ANALYTICAL SOLUTION OF SHORELINE RESPONSE FUNCTION

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1. Introduction

Recently, many coastal areas in Japan have been seriously with erosion problems. Therefore, many coastal structures have been built and the necessary of consideration about sediment transport through these structures has been concerned. On this occasion, the predictions of coastal morphology changes were demanded.

Despite the recent development of numerical models for predicting beach profile response, there remains a need for simple methods of analyzing beach erosion or accretion due to variable sediment, wave and water-level conditions. In such cases, however, engineering computations are not usually based on sophisticated or physical sediment transport models. Instead, they are based on simplified "engineering methods" of predicting the beach profile form based on a macroscopic approach without regard for the actual sediment transport processes, which take less time in calculation.

In this study, the effect of wave actions to the shoreline is considered. For general application, the beach is then modeled as a linear system such that the exponential beach response is convolved with a time-dependent erosion- forcing function to obtain the time-dependent erosion response.

2. Shoreline response function

With the assumption that the beach is a linear system, according to laboratory experiment of David et al[1)], the beach response to steady-state forcing conditions is approximately exponential in time. In this study, to figure out the shoreline response to wave action, the basic equation below has been applied:

$$y_s(t) = a\int_0^t f(\tau)\left\{1 - e^{-(t-\tau)/T_S}\right\}d\tau \qquad (1)$$

Where a: constant value, $f(t)$: input wave function and T_s: the time scale of the exponential response. The input wave function here was applied by C_s parameter found by Sunamura・Horikawa[2)]. C_s is defined as in the next equation:

$$C_S = \left(\frac{H_0}{L_0}\right) \Bigg/ \left\{(\tan\beta)^{-0.27}\left(\frac{d}{L_0}\right)^{0.67}\right\} \qquad (2)$$

Where H_0 : Deep water wave height, L_0 : Deep water wave length, d : grain diameter. Base on C_s value, beach profile erosion or deposition can be determined and the boundary value is known as C_0=18. In this study, by applying C_0-C_s as input wave function $f(t)$, the shoreline change in eq.(1) can then be rewritten as

$$y_s(t) = a\int_0^t \{C_0 - C_s(\tau)\}\left\{1 - e^{-(t-\tau)/T_S}\right\}d\tau \qquad (3)$$

From eq.(3), it is easy to recognize that when C_s=C_0, the shoreline will be stable, in case of C_s>C_0, $y_s(t)$<0: the shoreline retreats and when C_s<C_0, $y_s(t)$>0: the shoreline advances. Thus, base on C_s parameter, this model can reflect the characteristic of shoreline changes, same as Sunamura・Horikawa[2)].

3. Shoreline response due to idealize input wave condition:

Thought out a year, for the season which the wave is almost serenity, the shoreline will advance, on the contrary, for the season which the wave is extreme, the shoreline have tendency to retreat. The action of wave thought out a year seems to have periodicity. The ideal of this phenomenon can be thought as a sine function. The input periodic wave function with the above assumption can be presented as following equation:

$$C_0 - C_s(t) = A\sin\left(\frac{2\pi}{T_D}t\right) = A\sin(\sigma t) \qquad (4)$$

Where T_D is the duration of acting wave condition. Applying the input wave condition in eq.(4) into eq.(3) gives the following shoreline position:

$$y_s(t) = a\int_0^t A\sin(\sigma\tau)\left\{1 - e^{-(t-\tau)/T_S}\right\}d\tau \qquad (5)$$

This may be integrated directly to obtain a close-form solution for the shoreline position. In dimensionless form, this solution is then a function of just one parameter, β, as

$$y_s^*(t) = \frac{2\pi y_s(t)}{aAT_D} = 1 - \frac{1}{1+\beta^2}\left(\beta\sin\sigma t + \cos\sigma t + \beta^2 e^{-\sigma t/\beta}\right)$$
$$= 1 - \frac{\beta^2}{1+\beta^2}e^{-\sigma t/\beta} - \frac{1}{\sqrt{1+\beta^2}}\sin(\sigma t + \delta) \qquad (6)$$

Where π-δ is the phase-lag between input wave condition and shoreline response, δ was defined as below:

$$\sin\delta = \frac{1}{\sqrt{1+\beta^2}} \qquad (7)$$

And β is the ratio of the erosion or deposit time scale to the wave acting duration, as

$$\beta = \sigma T_S = 2\pi\frac{T_S}{T_D} \qquad (8)$$

Base on eq.(6), the predicted dimensionless shoreline changes due to the value of β which is an important factor in shoreline response. The result of shoreline changes in this equation is

