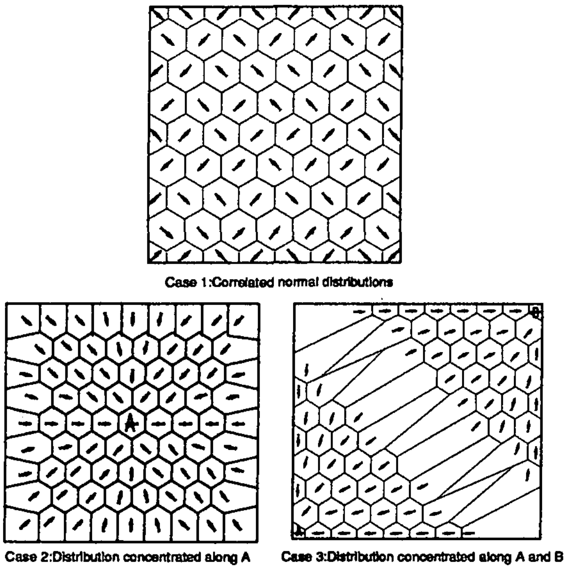

Fig.1 ex. Territories of Tilapia mossambica

3. Voronoi Polygon

Geometrical Method to Construct the Voronoi Polygon. The Voronoi polygons of a set V of points in the plane is defined as the dual graph of the Voronoi polygon of V, which is a subdivision of the plane into polygonal regions, each of which is associated with point P of V and is defined as the region closer to P than to any other points of V. An alternative characterization of a Voronoi polygon is given by so-called circle criterion. [5, 6]

Denote by P(x) a point in the N-dimensional Euclidean space R^N, where x is an N-dimensional vector (x1,x2,...,xn). For n distinct points $P_1(X_1)$, $P_2(X_2)$, ..., $P_n(X_n)$ given in R^N, stands for the set of points in R^N which are closer to $P_i(X_i)$ than any other $P_j(X_j)$ (j=i), where $\|\cdot\|$ denotes the euclidean distance. V_i is convex set because it is the intersection of half spaces.

This partition determines straight-forwardly a polyhedral complex, which is called voronoi polygon for the given n points $P_i(X_i)$'s. The partition is also called the Dirichlet tesselation. Figure 2 shows an example of the Voronoi polygon.

$$V_i = \bigcap_{j:j\neq i} \{x \in R^N \| \|x - x_i\| < \|x - x_j\| \| \} \quad (1)$$

$$(i = 0, \cdots, n, j = 1, \cdots, i-1, i+1, \cdots, n)$$

Following two geometral charecteristics can be found,between the void of every voronoi multi-angle whichare generated two-dimensionally.

1.Voronoi multi-angle as convex ulti-angle as convex multi-angle when the it is equally generated according to the mother-point.

2.Voroni point as the outer-center of the triangle connecting 3 sorrounding mother-points.(See Figure 3)

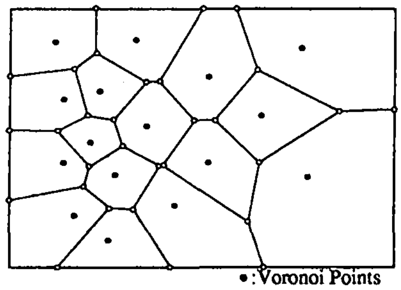

Fig.2 ex. Vonoroi diagram

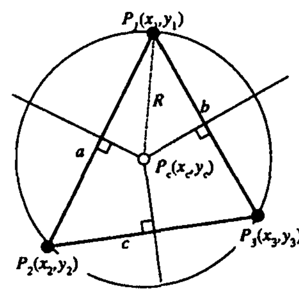

Fig.3 P_1, P_2, P_3 are the Voronoi points of the generator points P_c

4. Generation of The Generator Points

4.1 Input Data

Since the form of Voronoi region is dependent upon the distribution of the generator points, the generation of appropriate generator points is vital in the success in mesh generation. In order to reduce the number of input data, generator points can be chosen randomly. The following is a brief description of the essential procedures:

1. Determine Xmin. Xmax, Ymin and Ymax of the domain
2. The domain is divided into vp_{input} Sub-regions. where 'vp_{input}' is the number of generator points. Each sub-region has only one generator.

