SIMULATE SEDIMENT SCOUR IN RESERVOIR USING SMOOTHED PARTICLE HYDRODYNAMICS (SPH) METHOD

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1. INTRODUCTION

In the design of dams and reservoir, sediment transport in general or scour in particular is an important consideration. Sediment scoured from the reservoirs during floods increases the remaining sediment-storage capacity from which remains normal work condition. Thus, an accuracy numerical modeling has ability in prediction of hydrodynamics and local sediment scour problem in reservoir is necessary.

This paper presents a method for simulation sediment scour in reservoir using SPH method. The method is based on the SPH-description of the flow, coupled with the SPH-description of the interaction processes between water and solid boundary via advection diffusion equation. The method is tested with laboratory experimental data provided by CESI RICERCA.

2. NUMERICAL MODEL

In most previous models for sediment transport based on SPH method such as Gutfraind & Savage (1997), Falappi , et al., (2007), the water is modeled as a quasi-incompressible Newtonian fluid and the non-cohesive material is simulated as a single phase fluid with equivalent density ρ_{eq} by considering a Mohr-Coulomb yield criterion and using reproducing kernel technique. This study following the approach proposed by Krištof, et al., (2009), the water is represented by fluid particles carry mass m, velocity $\vec{v}$, and other fluid quantities, and the surface of reservoir bed is simulated by boundary particles. The equations governing the evolution of the fluid particles written in SPH form as follow:

$$P = B\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right] \tag{1}$$

$$\frac{d\rho_i}{dt} = \sum_j m_j \vec{v}_{ij} \vec{\nabla}_i W_{ij} \tag{2}$$

$$\frac{d\vec{v}_i}{dt} = -\sum_j m_j \left(\frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2}\right)\vec{\nabla}_i W_{ij} + \vec{g} + \sum_j m_j \left(\frac{4\upsilon_0 \vec{r}_{ij} \vec{\nabla}_i W_{ij}}{(\rho_i + \rho_j)\left|\vec{r}_{ij}\right|^2}\right)\vec{v}_{ij} \tag{3}$$

where, pressure P related to density ρ via state equation has Tait's form, with $\gamma = 7$, $B = c_0^2 \rho_0 / \gamma$, $\rho_0 = 1000\,kgm^{-3}$ is the reference density, c_0 is the speed of sound at the reference density, υ_0 is the kinetic viscosity of fluid flow ($10^{-6}\,m^2/s$), $\vec{g}$ is gravity force, and $\vec{\nabla}_i W_{ij}$ is the derivation of the smoothing kernel functions. Stability, accuracy and speed of the SPH method highly depend on the choice of the smoothing kernels. The kernel function adopted in this work is the classical 3rd order spline function.

Following the approach proposed by Krištof, et al., (2009), the interaction between water and sediment (between fluid particles and boundary particles, in another words) modeled by an erosion model based on applying the shear stress to a solid via a power-law model. That is, the shear stress τ related to the shear rate θ through the power equation:

$$\tau = K\theta^n \tag{4}$$

where, K is a constant, and n is the power-law index, a constant determined by the material of the solid. We use $n = 1/2$ in our erosion simulations.

The shear rate can be approximated from the velocity of the fluid particles relative to the boundary particles, v_{rel}, and the distance between a fluid particles and a boundary particle, l, over which the shear is applied:

$$\theta = \frac{v_{rel}}{l} \tag{5}$$

To determine the erosion rate we use the equation formulated by Partheniades (1965)

$$\varepsilon = K_\varepsilon(\tau - \tau_c) \tag{6}$$

where, ε is the erosion rate, K_ε is the erosion constant, and τ_c is the critical shear stress of the solid. Finally, the rate of change of mass at the boundary particle b due to erosion by fluid particles j within support domain is expressed as

$$\frac{dM_b}{dt} = -\sum_j L_b^2 \varepsilon(j) \tag{7}$$

where, L_b is the distance between boundary particles.

The sediment transportation modeled by using advection-diffusion equation for sediment transport has the following form

$$\frac{dC}{dt} = -\mathbf{v}_s \cdot \nabla C + \frac{1}{\rho}\nabla(D\nabla C) + J \tag{8}$$

where, C is sediment concentration values, J is sources or sinks. The advection term $-\mathbf{v}_s \cdot \nabla C$ in the right hand side of (8) is calculated by equation formulated by Krištof, et al., (2009):