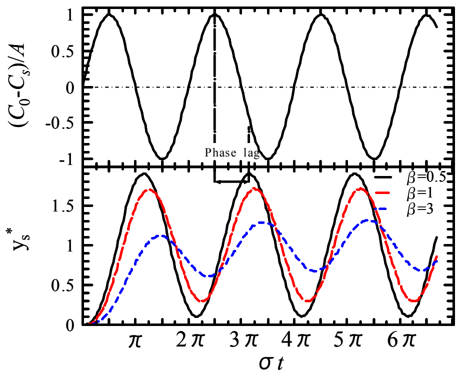

Figure-1　Shoreline response due to idealize sine wave

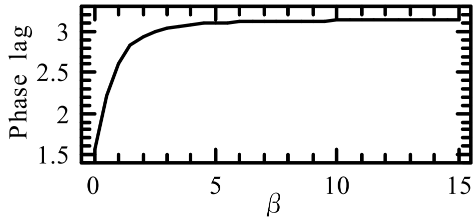

Figure-2　Relationship between phase lag and β

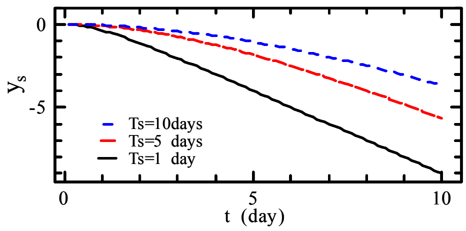

Figure-3 Shoreline response due to constant wave condition

shown in Fig.1. The upper part of Fig.1 shows the dimensionless input wave condition (C_0-C_s)/A and the lower part shows the dimensionless shoreline changes. Due to the value of input data C_0-C_s, the shoreline position, will retreat or advance, can be verified. In addition, the behavior of shoreline position with different β value can also be observed. In eq.(6), it is easy to recognize that when t reach ∞, the shoreline response then will become a sine function, the same as input function and the bigger β value is, the larger the maximum dimensionless shoreline change $y_s^*{}_{max}$ and the phase-lag between shoreline position and input C_0-C_s are (Fig.2).

Next, assuming that the extreme wave comes, and the wave condition all on a sudden changes from C_s=C_0 to a minus value of C_0-C_s, for example $-A_0$ (A_0> 0), and maintains that status for a period of time T_E. The wave function can then be defined as the following equation:

$$C_0 - C_s(t) = -A_0 \quad , \quad 0 < t < T_E \qquad (9)$$

Substitute this wave condition into eq.(3),

$$y_s(t) = -a\int_0^t A_0(1-e^{-(t-\tau)/T_S})d\tau \quad , \quad 0 < t < T_E \qquad (10)$$

and the solution of eq.(10) is as below:

$$y_s(t) = aA_0(T_S - t - T_S e^{-t/T_S}) \qquad (11)$$

The shoreline response in eq.(11) is shown in Fig.3. It can

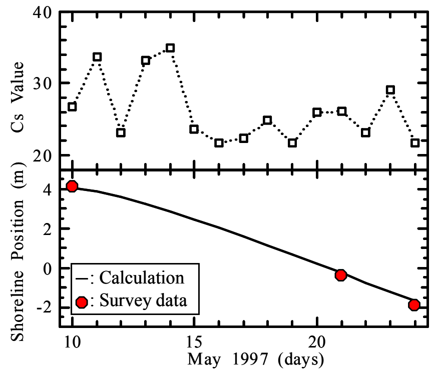

Figure-4　Wave and survey data in Sendai Coast

be found that, same as above case, for smaller value of T_s, the shoreline will retreat faster. For the time passes, the shoreline tends to become a linear and the trend of the straight line is defined by $-aA_0$.

4. Application to survey data in Sendai Coast:

Base on 15 day-data in Sendai Coast in May 1997, the wave condition in this period is really high than normal and the beach erosion had occurred during this period. These data-set are shown in Fig.4. Assuming that the wave in this period is stable with the average value C_s=26, due to eq.(11), the unknown parameter have been determined. As discussing above, for big value of t, the trend of the shoreline is defined by $-aA_0$, by applying some survey shoreline position in the end of this period, the unknown parameter a was determined around 0.06 m/day, the time scale T_s was then also defined around 2.87 day or nearly 69 hours. Applying these values into eq.(11) give a reasonable shoreline response compared with survey data (Fig.4). Moreover, according to numerical experiment by David et al[1], the time scale for erosion process varies from 5 hours to 80 hours depending on wave conditions. In this calculation, the time scale found around 69 hours. It is said that this computation has given a reliable time scale for erosion wave conditions.

5. Conclusions:

This analytical solution provides a simplified procedure for computing cross-shore beach-profile response to time-varying wave conditions with the assumption that the beach is a linear-dynamic system. From this result, the below conclusion have been made:

・Beach erosion (or deposit) response is determined as a function of input wave conditions (Horikawa and Sunamura's Cs parameter) and the characteristic exponential beach response to wave actions.

・Base on erosion survey data in Sendai Coast, it is found that at T_s=69 hours and a=0.06m/day, the analytical solution for erosion shoreline response is similar to survey data. The time scale of this solution is also a reliable result.

Reference

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Sunamura, T. and K. Horikawa（1974）: Two-dimensional shore transformation due to waves, Proc. 14th Conf. Coastal Engineering, pp.920-938.