3. The sub-region is a dx x dy rectangle, where dl and dy are the length of the sides in x and Y direction, respectively; dx and dy can be determined by the following formula:Equation (2)

If the complete generator points have been generated by the method described above, the Voronoi polygon in turn can be made.

$$\left(\frac{x}{dx}+1\right)\left(\frac{y}{dy}+1\right)=vp_{\text{input}} \quad (2)$$

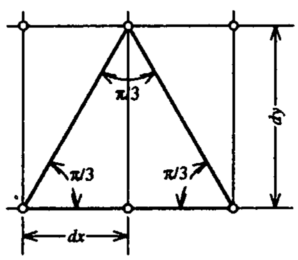

Fig. 4 Geometrical method to obtain the Virtual Voronoi points.

$$dy = dx \cdot \tan\pi/3$$

$$\{(1-vp_{\text{input}})\tan\pi/3\}\,dx^2 + (x\tan\pi/3+y)\,dx+xy=0 \quad (3)$$

If we use Equation Equation (3),generation width of x and y direction(dx,dy) can be found,This can be madde integers by calculating every generation number.

If the error is not settled within 1(See Figure 5,Figure 6)

4.2 Boundary Approximation

It is quite enough to move/replace the motherpoinit even in case of boundary approximation as the Voronoi joint is generated from the mother-point. While doing boundary approximation,the movement area according to the boundary line is fixed. Then the mother-point within the movement area is moved,and the rest are erased/eliminated.

The movement area is fixed as $\pm V_l/2$ of the boundary line . From the mother-points inthe movementy are,Verticall line is drawn in accordence with boundary line. Then the intersection point is considered as

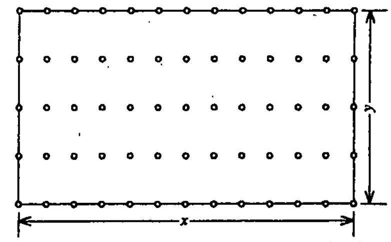

Fig.5 Arrangement of Virtual Voronoi Points

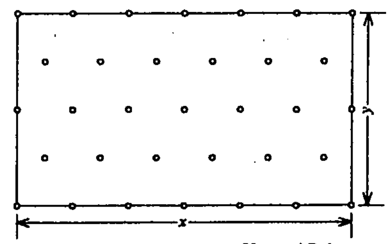

Fig.6 Arrangement of Voronoi Points

the coordinates after the the mother-point removess. (See Figure 7)

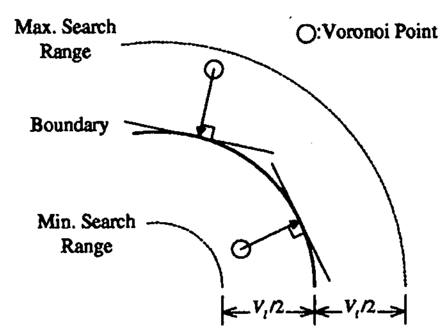

Fig.7 Boundary approximation by Voronoi points.

4.3 Element generation

Generation of voronoi joint takes place after the arrangement of mother-point is decided Voronoi joint is arranged at the center of moter point connecting circle Using the basic geometrial characteristics of voronoi theory. [8]Inside this circle,other motherpoints must be excluded. From this characteristics,the radius of the circle can be found by searching the least/minimum combination. Radius R can be found from Equation (4). Coordinates of the center of the circle can be found from equetion Equation (5). After the generation of all the Voronoi joints are completed,mother-point and Voronoi joints are replaced to the Finite Element Joints are replaced to the Finite Element Joints. After this ,trianguler data is generated connecting the joints of moter-point and Voronoi mesh. At last,this data is replaced to Finite Element joint data and elements are generated. (See Figure 3)

$$R = \frac{abc}{s(s-a)(s-b)(s-c)} \qquad (4)$$
$$s = \frac{1}{2}(a+b+c)$$
a, b, c : A triangle with sides

$$\left.\begin{aligned} x_c &= \frac{1}{2}\{(x_1+x_3)+\alpha(y_3-y_1)\} \\ y_c &= \frac{1}{2}\{(y_1+y_3)+\alpha(x_1-x_3)\} \\ \alpha &= \frac{(x_2-x_3)(x_2-x_1)+(y_2-y_3)(y_2-y_1)}{(x_1-x_3)(y_2-y_3)-(x_2-x_3)(y_1-y_3)} \\ x_i, y_i &: \text{ The coordinatesof mother point} \end{aligned}\right\} \qquad (5)$$

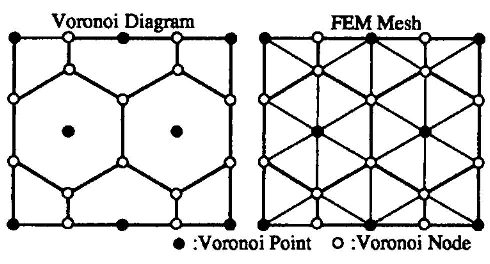

Fig. 8 Transformation of Voronoi diagram to FEM mesh.

