NUMERICAL SIMULATION OF BREAKING SOLITARY WAVE RUN UP

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1. INTRODUCTION

Shallow water equation (SWE) has been widely used for numerical simulation of solitary wave run up. Recent development has enhanced SWE model by coupling with k-ω model (Adityawan and Tanaka, 2010). However, breaking wave simulation using SWE is still a challenge since the equation itself does not have the appropriate terms. Boussinesq-type equations can be used to simulate breaking wave condition (Sato and Kabiling, 1994) by introducing a constant value of eddi viscosity in the shallower area. A more detail process of breaking wave is given by RANS type of model with turbulence closure, i.e. NEWFLUME (Lin et al., 1999). Nevertheless, these methods are far more complex than the SWE.

Breaking wave condition and discontinuity can be applied numerically to SWE model without the use of additional terms. SWE is commonly solved using finite difference scheme. Nevertheless, volume conservation problem arises due to discontinuities. A finite volume method has the advantage of solving the SWE and maintaining the volume conservation. The Godunov-type scheme with Riemann solver is known for its conserving and shock-capturing capability. A modification of Godunov-type scheme leads to a second order accuracy in space such as MUSCL scheme.

In this study, MUSCL and FORCE (Mahdavi and Talebbeydokhti, 2009) scheme is used to solve SWE. The scheme will provide numerical dissipation and shock capturing. Therefore, additional terms to accommodate breaking wave are not required. The method is used to simulate breaking solitary wave run up on a sloping beach. The results are compared to experimental data and a finite difference based SWE model.

2. MODEL DEVELOPMENT

2.1. *Governing Equation*

SWE consist of the continuity and momentum equation which can be written as.

$$\frac{\partial V}{\partial t}+\frac{\partial F}{\partial x}=S \tag{1}$$

The vector of conserved variable *V*, the flux variable *F* and the source term *S* are defined as

$$V=\begin{bmatrix} h \\ hU \end{bmatrix}; F(V)=\begin{bmatrix} hU \\ hU^2+\frac{1}{2}gh^2 \end{bmatrix}; S(V)=\begin{bmatrix} 0 \\ gh(S_0-S_f) \end{bmatrix} \tag{2}$$

where *t* is time, *x* is distance, *h* is the water depth, *U* is the depth averaged velocity, *g* is the gravity acceleration and S_o and S_f are the bed slope and friction slope, given as follows.

$$S_0=-\frac{dz_b}{dx}; S_f=\frac{n^2U|U|}{h^{4/3}} \tag{3}$$

where z_b is the bed elevation and *n* is the Manning roughness coefficient.

2.2. *Numerical Methods*

The governing equation, Eq,(1), can be rearranged into,

$$\frac{\partial V}{\partial t}=\underbrace{-F'(x_i)+S_i}_{L} \tag{4}$$

where *L* notates the right hand side of the equation.

Spatial derivation of flux F is approximated by conservative difference as

$$F'(x_i)=\frac{F_{i+(1/2)}-F_{i-(1/2)}}{\Delta x} \tag{5}$$

where Δx is the cell size.

The conservative variables (*h* and *hu*) discretization is conducted using the MUSCL scheme. The Surface Gradient Method (SGM) is applied, by calculating free surface elevation instead of depth to accommodate the effect of bed topography.

$$h=\eta-z_b \tag{6}$$

where η is the surface elevation.

The following discretization of conservative variable *h* is given as an example. The left hand side (-) and right hand side (+) of the interface are evaluated using linear reconstruction.

$$(\eta)^-_{i+1/2}=(\eta)_i+\frac{1}{2}\delta_i(\eta);(\eta)^+_{i+1/2}=(\eta)_{i+1}-\frac{1}{2}\delta_{i+1}(\eta) \tag{7}$$

The limited slopes from the above are as follows.

$$\delta_i(\eta)=\Psi_i\Delta_i(\eta);\Delta_i(\eta)=\frac{\Delta_{i+1/2}(\eta)+\Delta_{i-1/2}(\eta)}{2} \tag{8}$$

where ψ is the slope limiter which is given by a Superbee-type-non-linear slope limiter (Toro, 2001).