Keywords: SPH method, sediment scour, reservoir.
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$$-\mathbf{v}_s\cdot\nabla C_i=-\sum_j\begin{cases} m_j\dfrac{C_j}{\rho_j}(\mathbf{v}_s\cdot\hat{\mathbf{r}}_{ij})F(|\mathbf{r}_{ij}|,h),\mathbf{v}_s\cdot\mathbf{r}_{ij}\geq 0\\ m_i\dfrac{C_i}{\rho_i}(\mathbf{v}_s\cdot\hat{\mathbf{r}}_{ij})F(|\mathbf{r}_{ij}|,h),\mathbf{v}_s\cdot\mathbf{r}_{ij}<0\end{cases}\qquad(9)$$

where, $\vec{\nabla}_i W_{ij}=\hat{\mathbf{r}}_{ij}F(|\mathbf{r}_{ij}|,h)$, $\hat{\mathbf{r}}_{ij}=\mathbf{r}_{ij}/|\mathbf{r}_{ij}|$, $\mathbf{v}_s$ is the sediment settling velocity calculated by equation $\mathbf{v}_s=\dfrac{2}{9}r_s^2\dfrac{\rho_s-\rho}{\mu}\vec{g}f(C)$. $\rho_s=2650\,kgm^{-3}$ is the sediment density, $r_s=d_{50}=0.1mm$ is the radius of a sediment particle, μ is the viscosity of the fluid, $f(C)$ is the decreasing advection rate function calculated by using Richardson-Zaki's (1954) formulate $f(C)=1-(C/C_{\max})^5$.

The diffusion term $\dfrac{1}{\rho}\nabla(D\nabla C)$ in the right hand side of (8) is calculated by equation

$$\frac{1}{\rho}\nabla(D\nabla C_i)=\sum_j\frac{m_j}{\rho_i\rho_j}D(C_i-C_j)F(|\mathbf{r}_{ij}|,h)\qquad(10)$$

where, D is the coefficient of diffusion. We use $D=0.1$ in our simulation.

The deposition of sediment from fluid particles j onto the boundary particle b is calculated by

$$\frac{dM_b}{dt}=\sum_j\rho_s\frac{m_j}{\rho_j}\frac{dC_j}{dt}\qquad(11)$$

Finally, the reservoir bed evolution is calculated base on height change H of boundary particles.

$$H=\frac{4}{\pi L_b^2}\frac{m}{\rho_s}\qquad(12)$$

3. MODEL TESTING AND VERIFICATION

The test case is a laboratory flume schematizing a simplified vertical section of a reservoir with a bottom outlet and a sediment bed. By opening the bottom outlet, flume drawdown and local scour are induced. Experimental data provided by CESI RICERCA refer to a test case carried out in the laboratory flume shown in Fig.1. For more detail information see Falappi , et al., (2007).

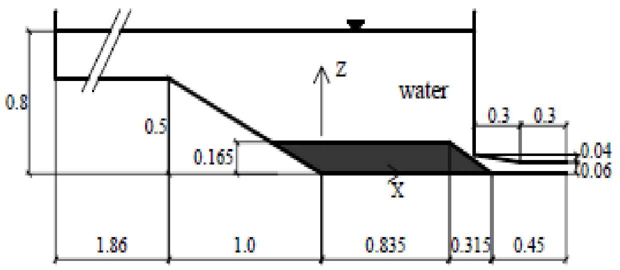
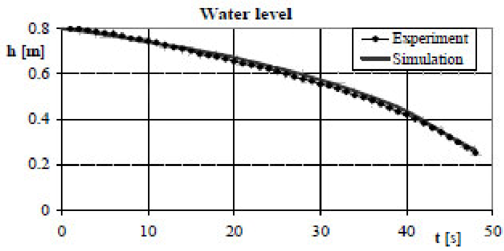
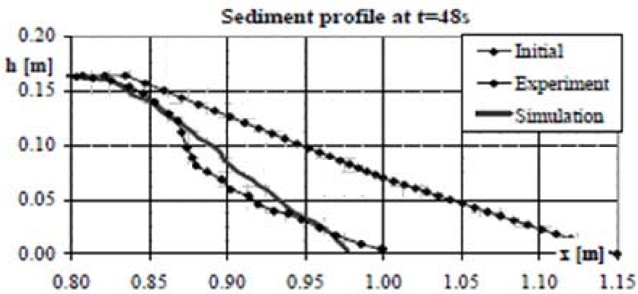


Fig.1 Reservoir geometry.

In creation of initial conditions, fluid is represented by a set of particles placed according to a regular grid with a step equal to 0.01 m. The surface of sediment is represented by boundary particles with equal distance to 0.005 m. The solid wall is simulated by semi-layer solid boundary particles. The total number of particles is close to 18000. The smoothing length h=0.0125 m is adopted. Boundary conditions for the solid walls are simulated using dynamics boundary conditions Dalrymple and Knio, (2000).

Fig. 2 and Fig. 3 show the comparison between calculated data and experimental data of water level and sediment profiles. The results are in close agreement (within 0.5%), confirming the accuracy of proposed method. In sediment profiles, results in reservoir bed are accurate in local with high erosion. However, errors are significance in deposition area.





4. CONCLUSION

The results are in reasonable agreement with the measured laboratory data. It means SPH suitable for sediment scour in reservoir sedimentation problems. The method proposed by Falappi , et al., (2007), needs more particles. Then, when extend to more realistic application in progress, possibly three dimensional, would bring about a huge number of particles and computer time. Moreover, that method derived based on considering sediment transportation as debris flows, it may have accurate in total of sediment movement. However, errors can sometimes be very large in prediction of local sediment scour. It contracts with our proposed method.

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