

3. Application of GA to evaluation of composite stresses

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[Abstract] The loading position where the absolute value of bending moment or shearing force is maximum can be obtained by using the influence-line. But the composite stress can't be obtained by using the method because it always has positive value. Therefore, Japan Specification for Highway Bridges (J.S.H.B) allows to use the simple method. However, it is certain that the result is smaller than quasi-maximum value. It is very important for designers to find the magnitude of difference between design value and quasi-maximum value obtained by using GA. This paper firstly explains about issues of running design method, theoretical background of this calculation method and its concrete procedure. Next, it shows existence of the intermediate loading situation which is larger than the composite stress specified in J.S.H.B by solving a typical problem which expresses a part of rahmen type bridge. Finally, a simple calculation method to obtain the composite stress which is comprised by bending moment and shearing force is presented.

[Keyword] Genetic Algorithm, Composite Stress, Influence-Line, quasi-maximum value

1. Introduction

Calculation method utilizing influence-lines can't find out the situation which the values of composite stress specified by Japan Specification of Highway Bridge (J.S.H.B)¹⁾ is maximum. But, composite stresses decided by the loading situation which bending moment or shearing force has maximum value are actually used as design values. These values are generally used for design of bridge based on the assumption that these design values must not almost be different from global maximum values as things stand. But it is apparent that global maximum value²⁾ which is larger than the present design value actually exists. Therefore, it is very important for designers to recognize the difference of them. The provision 8.8.4 of J.S.H.B regarding to composite stresses is as follows;

"If normal stress and shear stress simultaneously are occurred by bending moment and they exceed 45% of allowable stress specified by specification 2.2.1 at section where only bending moment and shearing force occurs, equation(8.2.4) must be fulfilled to this loading situation."

The equation (8.2.4) is as follows:

$$(\sigma_b / \sigma_a)^2 + (\tau_b / \tau_a)^2 \leq 1.2 \quad (1)$$

σ_b : normal stress occurred by bending moment
(kgf/cm²)

σ_a : allowable normal stress specified by
specification 2.2.1(kgf/cm²)

τ_b : shear stress occurred by bending moment
(kgf/cm²)

τ_a : allowable shear stress specified by
specification 2.2.1(kgf/cm²)

However, J.S.H.B appends the following provision with it. "Because enormous number of combinations of σ_b and τ_b exist, all combinations can't be verified. Therefore, this specification allows to verify the loading situation which bending moment and shearing force respectively shows the maximum."

The design utilizing just like the composite stress is done in many engineering fields. Especially, von Mises equivalent stress is frequently used in mechanical engineering field.

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In order to obtain the loading situation which the composite stress has maximum value, the real strength load is applied to various loading points with varying the length of loading and maximum value must be found out from all these combinations.

However, this is Knapsack problem which is one of NP-Difficult problems. It occurs the explosion of combination. Therefore, this paper utilizes genetic algorithm(GA)⁹⁾ because a special technique is required to obtain the solution.

This paper firstly explains about issues of running design method, theoretical background of this calculation method and its concrete procedure. Next, it shows existence of the intermediate loading situation which is larger than the composite stress specified in J.S.H.B by solving a typical problem which expresses a part of rahmen type bridge.

Finally, by considering above results, a simplified calculation method of composite stress is recommended.

2. Search of maximum composite stress by utilizing genetic algorithm

(1)Genetic Algorithm(GA)⁹⁾

Finding out high quality solution in the complicate and large scale searching space is frequently required in the various problems of artificial intelligence fields. Optimization problem regarding to combination such as traveling salesman problem and knapsack problem are the typical examples. But these problems are impossible to get the exact solution because of enormous calculation amounts if the scale is large. So, a method to search quasi-optimum solution is actually used in engineering field instead of obtaining the exact solution. If provident knowledge related to the problem has already obtained in this regard, fast search can comparatively be done. But if it can't be obtained, the procedure of search must be controlled with accumulating the knowledge about the search space. Such search problem is called as an adapting search problem and the control is generally difficult.