$$\Psi_i=\begin{cases} 0 & r_i\le 0 \\ 2r_i & 0\le r_i\le 1/2 \\ 1 & 1/2\le r_i\le 1 \\ \min(r_i,\frac{2}{1+r_i},2) & else \end{cases} \tag{9}$$

where *r* is the ratio of successive jumps in the conservative *h*

$$r_i=\frac{\Delta_{i-1/2}(hU)}{\Delta_{i+1/2}(hU)}; r_i=\frac{\Delta_{i-1/2}(\eta)}{\Delta_{i+1/2}(\eta)} \tag{10}$$

with

$$\Delta_i(\eta)=\frac{\Delta_{i+1/2}(\eta)+\Delta_{i-1/2}(\eta)}{2} \tag{11}$$

$$\Delta_{i+1/2}(\eta)=(\eta)_{i+1}-(\eta)_i;\Delta_{i-1/2}(\eta)=(\eta)_i-(\eta)_{i-1}$$

The depth at the evaluated locations can be calculated using Eq.(6). Same steps are conducted to evaluate conservative variable *hU*, without SGM.

The numerical flux (*F*) is evaluated using FORCE scheme which is a combination of Lax-Friedrichs (LF) and Lax-Wendroff (LW) scheme.

$$F^{FORCE}_{i+(1/2)}=\frac{1}{2}\left(F^{LF}_{i+(1/2)}+F^{LW}_{i+(1/2)}\right) \tag{12}$$

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Lax-Friedrichs flux (F^{LF}) is given as

$$F_{i+(1/2)}^{LF} = \frac{1}{2}\left(F_{i+(1/2)}^{-} + F_{i+(1/2)}^{+}\right) - \frac{1}{2}\frac{\Delta x}{\Delta t}\left(V_{i+(1/2)}^{+} - V_{i+(1/2)}^{-}\right) \quad (13)$$

and Lax-Wendroff flux (F^{LW}) is given as

$$F_{i+(1/2)}^{LW} = F(V_{i+(1/2)}^{LW}) \quad (14)$$

$$V_{i+(1/2)}^{LW} = \frac{1}{2}\left(V_{i+(1/2)}^{-} + V_{i+(1/2)}^{+}\right) - \frac{1}{2}\frac{\Delta t}{\Delta x}\left(F(V_{i+(1/2)}^{+}) - F(V_{i+(1/2)}^{-})\right)$$

where, V^{LW} is the intermediate state of conserved variables and Δt is the time step interval.

Time integration is solved using TVD Runge-Kutta.

$$V_i^{(1)} = V_i^t + \Delta t \times L(V_i^n)$$

$$V_i^{(2)} = 3/4 \times V_i^t + 1/4 \times V_i^{(1)} + 1/4 \times \Delta t \times L(V_i^{(1)}) \quad (15)$$

$$V_i^{t+1} = 3/4 \times V_i^t + 2/3 \times V_i^{(2)} + 2/3 \times \Delta t \times L(V_i^{(2)})$$

Time step is not constant and evaluated using the Courant-Friedrichs-Lewy (CFL) stability given by

$$\Delta t = c\frac{\Delta x}{\left(|U_i| + \sqrt{gh_i}\right)} \rightarrow 0 < c \le 1 \quad (16)$$

where c is the Courant number.

Wet dry moving boundary condition is applied by giving a minimum depth (h_{min}) to all physically dry cell. Water depth bellow the minimum depth will be assigned with zero momentum (dry).

3. RESULTS AND DISCUSSION

The model is applied to an experimental case (Synolakis, 1986) shown in Figure 1 with the ratio of the incoming wave height and water depth of 0.3 which corresponds to the breaking wave condition in the experiment.

