SIMULTANEOUS COUPLING OF SHALLOW WATER EQUATION WITH k-ω MODEL FOR WAVE PROPAGATION AND RUN UP SIMULATION

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1. INTRODUCTION

The coastal morphology changes due to sediment transport are an important phenomenon in nature. The sediment transport process is influenced by the bed stress which is related to the boundary layer beneath the wave motion during propagation and run up. Thus, boundary layer study beneath the wave motion is very important to understand the sediment transport process and the wave motion itself.

Shallow water equation (SWE) has been widely used in wave run up and propagation modeling (Adityawan, 2007). However, the stress term in the equation is commonly approach by using Manning method. This method is inaccurate in predicting the bed stress behavior beneath the wave (Adityawan et al., 2009). It fails to simulate the effect of rapid deceleration due to the assumption of square relation between velocity and bed stress. Turbulent model is known for its good performance in assessing the bed stress. There are various types of turbulent model, such as k-ε and k-ω. Past studies have shown that k-ω model is considered to be more accurate (Suntoyo et al., 2008). Nevertheless, wave run up modeling by using turbulent model is time consuming and required various treatments oppose to the SWE model.

The objective of this study is to obtain an accurate model by applying the wave boundary layer theory in SWE to simulate wave motion. The coupling is conducted by replacing the conventional Manning method used for bed stress term calculation in the SWE model with the calculated bed stress from the turbulent model. The k-ω model is used to simulate the flow within the boundary layer to obtain a more accurate bed stress. In this study, a 1D Depth Averaged Shallow Water Equation model is coupled with a 2D Vertical k-ω model. The coupled model is applied for solitary wave run up on a sloping beach (Synolakis, 1987).

2. MODEL DEVELOPMENT

The governing equations are Shallow Water Equation and k-ω equation (Wilcox, 1988). Both models are integrated and calculated simultaneously. The models are calculated separately at each time steps however their results are intertwine, allowing simultaneous calculation. The velocity obtained from the SWE model is applied as the upstream velocity boundary condition in the k-ω model. Furthermore, the bed stress obtained from the k-ω model is applied in the SWE model. At the wave front, the bed stress is calculated from the momentum equation in the SWE as proposed by Elfrink and Fredsöe (1993) due to the very low water depth.

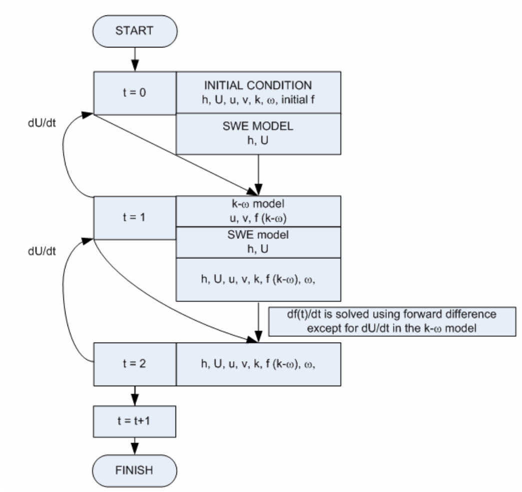

Figure 1. Model integration flow chart.

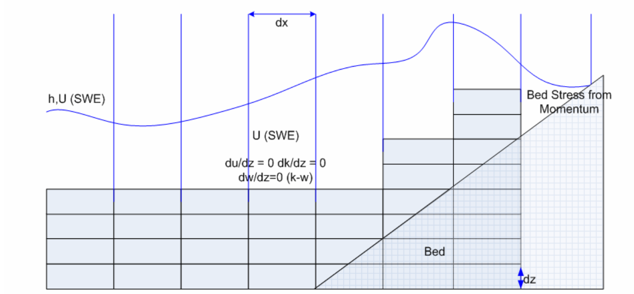

Figure 2. Model domain.

2.1. *SWE Model*

The SWE consists of the continuity equation and the momentum equation as follows.

$$\frac{\partial h}{\partial t}+\frac{\partial (Uh)}{\partial x}=0 \quad (1)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+gh\frac{\partial (h+z)}{\partial x}=-ghS_f \quad (2)$$

where h is the water depth, U is depth averaged velocity, t is time, g is gravity, z is the bed elevation, S_f is friction slope which can be approach as bed stress. Manning equation is commonly used to assess this parameter. The SWE governing equation is solved using McCormack predictor corrector scheme.

