# UNCERTAINTY ANALYSIS OF WATER BALANCE DOWNSTREAM THE CORDILLERA REAL IN BOLIVIA

Tohoku UniversityGraduate StudentO Freddy SORIATohoku UniversityFellow MemberSo KAZAMA

# 1. INTRODUCTION

A relevant advance in surface and subsurface hydrological research is the growing interest on assessing the final user on the degree of belief of modeling products (Montanari et al., 2009), as a form to bridge the gap between research and practice. We assess the predictive uncertainty of a semi-distributed water balance model (Collick et al., 2009) through Monte Carlo-generated response surfaces (Beven, 2004). The aim is to investigate the runoff response of a remote poorly-gauged basin in the Andes of Bolivia. The basin is a spatially heterogeneous (differenced hydrological response along the main river stream), high-elevation watershed (large difference in altitude between upstream and downstream). The objective is not only to address the model predictive uncertainty, but also to analyze the relevance of the assessment to increase the contribution to the current knowledge.

#### 2. STUDY AREA AND DATA

The study basin (1471 km<sup>2</sup>) is in the remote highlands of the Cordillera Real (15.8 to 16.3S, tropical Andes), upper Beni River basin (subbasin of the Amazon River basin). The high-elevation of the Cordillera determines variations in altitude of 5,500 m a.s.l. within horizontal distances of 50 km in average, which defines the spatial heterogeneity in topography and climate. Monthly precipitation data at eight stations situated between 4800 and 1196 m a.s.l. is provided by a local hydropower generation company COBEE. Climatic data is from SENAMHI.

## 3. METHODS

#### 3.1 Monthly semi-distributed water balance model

The model is semi-distributed in horizontal buckets. A saturation-excess runoff response is the basis of the *perceptual model*. The *conceptual model* is described by Equation 1 (Collick et al., 2009), where S [L] is the soil water storage volume, t is time,  $\Delta t$  is time step, P [L/T] is rainfall intensity, Rse [L/T] is saturation excess runoff rate, Perc [L/T] is percolation, Ea [L/T] is the actual evapotranspiration obtained from the potential evapotranspiration PET multiplied by a fraction of rain days (raindays [days]).

$$S_t = S_{t-\Delta t} + (P - Rse - Ea - Perc)_t \Delta t \tag{1}$$

When P < Ea, the soil water depth above saturation  $S_E$  is zero. When P > Ea,  $S_E$  [L] (that becomes either *Rse* or *Perc*) is calculated with Equation 2 (Collick et al., 2009), where *Csc* [non-dimensional] calibrates

the threshold when surface runoff occurs (i.e., the difference between the maximum soil storage  $S_{Tmax}$  [L] and the soil storage at wilting point  $S_{wilt}$ ). *Cse* [non-dimensional] decides the proportion of water that is converted into *Perc* or *Rse* (Equations 3 and 4).

$$S_{E,t} = S_{t-\Lambda t} + (P - Ea)_t \Delta t - Csc(S_{T max} - S_{wilt})$$
(2)

$$Perc = Cse * S_E / \Delta t \tag{3}$$

$$Rse = (1 - Cse) * S_E / \Delta t \tag{4}$$

The contribution of *Perc* to the groundwater storage ( $S_{GW}$  [L]) is calculated with Equation 5. The contribution to groundwater flow over a unit of surface area ( $R_{GW}$  [L/T]) is calculated with a linear reservoir model (Equation 6), where k [non-dimensional] is the recession constant.

$$S_{GW,t} = S_{GW,t-\Delta t} + (Perc_t - R_{GW,t-\Delta t}) \cdot \Delta t$$
 (5)

$$R_{GW,t} = S_{GW,t-\Delta t} \left(1 - e^{-k_t}\right) / \Delta t \tag{6}$$

#### **3.2 Numerical experiments**

To assess the water balance model predictive uncertainty, the model parameters Csc,  $S_{Tmax}$ ,  $S_{wilt}$  and Cse are assumed uncertain. In addition, to assess the predictive performance of the water balance model under an imperfect measuring network, observed P, PET, and "observed" raindays are assumed uncertain. The assumption of input data as uncertain information may serve as a guide to assess the impacts of changes in climatic conditions; however, considering the complexity of such affirmation, the results are not strictly analyzed from such perspective.