GA is especially effective search algorithm for such problem which the search space can't be nicely narrowed and it is so large that the explosion of combination can't be avoided because inherent knowledge of the problem can't be used.

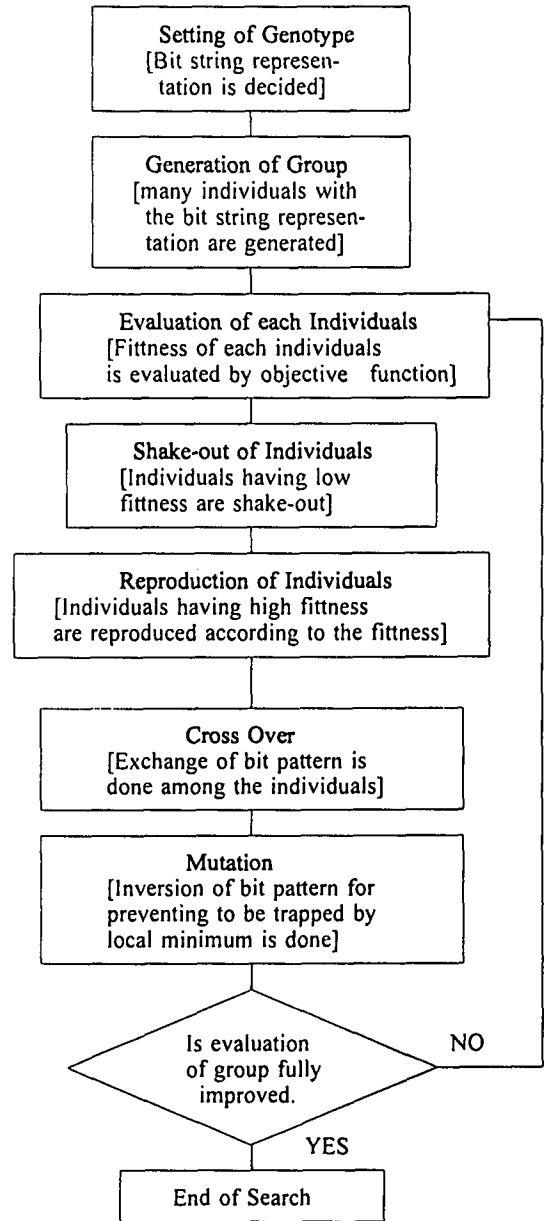


Figure.1 Basic procedure of genetic algorithm

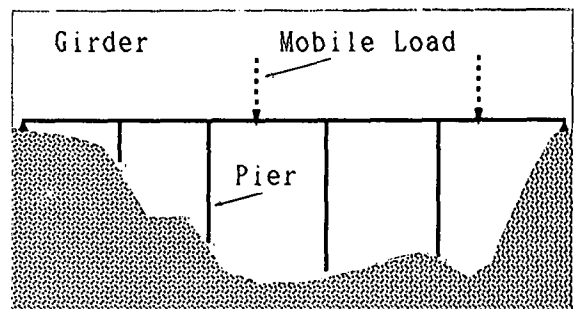


Figure.2 Analytical model for investigating into composite stress

GA is executed according to the following procedure. Figure.1 shows the basic process of the procedure.

- (a)Target system is expressed as gene(chromosome as the aggregation).It is generally represented with a length of row of bit.
- (b)Large number of individuals having this chromosome are generated. The information is equivalent to phenotype.
- (c)Group of the individuals lives in an environment and is evaluated whether it is fit in with the environment or not.
- (d)It is applied to shake out in accordance to the evaluation. That is to say, individuals which are evaluated lower than a criterion are killed and others can survive.
- (e)(f)(g)Individuals which can survive in struggle for existence can find out spouses and remain some descendants according to the evaluation. And they can exchange their information of chromosome and occur mutation at the same time.
- (h)The cycle of (b)(c)(d)(e) is repeated many times.
- (i)When the environment changes, better individuals are selected according to the evaluation. At last, it can be expected that some new individuals which didn't exist at the beginning stage appear.

