# STUDY OF BOTTOM SHEAR STRESS UNDER SKEW WAVE


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## 1. INTRODUCTION

The sediment transport process in near shore area is related to the bottom boundary layer structure in wave movement. Strong asymmetric shape often observed in this region due to wave breaking and other coastal process.

Numerous studies on numerical model have been conducted to understand the turbulent boundary layer under the wave motion. Sana and Tanaka (2000) have compared the Direct Numerical Simulation (DNS) data for sinusoidal oscillatory boundary layer over smooth bed to several 1D low Reynolds number $k$–$\varepsilon$ two equation model with. However, these models were not applied to asymmetric wave profile. A detail study of the asymmetric wave profile was given by Suntoyo and Tanaka (2006). They conducted laboratory experiment on oscillatory wave boundary layer using skew and sinusoidal wave. The results are compared to several 1D turbulent model. Furthermore, Suntoyo et al. (2008) studied the characteristic of turbulent layer under saw tooth wave and its application to sediment transport. The study involved experimental wind tunnel and 1D numerical model.

The 1D model is based on the assumption of zero convective acceleration. However during the wave run up and propagation, the convective acceleration will play an important role and therefore, the 1D model is no longer adequate to assess the boundary layer.

There are several types of model for turbulence flow. The two equation models are commonly used due to its reliability. The $k$–$\varepsilon$ model was one of the first two equation models used to study turbulence. $k$–$\omega$ model is a further development from the $k$–$\varepsilon$ model. Past studies have shown that the $k$–$\omega$ is superior to $k$–$\varepsilon$ model. In this study, a 2D vertical model is developed base on the Reynolds-averaged equation. The turbulence model is governed by $k$–$\omega$ equation proposed by Wilcox (1988).

The present study will focus on the development of 2D vertical model for assessing turbulence boundary layer flow under skew wave. The $k$–$\omega$ model is applied to the experimental studies of oscillatory wave. The experiment was conducted by Suntoyo (2006). His study also covered the use of a 1D $k$–$\varepsilon$ model to assess the properties of the turbulent boundary layer. The results from the model in this study are compared to his 1D model and experimental data.

## 2. NUMERICAL METHOD

The governing equation for the model is based on the Reynolds-averaged equations of continuity and momentum. The equation can be written in non-dimensional form and Cartesian tensor notation as follows:

$$\frac{\partial U_i}{\partial x} = 0 \tag{1}$$

$$\rho\frac{\partial U_i}{\partial t} + \rho U_j\frac{\partial U_i}{\partial x_j} = \frac{\partial P}{\partial x_i} + (2\mu S_{ij} - \rho\overline{u_i^{'}u_j^{'}}) \tag{2}$$

Where $U_i$ and $x_i$ denotes the mean velocity and location in the grid, $u_i'$ is the fluctuating velocity in the $x$ (i = 1) and $y$ (i = 2) directions, $P$ is the static pressure, the pressure gradient itself is a function of the free stream acceleration. $\nu$ is the kinematics viscosity, $\rho$ is the density of the fluid. And $S_{ij}$ is the strain-rate tensor and can be calculated from the following equation:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{3}$$

Where $-\rho\overline{u_i^{'}u_j^{'}}$ is the Reynolds stress tensor. The Reynolds stress tensor is given through eddy viscosity by Boussinesq approximation:

$$-\overline{u_i^{'}u_j^{'}} = \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij} \tag{4}$$

With $k$ is the turbulent kinetic energy and $\delta_{ij}$ is the Kronecker delta.

The turbulence model is the $k$–$\omega$ model. According to Wilcox (1988), $\omega$ is the ratio of turbulence dissipation rate ε to the turbulent kinetic energy $k$ and may be regarded as a characteristic time scale of the turbulence. The $k$–$\omega$ model equations are with the following equation:

$$\frac{\partial k}{\partial t} + U_j\frac{\partial k}{\partial x_j} = \tau_{ij}\frac{\partial U_i}{\partial x_j} - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[\left(\nu + \sigma^*\nu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{5}$$

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$$\frac{\partial \omega}{\partial t}+U_j\frac{\partial \omega}{\partial x_j}=\alpha\frac{\omega}{k}\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta\omega^2+\frac{\partial}{\partial x_j}\left[\left(\nu+\sigma\nu_t\right)\frac{\partial \omega}{\partial x_j}\right] \qquad (6)$$

The eddy viscosity is given by:

$$\nu_t=\frac{k}{\omega} \qquad (7)$$

The values of the closure coefficients are given by Wilcox (1988). They are $\beta$= 3/40, $\beta$* = 0.09, $\alpha$ = 5/9, and $\sigma$ = $\sigma$* = 0.5.

Finite difference central scheme is applied to solve the governing equations in time and space.

## 3. SIMULATION

The model is used to assess the velocity profile and bed stress under skew wave profile given by Suntoyo (2006). The wave profile is given in Figure 1.

