

Studying the evolutions of finite river delta coastlines

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1. INTRODUCTION

Larson et al. (1987) introduced an analytical solution to rapidly and economically study the formation of a river delta. However, this solution is applicable for infinite river delta shorelines. Therefore, Duy et al. (2018) provided another analytical solution which can be used to examine the formation of a river delta with finite shorelines. Although the solution provided by Duy et al. (2018) was already validated using experimental data, no application of this solution to a real case study has been made. Therefore, an application of the model provided by Duy et al. (2018) will be performed based on shoreline positions obtained in Funatsu River delta, Lake Inawashiro.

2. STUDY AREA AND DATA COLLECTION

This study focuses on a finite shoreline on the left of Funatsu River mouth in Lake Inawashiro (Figure 1). A series of aerial and satellite images from 1982 to 2015 was utilized. All the images were rectified to a same coordinate system. The image analysis method in this study was already presented by Duy et al. (2016).

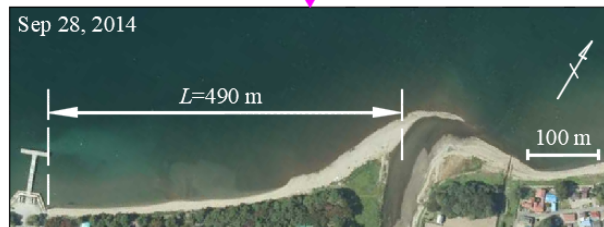
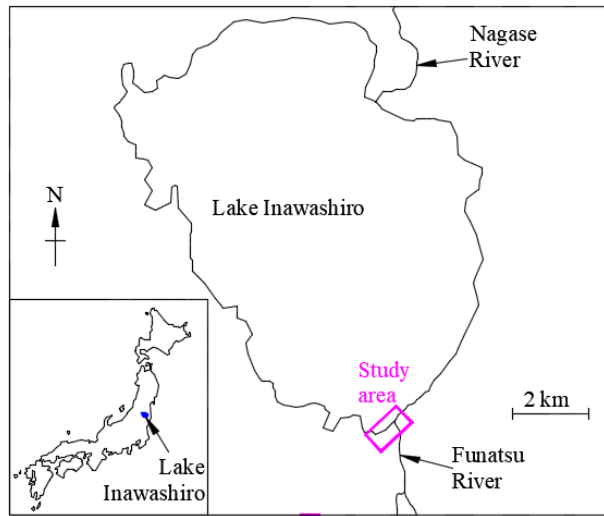


Figure 1. Lake Inawashiro and the finite shoreline on the left of Funatsu River mouth in Lake Inawashiro.

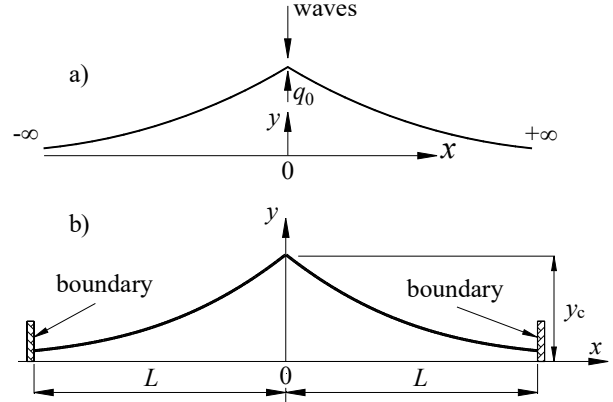


Figure 2. Schematic diagram of (a) infinite delta shorelines and (b) finite delta shorelines.

3. ANALYTICAL SOLUTION FOR THE DEVELOPMENTS OF RIVER DELTAS

Analytical solutions for developments of the infinite and finite shorelines as schematized in Figure 2 can be expressed as:

For infinite delta shorelines (Larson et al., 1987) (Figure 2a):

$$y = \frac{q_0}{D} \sqrt{\frac{t}{\pi \varepsilon}} e^{-x^2 / (4 \varepsilon t)} - \frac{q_0}{D} \frac{|x|}{2 \varepsilon} \operatorname{erfc} \left(\frac{|x|}{2 \sqrt{\varepsilon t}} \right) \quad (1)$$

in which, q_0 is the sediment supply rate from the river, $D = D_B + D_C$ (D_B : berm height, D_C : depth of closure), ε is the diffusion coefficient, x is the alongshore distance, y is the offshore distance, t is the time, erfc is the complementary error function.

For the development of finite delta shorelines (Figure 2b), Duy et al. (2018) provided:

$$y = \frac{q_0}{2 \varepsilon D L} \left[\frac{x^2}{2} - L|x| + \frac{L^2}{3} + \varepsilon t - \frac{2L^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-n^2 \pi^2 \frac{\varepsilon t}{L^2}} \cos \left(\frac{n \pi x}{L} \right) \right] \quad (2)$$

in which, L is the shoreline length, y_C is the maximum shoreline position determined as a distance between the x -axis and the delta's tip.