(2)Search on the loading situation showing maximum value of composite stress

At first, mobile load expressed by real load(concentrated or distributed load) is applied to the analytical model. And observing components of section forces at each loading situation are obtained by using the same way with the method specified in J.S.H.B.

Here, when bending moment or shearing force has maximum value, composite stress coincides with the result of method using influence-line. However, it always doesn't be guaranteed at all loading situation. The composite stress can't easily be obtained by such as the calculation method using influence-line because it is expressed by equation (1).

It can be considered that the loading situation which the composite stress has maximum value is decided by the combination of loading pattern(for example, concentrated load, partial distributed load ,,etc). That is to say, some regions of loading points are loaded and others are not loaded.

The situation is expressed by the following equation(2).

$$M_j = \left| \sum_{i=1}^m M_{ji} \right|, \quad S_j = \left| \sum_{i=1}^m S_{ji} \right| \quad (2)$$

M_j :bending moment at point j

S_j : shearing force at point j

M_{ji} :bending moment at observing point j in case of being loaded at point i

S_{ji} :shearing force observing point j in case of being loaded at point i

$i(=1,2,,m)$ shows the nodal numbers of all loading points and $j(=1,2,,n)$ shows the observing points. Especially, α_{ij}, β_{ij} are coefficients which indicate whether the nodal point is now loaded or not. If they are now loaded, these parameters(α_{ij}, β_{ij}) have 1.0. But the maximum composite stress can't be easily obtained by using the combination of many loading points because these can have enormous patterns. Enormous patterns are calculated by the following equation(3).

$$P = \{P_1, P_2, P_i, P_N\} \quad (3)$$

Here,

N: all loading point

P: Load vector

P_i : Load at point i

And, bending moment and shearing force corresponding to each load P_i at a observing point j is defined by following equation(4).

$$M_j = \{M_{j1}, M_{j2},, M_{jN}\} \quad S_j = \{S_{j1}, S_{j2},, S_{jN}\} \quad (4)$$

Now, loading situation which composite stress has maximum value is found out by selecting one pattern from loading patterns based on above situation. More specifically, partial confluence P_n, M_n, S_n is constructed. n shows the number of loading position where is selected from the number of all loading point N. Number of the combination of each component is ${}_N C_q (q=1,2,,N)$ and k-th partial combination in them is defined as ${}_N C_k$. Now, if sum of the

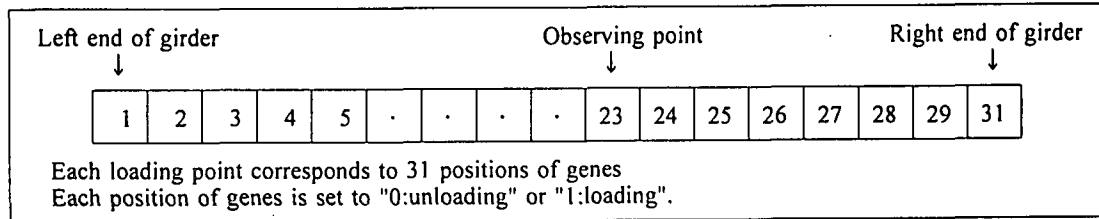


Figure.3 Genetic coding expressing the loading situation of each loading point

Table.1 Variation of composite stress due to the position of observing point (left end has simple support and other end has cramped support)

x/L	M_{max}/X	S_{max}/X
0.13	1.00	0.88
0.26	1.00	0.71
0.50	1.00	0.53
0.63	1.00	0.61
0.70	0.97	0.69
0.73	0.94	0.94
0.77	0.93	1.00
0.83	1.00	1.00
0.90	1.00	1.00

G:Parameter $n^2/L^2=0.0022$
 Span length:L=30m
 M_{max} :Composite stress comprised of bending moment and shearing force at showing maximum bending moment
 S_{max} :Composite stress comprised of bending moment and shearing force at showing maximum shearing force
 X:Maximum search value