2.2. *k-ω Model*

The governing equation for the model is based on the Reynolds-averaged equations of continuity and momentum, which can be written as follows:

$$\frac{\partial u_i}{\partial x}=0 \quad (3)$$

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$$\rho\frac{\partial u_i}{\partial t}+\rho u_j\frac{\partial u_i}{\partial x_j}=\frac{\partial P}{\partial x_i}+(2\mu S_{ij}-\rho\overline{u_i^{'}u_j^{'}}) \tag{4}$$

where u_i and x_i denotes the mean velocity and location in the grid, u_i' is the fluctuating velocity in the x (i = 1) and y (i = 2) directions, P is the static pressure, ν is the kinematics viscosity, ρ is the density of the fluid, $\rho u_i^{'} u_j^{'}$ is the Reynolds stress tensor, and S_{ij} is the strain-rate tensor from the following equation:

$$S_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

The Reynolds stress tensor is given through eddy viscosity by Boussinesq approximation:

$$-\overline{u_i^{'}u_j^{'}}=\nu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{2}{3}k\delta_{ij} \tag{6}$$

with k is the turbulent kinetic energy and δ_{ij} is the Kronecker delta. The k–ω model equation is given as follows:

$$\frac{\partial k}{\partial t}+u_j\frac{\partial k}{\partial x_j}=\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta^* k\omega+\frac{\partial}{\partial x_j}\left[(\nu+\sigma^*\nu_t)\frac{\partial k}{\partial x_j}\right] \tag{7}$$

$$\frac{\partial \omega}{\partial t}+u_j\frac{\partial \omega}{\partial x_j}=\alpha\frac{\omega}{k}\tau_{ij}\frac{\partial u_i}{\partial x_j}-\beta\omega^2+\frac{\partial}{\partial x_j}\left[(\nu+\sigma\nu_t)\frac{\partial \omega}{\partial x_j}\right] \tag{8}$$

The eddy viscosity is given by:

$$\nu_t=\frac{k}{\omega} \tag{9}$$

The values of the closure coefficients are given by Wilcox (1988) as β= 3/40, β* = 0.09, α = 5/9, and σ = σ* = 0.5.

3. RESULTS AND DISCUSSION

The model is applied to an experimental case (Synolakis, 1987) shown in figure 3 with the ratio of wave height and water depth of 0.019. The results are shown in figure 4. It is observed that the new method gives better comparison to experimental data (fig. 4a).

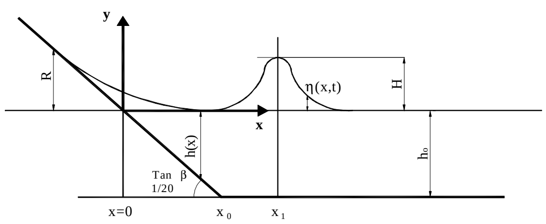


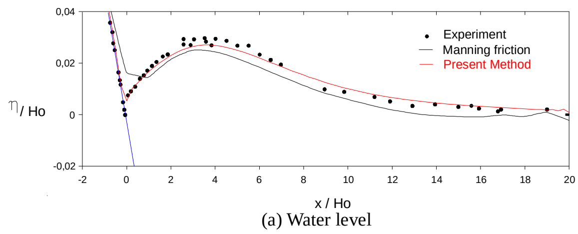
Figure 3. Wave setup.

The bed stress relation in the conventional manning method is assumed linear to the square of upstream velocity as shown bellow.

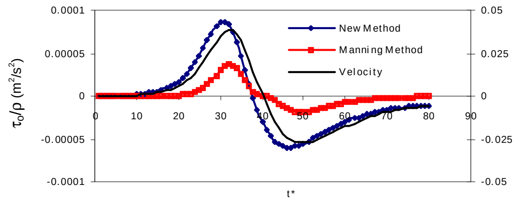
$$\frac{\tau_0}{\rho}=gn^2/R^{1/3}\times U\times|U| \tag{10}$$

where τ_0 is the bed stress, g is velocity, n is manning roughness value, R is the hydraulic radius or the ratio of area and perimeter.

The bed stress profile obtained from the calculation with the new method shows different profile especially during the deceleration phase. A slightly phase different of the bed stress peak is also observed, (fig. 4b)



(a) Water level



(b) Bed Stress and velocity at the beginning of the sloping beach

Figure 4. Simulation Result

4. CONCLUSION

A new method for simulating wave motion has been developed by coupling SWE model with k-ω model. This method has shown to give better comparison than the conventional Manning method commonly used in SWE model. This method also provides a better understanding of bed stress due to the wave motion. Thus, the model would be very useful in developing an accurate model for sediment transport analysis. Further development is required to enhance the model applicability.

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