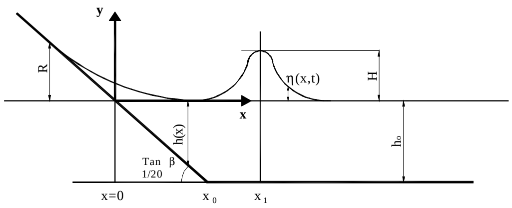

Figure 1. Wave setup.

Another SWE model, solved using finite difference Mac Cormack predictor corrector scheme (Adityawan, 2007) is also simulated for comparison. For both model, we used the same grid system. The model domain was divided into 470 grids in x direction with grid spacing equals to $0.2\Delta x^*$.

The results are shown in Figure 2. It is observed that the FORCE MUSCL scheme provides better comparison to experimental data. The Mac Cormack based model fails to provide an accurate water profile. It is observed that the finite difference scheme gives an unrealistic profile at the early stage of run up, noted with a pointy shape at the wave top. This also occurs as the breaking wave moves over a dry bed.

4. CONCLUSION

We have shown a type of finite volume model, based on FORCE MUSCL scheme, which we applied to solve SWE. The model was used to simulate breaking solitary wave run up on a sloping beach. Comparison between the finite volume scheme, finite difference scheme and experimental data has shown the superiority of the finite volume method compares to the finite difference method. Hence, the model would significantly improve the accuracy of SWE model in wave breaking simulation without the necessities to modify the equation itself.

The model can be further enhanced by coupling with a two equation model for assessing bed stress directly from the boundary layer.

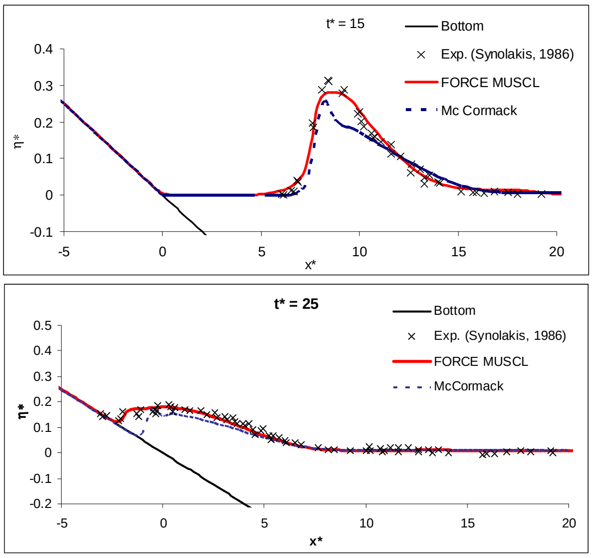

Figure 2. Simulation Result

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REFERENCES

Adityawan, M., B., (2007), 2d Modeling of Overland Flow Due to Tsunami Wave Propagation, *Master Thesis*, Institut Teknologi Bandung, Indonesia.

Adityawan, M., B., and Tanaka, H., (2010), Coupling between shallow water equation and k-ω model for simulating solitary wave run-up, *Annual Journal of Coastal Engineering*, B2-66(1), pp.91-95. *(in Japanese)*

Lin, P., Chang, K-A., and Liu, P. L-F., (1999), Runup and rundown of solitary waves on sloping beaches, *Journal of Waterway, Port, Coastal, and Ocean Engineering*, Vol. 125, No.5 , pp. 247-255.

Mahdavi, A., and Talebbeydokhti, N., (2009), Modeling of non-breaking and breaking solitary wave run-up using FORCE-MUSCL scheme, *Journal of Hydraulic Research*, Vol. 47, No. 4, pp. 476–485.

Sato, S., and Kabiling, M., B., (1994), A numerical simulation of beach evolution based on a non linear dispersive wave-current model, *Proc. 24th Conf. on Coast. Engrg.*, pp. 2557-2570.

Synolakis, C., E.,(1987), The run up of solitary waves, *Journal of Fluid Mechanics*, Vol. 185, pp.523-545.

Toro, E., F., (2001), Shock-capturing methods for free-surface shallow flows, *Wiley*, Chichester UK.