Table.2 Variation of composite stress due to the position of observing point (both ends have cramped support)

x/L	M_{max}/X	S_{max}/X
0.50	1.00	0.59
0.57	1.00	0.53
0.63	1.00	0.54
0.70	1.00	0.60
0.73	1.00	0.64
0.77	0.99	0.72
0.80	0.92	0.93
0.83	0.89	1.00
0.87	0.95	1.00

G:Parameter $n^2/L^2=0.0022$
 Span length:L=30m
 M_{max} :Composite stress comprised of bending moment and shearing force at showing maximum bending moment
 S_{max} :Composite stress comprised of bending moment and shearing force at showing maximum shearing force
 X:Maximum search value

Table.3 A example of loading situation(condition of calculation:span length of girder L=30m observing point S=22m from left end, $n^2=1.5$)

GA	Loading situation	01111111111111111111111111111110
	Composite stress	$X_{max}=137.4$ ($M=55.6,S=102.5$)
M_{max}	Loading situation	00000000000000001111111111111111
	Section force	$M_{max}=117.3$ ($M=117.3,S=45.1$)
	Composite stress	$M_{max}=\sqrt{(117.3^2+1.5*45.1^2)}=129.7$
S_{max}	Loading situation	1111111111111111111111111000000000
	Section force	$S_{max}=105.0$ ($M=13.6,S=105.0$)
	Composite stress	$S_{max}=\sqrt{(13.6^2+1.5*105.0^2)}=129.3$

1:Loading, 0:Unloading, Composite stress: $X=\sqrt{M^2+S^2}$, Span length:L=30m
 X:Quasi-maximum value, M_{max} :Composite stress comprised of bending moment and shearing force at showing maximum bending moment.
 S_{max} :Composite stress comprised of bending moment and shearing force at showing maximum shearing force, X:Maximum search value
 FL:Full loading situation, observing point:S=22m
 Max:Maximum absolute value \rightarrow Max=MAX{ M_{max} , S_{max} , FL}

defined as V_{nk} , this is converted to combination search problem as shown in equation(5).

$$V_{nk} \rightarrow \text{Maximum} \quad (5)$$

And the sum of combination patterns is expressed by following equation.

$$H = N_{c1} + N_{c2} + \dots + N_{cN-1} + N_{cN} = 2^N \quad (6)$$

By the way, the equation (2) converted to force-form from stress-form of equation (1) is used in this paper.

$$\begin{aligned} & (\sigma_b / \sigma_a)^2 + (\tau_b / \tau_a)^2 \\ & = \{ (M_b)^2 + (Z \sigma_a / A \tau_a)^2 (S_b)^2 \} / (Z \sigma_a)^2 \end{aligned} \quad (7)$$

Here,

M_b : bending moment obtained by GA(kgf·cm²)

S_b : shearing force obtained by GA(kgf·cm²)

σ_a : allowable stress(kgf/cm²)

τ_a : allowable shear stress(kgf/cm²)

Z : section coefficient(cm³)

A : section area (cm²)

σ_b : normal stress(kgf/cm²)

τ_b : shear stress(kgf/cm²)

3. Specification using non-deterministic beam

A model cutting off a part from rahmen type bridge which is observed at highway in city is used as shown in Figure.2. The model is non-deterministic beam with the structure which the girder is rigidly connected with the pier. Now, two non-deterministic models that are supported on S-F, F-F condition (S indicates simple support condition, F indicates cramped support condition) are adopted. The load being applied to the model as mobile load is concentrated force of 10.0 tf. And the observing point is S(m) position from left end.

When mobile load is used for design, it is generally applied to the region so that it gives rise to maximum section force. That is to say, it is the most disadvantageous situation. In the case of considering only bending moment and shearing force, it is easy to decide the loading region where it has

maximum value by considering shape of influence-line. However, it is difficult to decide the loading region because the composite stress has always positive value. Therefore, it is necessary to obtain quasi-maximum value by using GA.