Monte Carlo experiments are carried for the period September 1981-August 1982 (sample size 2048), assuming an ignorance on the model predictive response (uncertain variables follow uniform distributions). Response surfaces (Beven, 2004) are employed to analyze the outcomes. Having a heterogeneous study basin, the assessment is carried on representative buckets: Z1 (rainforest) and Z9 (sparse vegetation, shallow soil depths). Uncertainty bounds fall within a range +/- 20% in reference to "calibrated" and "observed" values shown in **Table 1**.

| <b>Table</b> | 1 | Calibrated | and | observed | values | (wet | months). |
|--------------|---|------------|-----|----------|--------|------|----------|
|              |   |            |     |          |        |      |          |

| Bucket;                            | Calibrated values         |            |                           |            | Observed values |                |                           |
|------------------------------------|---------------------------|------------|---------------------------|------------|-----------------|----------------|---------------------------|
| altitudinal<br>range<br>[m a.s.l.] | S <sub>wilt</sub><br>[mm] | Csc<br>[-] | S <sub>Tmax</sub><br>[mm] | Cse<br>[-] | P<br>[mm]       | PET<br>[mm]    | <i>raindays</i><br>[days] |
| Z1; 500-1000<br>Z9; 4001-450       | 10.0<br>(5.0              | 0.2<br>0.2 | 100.0<br>40.0             | 0.2<br>0.8 | 610.0<br>247.0  | 141.0<br>141.0 | 30.0<br>10.0              |

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Tohoku University, 6-6-06 Aoba, Sendai 980-8579, Japan. Tel & Fax: +81-22-795-7451

The uncertainty in model inputs is assessed for two observations: the wettest observation (January 1982) and the driest observation (May 1982). The wettest month is assumed a relevant uncertainty indicator of the model adequacy to represent catchment runoff responses, because during such wet period strong relationships between model response and hydroclimatic conditions occur (Yapo et al., 1996). On the other hand, the driest month is assumed a relevant indicator of the model inadequacy, because during such periods it is expected the lowest model predictive performance.

#### 4. RESULTS

The model predictive uncertainty varies according to the geographical location where the water balance (WB) is estimated. As suggested by the identifiability in *P* and *Cse* patterns (**Fig. 1**), the WB during wet periods is dominated in humid regions (Z1) by variations in precipitation *P*, whereas in arid regions (Z9) the watershed runoff response is dominated by saturation-excess runoff processes that constitute the basis of the perceptual model (conceptually described by parameter *Cse*). During dry periods, the relevance of the soil storage capacity (*Cse*) over the input *P* and evapotranspiration (*PET*) seems likely.

The *conceptual model* predictive uncertainty is assessed through the trends of inputs P and PET (Fig. 1). The identifiability in P trends, more emphasized in wet regions (Z1), suggests an adequate model performance during wet periods (January) and an uncertain behavior during low flows. During dry periods, such uncertainty is more relevant in humid





regions (Z1) than in arid regions (Z9), suggesting the need for additional field data when the modeler's interest is focused on low flows. The uncertainty in *PET* inferences is demonstrated to be higher in humid regions (Z1) in proportion to the amount of water available for those processes.

Parameters *Csc*,  $S_{Tmax}$ ,  $S_{wilt}$  are influential on initial conditions, as suggested by the scatter patterns drawn (*Csc* patterns in **Fig. 1** are representative for  $S_{Tmax}$  and  $S_{wilt}$ ). The latter mentioned behavior is in general desired in most models, because in that form the influence of calibrable parameters is minimized.

## 5. CONCLUSIONS

Two-dimensional response surfaces demonstrate the relevance of the input precipitation and the soil storage capacity in humid and arid regions where saturation-excess is assumed to be the most likely runoff mechanism. Thus, this research suggests that:

- The uncertainty contribution from an imperfect precipitation observation is likely to be as relevant as the uncertainty contribution from the conceptual model itself, especially during wet catchment states.
- In humid regions, the uncertainty in precipitation data is unlikely to propagate to other months, which does not seem to occur in arid regions, where the input data uncertainty contributions are likely to have an effect on subsequent time steps.
- From a modeler and a practitioner's perspective, to draw adequate perceptions of a system during dry states is likely to demand "specific" field data in addition to the information granted as sufficient to describe wet states of the catchment. The reason is the high predictive uncertainty in dry-catchment states compared to the uncertainty expected during wet catchment states.

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