4. RESULTS AND DISCUSSION

Figure 3 shows the evolution of the Funatsu River delta from 1982 to 2015 and the coordinate system used in the analysis. From several photos in Figure 3, it can be seen that there is no delta shape in 1982 at Funatsu River mouth. Therefore, the year 1982 is used as the

initial year from which the delta started to form owing to sediment supply from the river. Using the “delta-fitting” method presented by Duy et al. (2016) and the equation provided by Duy et al. (2018), the parameters required for calculating the development of the Funatsu River delta are obtained in Table 1. Figure 4 shows the final result of the fitting process. In which, the values of q_0 and ε in Table 1 are changed to calculate different shoreline positions using Eq. (2). The theoretical shoreline positions are compared with the measured shoreline in 1982. The root-mean-square error (RMSE) is calculated and the fitting process will stop when the smallest value of RMSE is obtained. In this case, RMSE=1.92 m.

After confirming the values of ε and q_0 , the parameters in Table 1 are used to calculate the development of the Funatsu River delta. In this step, the calculation is done using both (i) the analytical solution with the effect of the boundary (Eq. 2) and (ii) the solution provided by Larson et al. (1987) (Eq. 1). The shoreline evolutions near the boundary ($x=-480$ m) are compared between the theoretical results and the measured data to see the effect of the boundary as shown in Figure 5. As can be seen from this figure, the new solution (blue line) shows better agreement with the measured shoreline positions near the boundary ($x=-480$ m). This result indicates that the analytical solution provided by Duy et al. (2018) (Eq. 2) is applicable for studying the development of finite river delta shorelines.

5. CONCLUSION

The analytical solution provided by Duy et al. (2018) has been used together with measured shorelines in Funatsu River delta to investigate the time evolutions of the finite river delta shoreline on the left of Funatsu River mouth. From the analysis results, it can be said that the analytical solution provided by Duy et al. (2018) is suitable to examine the developments of finite river delta shorelines.

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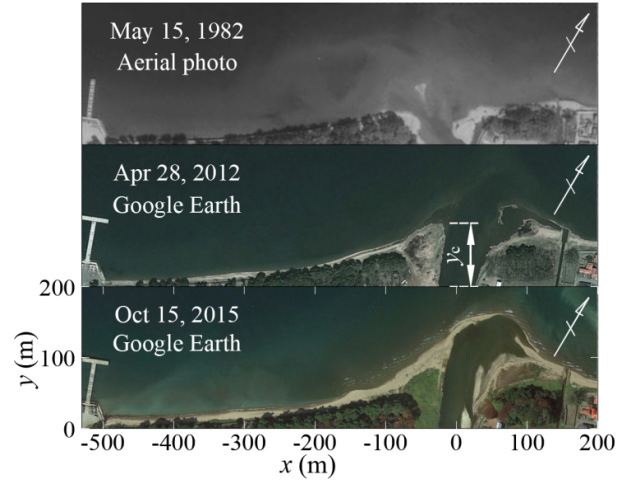


Figure 3. Evolution of Funatsu River delta and the coordinate system used in the analysis.

Table 1. Parameters used for calculating the development of Funatsu River delta.

Formation time (from 1982 to 2012)	$t_0=30$ years
Depth of closure (Fujita and Tanaka, 2004)	$D_C=1.36$ m
Beach length	$L=490$ m
Maximum shoreline position	$y_C=84$ m
Diffusion coefficient	$\varepsilon=3$ m ² /day
Sediment supply from the river	$q_0=1,350$ m ³ /y

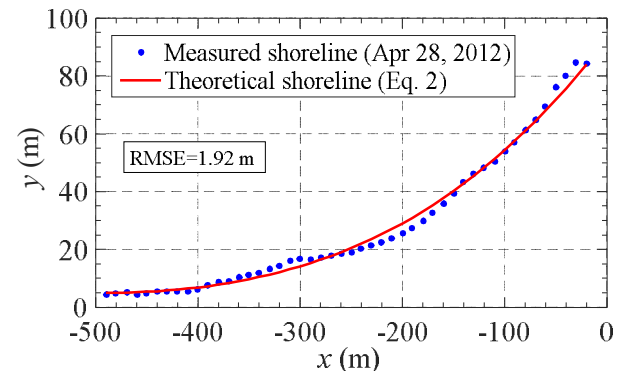


Figure 4. Fitting the measured shoreline and the theoretical shoreline to estimate q_0 and ε .

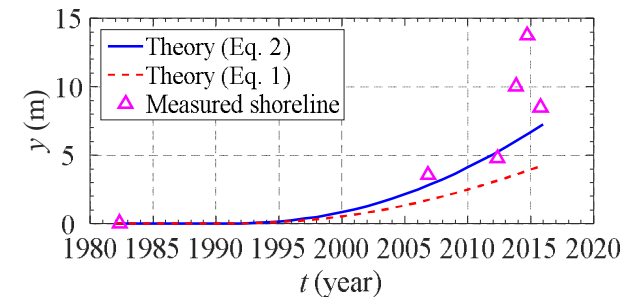


Figure 5. Shoreline evolutions at $x=-480$ m.