MOMENTUM BALANCE UNDER BREAKING SOLITARY WAVE RUNUP

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1. INTRODUCTION

Tsunami is well known as a major threat to coastal area. Although inundation area is a major concern, yet coastal morphology may change drastically due to a single tsunami event, as it had been shown in the case of The Great East Japan 2011 tsunami. Therefore it is important to understand the process of tsunami wave runup.

Solitary wave has been widely employed in tsunami wave studies due to their similarity. Synolakis (1986) conducted a series of laboratory experiment for solitary wave runup, and provided analytical results, referred to as the canonical problem. The study is commonly used for benchmarking numerical model. Nevertheless, the runup mechanism has not been widely examined.

Simultaneous Coupling Method (SCM) was developed to apply boundary layer theory in wave run up modeling. Recent development has broadened the range of SCM applicability, covering breaking wave simulation (Adityawan and Tanaka, 2011b). It had been used to investigate bed stress under solitary wave.

In this study, SCM is applied to assess momentum balance under breaking solitary wave runup. Furthermore, bed stress significance domain is investigated.

2. METHOD

The governing equations used in the model are SWE and k−ω equation. Both models are coupled and solved simultaneously, hence its name as Simultaneous Coupling Method (SCM). The basic idea of SCM is to upgrade the SWE model by replacing the Manning approach with a more accurate method to assess the bed stress term within the momentum equation. The commonly used Manning approach will be replaced by direct approach of bed stress in the near bed region using k−ω model.

The velocity (U) obtained from the SWE model is applied as the free stream velocity boundary condition in the k−ω model. Furthermore, the bed stress (τ_o) obtained from the k−ω model is applied in the momentum equation of SWE model. The process continues until the end of simulation time. Further detail regarding coupling method and its verification can be found in previous study (Adityawan and Tanaka, 2010).

SCM applies shock capturing scheme, FORCE MUSCL (Mahdavi and Talebbeydokhti, 2009), to extend SWE capability for breaking wave computation. This scheme employs slope limiter function to handle shock and discontinuity due to wave breaking. The FORCE-MUSCL scheme is a combination of several schemes with the application of slope limiter function. The scheme uses TVD Runge Kutta for solving time derivatives to achieve higher stability with non-fix times step, following the Courrant number criteria 0.4-0.8. Implementation of the scheme had been shown to enhanced the capability of SCM for breaking wave simulation.

In this study, the model is used to investigate momentum balance under breaking solitary wave run up (Synolakis, 1986).

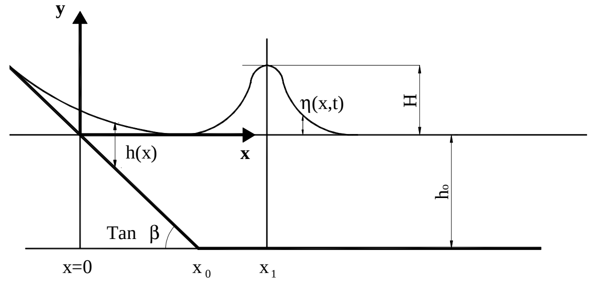

Figure 1 Solitary wave run up on sloping beach

The model setup in x y coordinate system for the case is shown in Fig.1. Here, η is the water level. H is the wave height, h is the water depth with h_o is the initial water depth. The beach slope ($Tan\ \beta$)is 1:20. The breaking wave condition is given for the ratio of H h_o = 0.3. The wave Reynolds number (Re) is calculated by the following equation.

$$R_e = \frac{U_c a_m}{\nu} \tag{1}$$

where a_m is half of stroke of water displacement and ν is viscosity. U_c is the incoming wave maximum velocity given as.

$$U_c = (g/h_0)^{0.5} H \tag{2}$$

where g. is the gravity. The simulated case Re is 18,000 which is bellow the transition condition criteria ($2\times10^5<Re<5\times10^5$), as proposed by Sumer et al. (2010). Hence, the condition for all cases is laminar boundary layer. It should be noted here that higher Re values may be achieved as the wave travels to the shallower area.

Momentum balance is investigated in each case based on the momentum equation in SWE as follow.

$$\underbrace{\frac{\partial U}{\partial t}}_{A} + \underbrace{U\frac{\partial U}{\partial x}}_{B} + \underbrace{g\frac{\partial (h+z_b)}{\partial x}}_{C} + \underbrace{\frac{\tau_o}{\rho h}}_{D} = 0 \tag{3}$$

with t is time, ρ is fluid density, z_b is the bed elevation. The terms are analyzed separately with A is the local acceleration, B is the convective acceleration, C is the pressure gradient and D is the bed stress.

3. RESULTS AND DISCUSSION

The evolutions of water surface and velocity are shown in Fig.2. η^* is the non-dimensional elevation (η/h_o) and U^* is the non-dimensional velocity (U/U_c). The slope starts at x^*=20, where x^* is the non-dimensional distance (x/h_o). Shoreline position corresponds to x^*=0. The maximum run up height occurs around t^*=54. t^* is the non-dimensional time ($t(g/h_o)^{0.5}$). Run down movement has started even though it has not reached the maximum runup (t^*=48). Here, part of the flow moves in seaward direction, marked with negative velocity. Furthermore, the effect of the wave propagation to the velocity around the shoreline seems to diminish after breaking (x^*=10).