5. Specification of stress concentration part

This research is based on the idea that stress concentration is remarkable at the greater/bigger part of non-continuous element stress value. Stress Slope is used here as an index.

5.1 Optimizing of Mesh

In this chapter, two general algorithms for the optimization of meshes are presented. The first algorithm is the stress gradient algorithm and the second the mesh refinement algorithm. Both algorithms are based on the stresses of each element: therefore, prior. to the optimizing process, the stress data in each element should be computed by the finite element method. In this study the stress data are provided froth the. output of ,COSMOS/M. The stress at each node can be computed by the average of the nodal stresses of all elements meeting at the common node.

5.2 Stress Gradient Algorithm

stress gradient algorithm is based on the policy that element area or the distance between two nodes should be reduced or increased according to whether the stress intensity is high or low.

Mesh stress Slope SG_{ij} is calculated from Equation Equation (6), Here i means voronoi mesh and j means the neighhboring mesh. (See Figure 9)

$$SG_{ij} = \frac{\sigma_i - \sigma_j}{d_{ij}} \qquad (6)$$

σ_i, σ_j : i, j Voronoi stress

d_{ij} : i, j An interval mother point

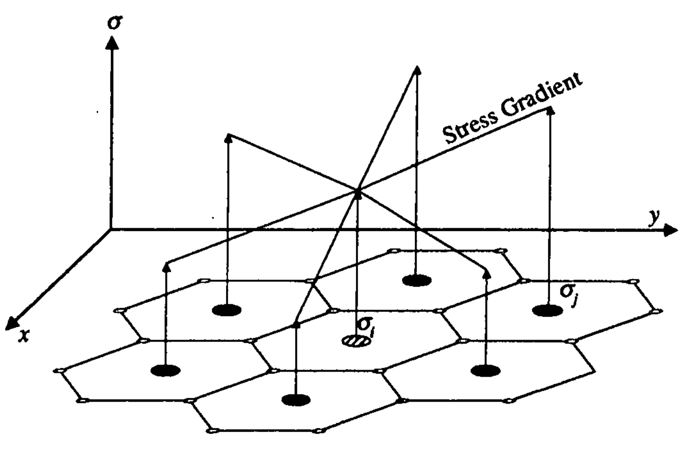

Fig.9 Stress Gradient between Voronoi regions

5.3 Stress Sloop inside the mesh

When the absolute value of positive and negative slope are almost equal, they are set off and sometimes overlooked/omitted. The stress slope Sgi between the elements of highest and lowest stress inside the mesh is calculated by Equation (7) (See Figure 10)

$$sg_i = \frac{\sigma_{\max} - \sigma_{\min}}{d} \qquad (7)$$

$\sigma_{\max}, \sigma_{\min}$: max min element stress

d : The center of gravity

6. Element Re-generation

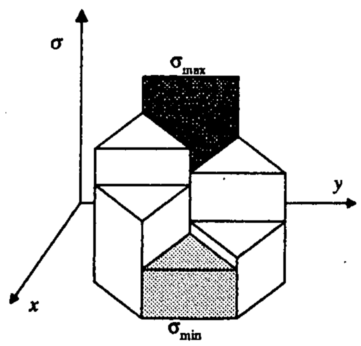

Fig.10 Stress Gradient between elements in Voronoi region

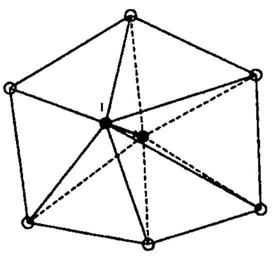

Fig.11 Determin new node i

The error of separation is made less by subdividing the elements at the center of specified stress concentrated part. Optimization of element form takes place using the joint replacement. (See Figure 11,Figure 12)

7. Judgement of converge

The judgment of convertion takes place comparing the highest stress value and the former value. To put it concretely,it is be caluculated by Equation (8). If the converge is not satisfactory,the elements are regenerated . This method is highly effective as it can increasa the number of elements with more efficiency. (See Figure 13)

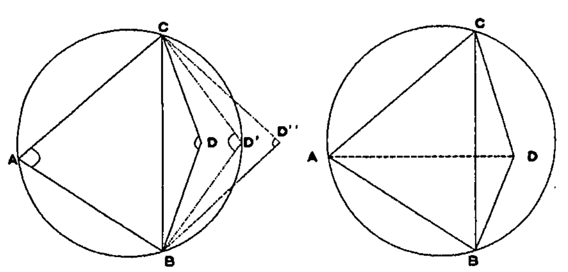

Fig.12 Modification of element

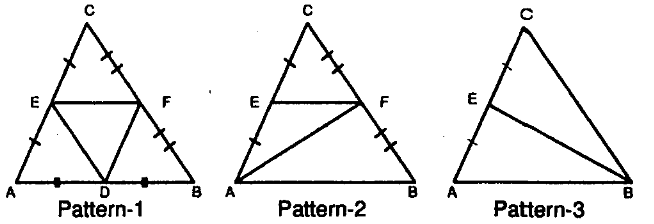

Fig.13 Patterns of refinement of mesh

$$\frac{\sigma_{1max} - \sigma_{2max}}{\sigma_{1max}} \cdot 100 < 5\% \qquad (8)$$