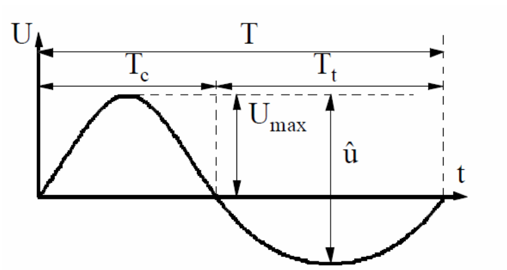


**Figure 1. Skew Wave Profile (Suntoyo 2006)**

**Table 1. Skew Wave Parameters (Suntoyo 2006)**

| T (s) | $U_{max}$ (m/s) | $N_i$ ($U_{max}$/U) | a | $a_m/k_s$ | Re |
|---|---|---|---|---|---|
| 3 | 3.63 | 0.67 | 0.393 | 115.6 | $4.34 \times 10^5$ |

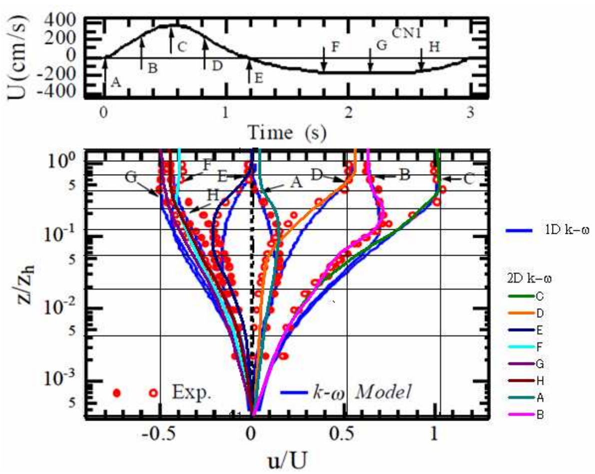


**Figure 2. Velocity Profile Comparison**

The 2D model requires a definition of the model domain. It is assumed that the model domain is equal to the wave length approached by shallow water wave theory because the experiment was under the assumption of a very long wave.

Periodic boundary condition is applied to the open end of the domain. The upper boundary condition will be given as the pressure gradient according to the wave profile No slip bottom is applied.

The grids spacing for $x$ axis is 0.05 meter. And the grids spacing for $y$ axis is 0.005 meter. The smaller grid spacing in $y$ axis is required to capture the boundary layer profile.

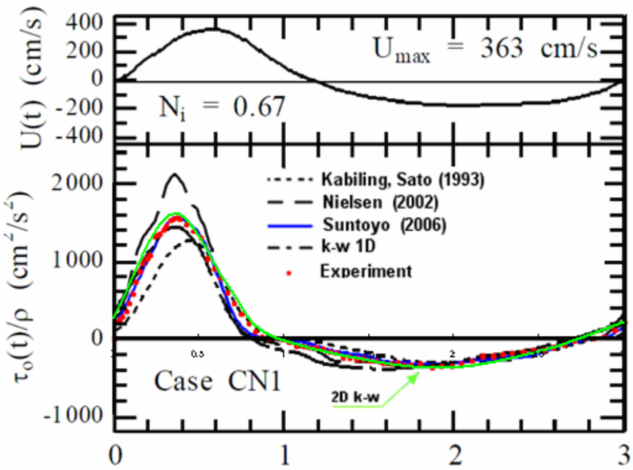


**Figure 3. Bed Shear Stress Comparison**

The mean velocity profile is shown in Figure 2. The simulation result shows good comparison with the experimental data. The 2D model shows good comparison with the 1D model and experimental data.

The bed shear stress comparison is shown in Figure 3. Comparison is made to 1D model, experimental data and several empirical formulas. Comparison result is satisfactory. The 2D model is able to provide good comparison with the experimental data and the 1D model.

## 4. CONCLUSION

The 2D $k$-$\omega$ has been used to asses the turbulent boundary layer in skew wave. The velocity profile and bed shear stress from the computation results is are compared to the experimental data and 1D model from previous study. The model results show good comparison.

## REFERENCES

Suntoyo, Tanaka, H, Sana, A.: Characteristic of turbulent boundary layers over a rough bed under saw tooth wave and its application to sediment transport, *Coastal Engineering*, Elsevier 55(2008), pp. 1102-1112, 2008.

Sana, A. and Tanaka, H.: Review of k–ε model to analyze oscillatory boundary layers, *Journal of Hydraulic Engineering*, 126(9), pp. 701-710, 2000

Suntoyo: *Study on turbulent bottom boundary layer under non-linear waves and its application to sediment transport*, Ph.D Dissertation, Tohoku University, Japan, 2006

Wilcox, D.C.: Reassessment of the scale determining equation for advanced turbulence models, *AIAA Journal*, Vol. 26, No. 11, pp. 1299–1310, 1988.