When GA is used for this problem, genetic coding shown in Figure.3 must be done. 31 loading points on the non-deterministic beam are defined as one chromosome. If it is defined as 0, the load is not applied to the loading point. If it is defined as 1, the load is applied to there.

Here, the calculation using the beam model above mentioned must be done in order to specify the loading region where the difference between composite stress obtained by using GA and it obtained by using influence-line becomes maximum. Span length of beam (L= 30m) and parameter ($n^2 = 1.5$) is used as the calculation condition. The results are shown in Table.1 and 2.

If the support condition is S-F, ratio(M_{max}/X) shows about 1.0 at neighborhood of both ends and shows smaller than 1.0 at neighborhood of center of span ($x/L=0.70 \sim 0.77$). Next, ratio (S_{max}/X) converges to about 1.0 at neighborhood of right end and shows smaller than 1.0 from center of span to neighborhood of left end ($x/L=0.13 \sim 0.73$). Here, X is composite stress obtained by using GA. M_{max} is a composite stress obtained by using bending moment and shearing force when bending moment shows maximum. S_{max} is a composite stress obtained by using bending moment and shearing force when shearing force shows maximum. And M_{max} and S_{max} are composite stress which is adapted in J.S.H.B.

On the other hand, in the case of F-F, the part from center of span to right end is considered. Ratio(M_{max}/X) shows smaller than 1.0 from ($x/L=$)0.77 to 0.87 and ratio(S_{max}/X) shows smaller than 1.0 from ($x/L=$)0.50 to 0.80. Therefore, the calculation is done regarding to 0.73L in the case of S-F and 0.80L in the case of F-F. The loading situation indicating M_{max} , S_{max} and quasi-maximum value obtained by using GA is shown in Table.3. Consequently, the quasi-maximum value of composite stress obtained by using GA can be recognized that the loading situation is close to full loading situation. From the above reason, the full loading situation is also calculated at the same time.

Now, the calculation using GA with varying n^2 is done on

condition that the span length is set to 15m,30m and the observing point is set to 11m,22m in the case of S-F. And in the case of F-F, the observing point is set to 12m,24m. These results are shown in Table.5 to 8.

Here, the parameters used in these tables are as follows;

$$X: \{(M_b)^2 + n^2 (S_b)^2\}^{1/2} (= \text{quasi maximum value given by GA})$$

$$n^2: (Z \sigma_y / \tau_y)$$

M_b : bending moment obtained by using GA

S_b : shearing force obtained by using GA

Table.5 to 8 shows the following things.

(a) Composite stress obtained by using GA is almost in accordance with the results using influence-line on condition that n^2 is 0.0 (bending moment shows maximum and effectiveness of shearing force hardly exists) and n^2 is very large (shearing force shows maximum and effectiveness of bending moment hardly exists). Therefore, it can be concluded that boundary condition (n^2 is 0 and n^2 is very large) of the problem is gratified.

(b) The region where ratio (FL/X) is greater than 1.0 digresses from the region where both M_{max}/X and S_{max}/X are smaller than 1.0. Here, FL indicates a solution on condition that full loading situation is applied and X indicates solution of GA.

(c) The maximum values of both (M_{max}/X) and (S_{max}/X) has 1.0. M_{max}/X is 1.0 when parameter n^2 is close to 0.0 and it diminishes according to n^2 being smaller. Therefore, it can be recognized that the idea of J.S.H.B in the case that these are larger than 1.0 has adequacy. However, the idea is not always correct because both values are smaller than 1.0 when n^2 has medium value.

As a result, all behaviors are marshaled as follows;

(a) Both influence-lines of bending moment and shearing force at one observing point have inversion region. That is to say, it can be specified that the region exists between 0.7L and 0.8L from cramped support end.

(b) $n^2 (= \{Z \sigma_y / \tau_y\}^2)$ calculated by using bending and shearing stiffness is a basic parameter in the calculation. And it gives much influence to the result.