Keywords: momentum balance, solitary wave, runup, breaking wave, numerical simulation
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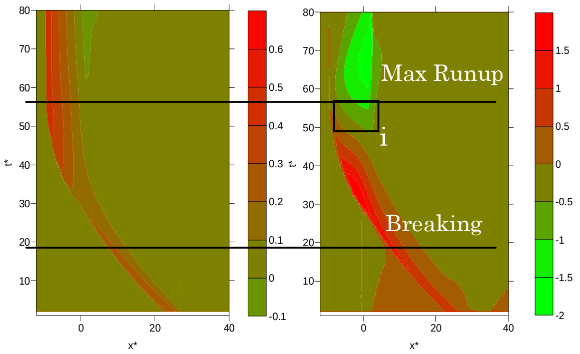
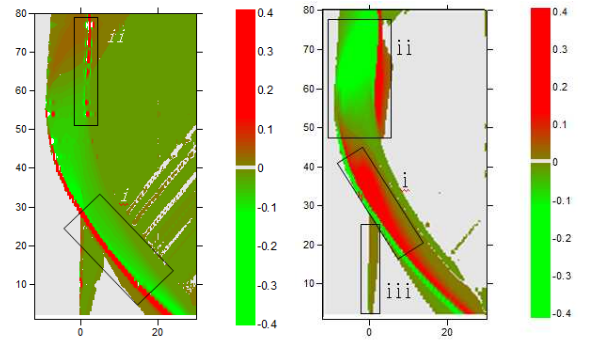
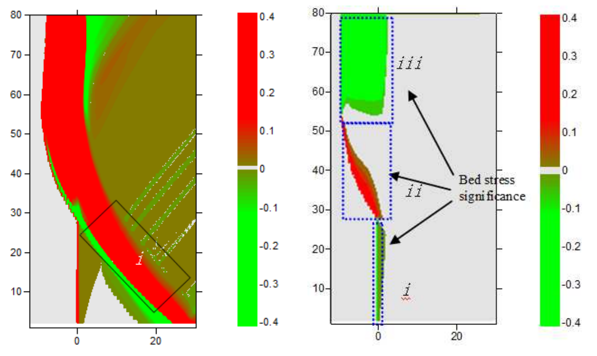


(a) *η** (b) *U**

Figure 2 Water surface and velocity evolution of breaking solitary wave run up



(a) *Local acceleration (m/s²)* (b) *Convective acceleration (m/s²)*



(c) *Pressure gradient (m/s²)* (d) *Bed Stress (m/s²)*

Figure 3 Momentum balance evolution

The momentum terms (A, B, C, and D) evolution are shown in Fig. 3 (a) to (d), respectively. In the deeper area, local acceleration and pressure gradient term have significant effect on the process (Fig. 3 (a) and (c)), with some influence from the convective acceleration (Fig. 3 (b)). The convective acceleration term B is not negligible as in the non breaking wave case (Adityawan and Tanaka, 2011a). It is also noted that around the shoreline during the run down process the local acceleration term A fluctuates (Fig. 3 (a) box ii) in respond to the hydraulic jump-like behavior as shown in previous study (Adityawan and Tanaka, 2011b). Moreover, in the case of breaking wave, nonlinear acceleration gives significant contribution to the overall momentum balance. The effect of this term is greatly emphasized during run up after breaking wave occurs (Fig.3 (b) box i) and during the run down around the shoreline (Fig. 3 (b) box ii).

Bed stress is significant during the early stage of run up near the shoreline area, run up, and run down as shown in Fig. 3 (d). Bed stress influence on the process starts at the early stage of run up, before the main body of the wave reaches the shoreline. It starts immediately up to t*=29 (Fig. 3 (d), box (i)). It is also observed that the occurrence of bed stress in the early stage of run up coincides well with the occurrence of convective acceleration (Fig. 3 (b) box iii). As the wave approaches the shoreline and run up occurs. Bed stress becomes more important in the process as shown in Fig 3 (d) box ii. The influence is greatly emphasized during run down process as shown in Fig 3. (d) box iii. It is noted here that the bed stress transformation from run up motion to run down motion is instant.

3. CONCLUSSIONS

Investigation of momentum balance under non breaking solitary wave run up has been conducted. The momentum balance during the wave propagation in the deeper area is mainly governed by the local acceleration and pressure gradient with some influence from the convective acceleration, with no effect from bed stress term.

Based on the bed stress significance on the runup process, 3 phases were classified in the breaking wave case. The first phase occurs around the shoreline, although the main body of the wave is still in the deeper area. The second phase occurs during runup. The last phase occurs during run down. This information can be used to simplified runup simulation, in which, the domain with no significant bed stress may use simplified method for bed stress estimation. Thus the overall efficiency of the simulation can be increased.

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