σ_{1max} : After max stress

σ_{2max} : Before max stress

8. Examples of application

The effects of this model are examined using the model shown at (Figure 14 Here Young modulas $E = 2.06 \times 10^5$kN/mm^2,poisson Ratio $\nu = 0.3$, Thickness of plate 10mm, height h = 100mm,width b = 50mm,radius r = 25mm, Tensile force q = 4kN.

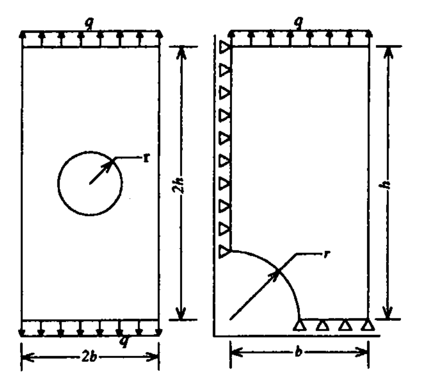

Fig. 14 Analysis model of thin plate with hole and 1/4 model

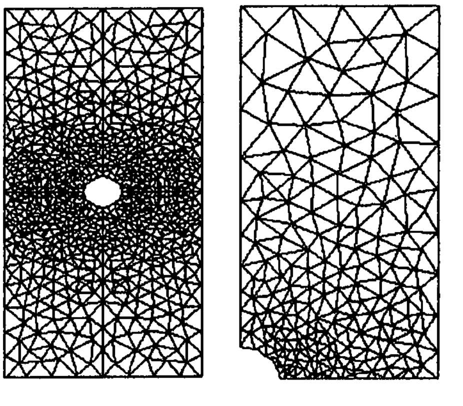

Fig.15 Optimum Voronoi diagram and mesh

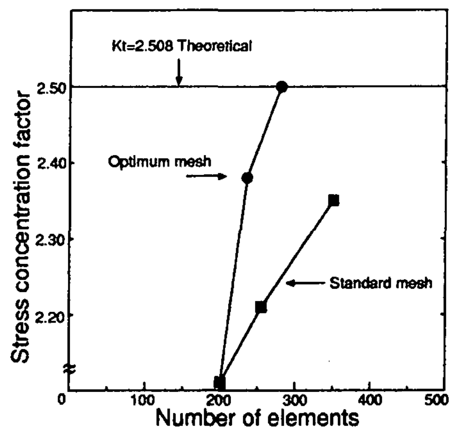

Fig.16 Comparison of results by optimum and standard mesh

Voronoi Figure and Element distribution /arrangement figure after re-generation are shown at (Figure 14).

Figure 16shows the number of elements required for calculating stress concentration Index Kt. Here it is converged by the number of elements which is small in number comparing with general method.

Figure 17 shown the comparative result theoretical Value with the caluculation Value of stress concentration Index after the radius is changed.

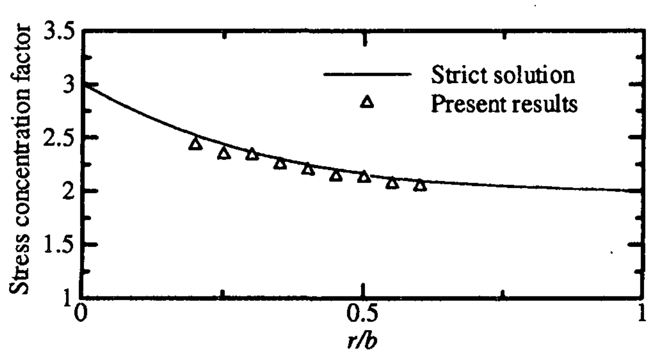

Fig.17 Graph of Stress concentration factor

9. CONCLUSIONS

In this paper, the Voronoi polygon has been used to generate the automatic mesh. This method is quite tractable and yields a good result in computing the stress intensity factor. The element-shape generated by the Voronoi polygon is dependent on the location of the generator points. It should be concern in the next research to optimize the location of the generator points. By smoothing process of elements, the element-shapes become nearly equiangular or equilateral triangle, and the elements vary smoothly. For the non-convex domain, the Voronoi polygon has some restriction, and hence the modification of the boundary points are needed. This is the problem to be addressed in the future.

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