(c) It can be considered that the influence of support condition is large. In the case of having S-S condition, the region where

the influence of bending moment and shearing force recoils don't exist. Therefore, such problem discussing so far doesn't occur. Furthermore, it is considered that the non-deterministic beam which the both ends have cramped support remarkably makes the problem occur. The non-deterministic beam which one side has simple support and other side has cramped support shows intermediate behavior. And it has a large relationship with the stiffness of the part of fixed end, that is to say, the magnitude of stiffness of spring. For example, in the case of rahmen type bridge which the girder and pier are rigidly connected, the both ends of girder are supported by piers. It is assumed that both ends of the non-deterministic beam are fixed by different stiffness of spring. That is to say, the evaluation method of composite stress depends on the variant of stiffness of spring. (d) The region where the ratio (FL/X) of composite stress on full loading situation and quasi-maximum value has 1.0 deviates from the region where both of M_{max}/X and S_{max}/X become smaller than 1.0. Therefore, it should also sufficiently be considered about the full loading situation.

4. Recommendation of simplified calculation method

Based on the above discussion, composite stress comprised of normal stress and shear stress can be obtained in accordance to the following procedure.

(a) Shearing force and bending moment at M_{max} , S_{max} and full loading situation should be calculated.

(b) Composite stress is calculated by using equation(8).

(c) One of the loading situations which composite stress has maximum value is selected.

(d) A premium value β is multiplied to the maximum value.

The concrete equation is expressed as follows:

$$C = \{M_c^2 + (Z \sigma_y / A \tau_y)^2 \cdot S_c^2\}^{1/2} / (Z \sigma_y) \cdot \beta \quad (8)$$

M_c : each bending moment at M_{max} , S_{max} and full loading situation

S_c : each shearing force at M_{max} , S_{max} and full loading situation

Table.4 Variation of composite stress due to parameter ($n^2/L^2, L=15m$)(both ends have cramped support)

$G=n^2/L^2$	M_{max}/X	S_{max}/X	FL/X	Max
0.0000	1.02	0.62	0.91	1.02
0.0018	1.01	0.63	0.98	1.01
0.0036	0.96	0.76	0.99	0.99
0.0053	0.92	0.83	0.99	0.99
0.0067	0.90	0.86	0.99	1.00
0.0071	0.89	0.87	1.00	0.99
0.0080	0.88	0.89	0.99	0.99
0.0089	0.87	0.91	0.99	0.99
0.0098	0.86	0.92	0.99	0.99
0.0107	0.85	0.93	0.99	0.99
0.0124	0.83	0.95	0.99	1.00
0.0142	0.82	0.97	1.00	0.99
0.0160	0.81	0.98	0.99	1.00
0.0178	0.80	0.99	1.00	0.99
0.0222	0.77	1.00	0.99	1.00

Span length: =15m, Observing point:S=12m
X:Quasi-mximum value
 M_{max} :Composite stress comprised of bending moment and shearing force at showing maximum bending moment
 S_{max} :Composite stress comprised of bending moment and shearing force at showing maximum shearing force
X:Maximum search value
FL:Full loading situation
Max:Maximum absolute value
Max=MAX{ M_{max} , S_{max} , FL}

Table.5 Variation of composite stress due to parameter ($n^2/L^2, L=30m$) (both ends have cramped support)

$G=n^2/L^2$	M_{max}/X	S_{max}/X	FL/X	Max
0.0000	1.00	0.59	0.57	1.00
0.0006	1.01	0.57	0.77	1.01
0.0009	1.00	0.69	0.85	1.00
0.0011	1.00	0.76	0.89	1.00
0.0013	0.99	0.82	0.93	0.99
0.0017	0.97	0.88	0.97	1.00
0.0020	0.95	0.92	1.00	1.00
0.0022	0.92	0.93	1.00	1.00
0.0024	0.90	0.94	1.00	1.00
0.0027	0.88	0.95	1.00	1.00
0.0031	0.85	0.97	1.00	1.00
0.0036	0.82	0.97	1.00	1.00
0.0040	0.79	0.98	1.00	1.00
0.0044	0.77	0.99	1.00	1.00
0.0056	0.73	1.00	1.00	1.00

Span length:L=30m, Observing point:S=24m
X:Quasi-maximum value
 M_{max} :Composite stress comprised of bending moment and shearing force at showing maximum bending moment
 S_{max} :Composite stress comprised of bending moment and shearing force at showing maximum shearing force
X:Maximum search value
FL:Full loading situation
Max:Maximum absolute value
Max=MAX{ M_{max} , S_{max} , FL}

Table.6 Variation of composite stress due to parameter ($n^2/L^2, L=15m$) (Left end has simple support and other end has cramped support)

$G=n^2/L^2$	M_{max}/X	S_{max}/X	FL/X	Max
0.00000	1.00	0.12	0.46	1.00
0.00044	1.00	0.57	0.70	1.00
0.00089	0.99	0.76	0.85	0.99
0.00133	0.98	0.90	0.96	0.98
0.00156	0.96	0.94	0.99	0.99
0.00178	0.92	0.95	0.99	0.99
0.00222	0.86	0.97	1.00	1.00
0.00444	0.70	1.00	1.00	1.00
0.00889	0.59	1.01	1.00	1.01
0.01333	0.53	1.00	0.98	1.00
0.04444	0.46	1.00	0.97	1.00

Span length:L=15m, Observing point:S=11m
X:Quasi-maximum value
 M_{max} :Composite stress comprised of bending moment and shearing force at showing maximum bending moment
 S_{max} :Composite stress comprised of bending moment and shearing force at showing maximum shearing force
X:Maximum search value
FL:Full loading situation
Max:Maximum absolute value
Max=MAX{ M_{max} , S_{max} , FL}

Table.7 Variation of composite stress due to parameter ($n^2/L^2, L=30m$)(left end has simple support and other end has cramped support)

$G=n^2/L^2$	M_{max}/X	S_{max}/X	FL/X	Max
0.00000	1.00	0.11	0.47	1.00
0.00056	1.00	0.62	0.75	1.00
0.00111	0.99	0.83	0.91	0.99
0.00167	0.94	0.94	1.00	1.00
0.00222	0.86	0.96	1.00	1.00
0.00278	0.80	0.97	1.00	1.00
0.00333	0.76	0.98	1.00	1.00
0.00389	0.73	0.99	1.00	1.00
0.00444	0.70	1.00	1.01	1.01
0.00500	0.68	1.00	1.00	1.00
0.00556	0.66	1.00	1.00	1.00

Span length:L=30m, Observing point:S=22m
X:Quasi-maximum value
 M_{max} :Composite stress comprised of bending moment and shearing force at showing maximum bending moment
 S_{max} :Composite stress comprised of bending moment and shearing force at showing maximum shearing force
X:maximum search value
FL:Full loading situation
Max:Maximum absolute value
Max=MAX{ M_{max} , S_{max} , FL}

Z: section coefficient

σ_s : allowable normal stress

τ_s : allowable shear stress

A: section area

β : premium factor

C: composite stress

The results calculated with above equation are shown in Table.4 to 7. It is recognized that these shows values of 99% ~100% compared with quasi-maximum X. Therefore, the equation can evaluate the safe composite stress by setting the adequate β value to the equation.

5. Conclusion

Conventional calculation method using influence-line can't obtain composite stress because it is obtained by squaring bending moment and shearing force. And it has always positive value.

The simplified calculation method based on businesslike judgment is recommended in J.S.H.B but it can't regretfully offer maximum value.

This paper presents a calculation method which can offer maximum composite stress by using GA and shows how J.S.H.B evaluates composite value at present by solving some simple examples.

As a result, it is apparent that the calculation method adapted by J.S.H.B underestimates composite stress although the amount of underestimation which depends on the support condition of beam at both ends and parameter n^2 is not so large. But it is difficult to apply GA for all practical design in order to solve these problems. Therefore, this paper recommends a practical simplified calculation method which bridge designer can easily use.

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