

ESTIMATION METHODS OF BOTTOM SHEAR STRESS OVER SMOOTH BED UNDER SOLITARY WAVE

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1. INTRODUCTION

Estimation of bottom shear stress under non-linear wave motion is an important step needed in sediment transport modeling for practical application purposes. In the present study, boundary layers characteristics under solitary wave over smooth bed are investigated through laboratory experimental performed in a conduit water tunnel by mean of Laser Doppler Velocimeter (LDV) for the velocity properties measurement. This result was examined by the BSL $k-\omega$ model proposed by Menter (1994). Moreover, a new calculation method of bottom shear stress under solitary wave motion is proposed. And it will be examined with two existing bottom shear stress calculation methods, the available experiment data and the BSL $k-\omega$ turbulent model. The new proposed method agrees well with the available experimental data and the BSL $k-\omega$ turbulent model. Therefore, it can be concluded that the new proposed method might give the significant improvement on the sediment transport rate modeling under solitary wave.

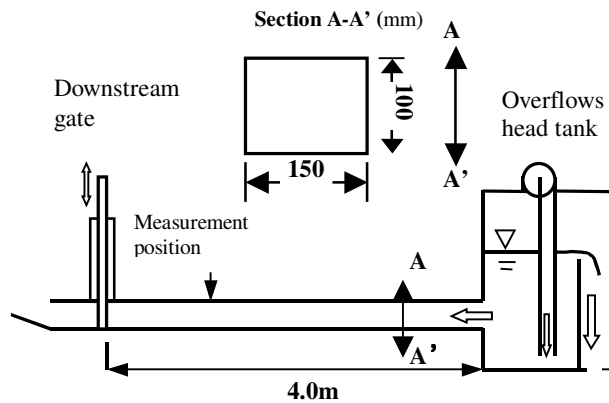


Fig. 1. Sketch of the experimental set-up

2. EXPERIMENTAL STUDY

Boundary layer flows experiments under solitary wave over smooth bed were performed in a conduit water tunnel. The velocity was measured at 17 points in the vertical direction by means of Laser Doppler Velocimeter (LDV) installed with distance 1.3 m from the downstream gate. A general sketch of the experimental set-up is given in Fig. 1. The experimental set-up consisted of overflows head tank, downstream gate, flow velocity measurement device and conduit water tunnel. The conduit has a length 400 cm, a width of 15 cm and a depth of 10 cm. The overflows head tank keeps a constant pressure head and then flows into measurement section along a conduit part. The downstream gate was made of acrylic fiber board that can be raised up and dropped down regularly using a mechanism of rotating the disk connected with the motor and moving the disk from center at fixed position according to the slot where the bearing parallel to an acrylic fiber board. The moment when flow velocity in the conduit becomes zero, wave motion similar to the solitary wave can be generated well. The measured flow velocity record was collected by means of an

A/D converter at 10 millisecond intervals, and the mean velocity profile variation was obtained by averaging over 50 wave cycles. The experiment condition is given in Table 1. Where, U_c is the velocity amplitude under wave crest, ν is kinematics viscosity, $R_E = U_c^2 / \sigma \nu$ is the Reynolds number expression and P_h is the pressure head.

Table 1. Experimental conditions

$T(s)$	$\nu(\text{cm}^2/\text{s})$	$U_c(\text{cm/s})$	R_E	P_h
16.48	0.01	27.01	1.91×10^5	7.5

3. TURBULENCE MODEL DESCRIPTION

Turbulence models can be used to predict the turbulent properties under waves motion. The base line (BSL) $k-\omega$ model proposed by Menter (1994) is one of two-equation turbulence models. The basic idea of the BSL $k-\omega$ model is to retain the robust and accurate formulation of the Wilcox (1988) $k-\omega$ model in the near wall region, and to take advantage of the free stream independence of the $k-\epsilon$ model in the outer part of boundary layer. The BSL model gives results similar to the $k-\omega$ model of Wilcox (1988) in the inner of boundary layer but changes gradually to the Jones-Launder (1972) $k-\epsilon$ model towards to the outer boundary layer and the free stream velocity. For brevity here, complete description of the turbulence model, governing equations, numerical technique, boundary conditions and model parameters are provided in Suntoyo (2006).

4. BOTTOM SHEAR STRESS ESTIMATION

Local bottom shear stress can be estimated from the logarithmic relation between the friction velocity and the variation of velocity with height given by Schlichting (1979) for smooth bed turbulence, as follows:

$$u = \frac{U^*}{\kappa} \ln \left(\frac{U^* z}{\nu} \right) + B \quad (1)$$

where, u is the flow velocity in the boundary layer measured by LDV in a conduit water tunnel, κ is the von Karman's constant (≈ 0.4), z is the cross-stream distance from bed level and B is a constant (≈ 0.56). U^* is the friction velocity related to the bottom shear stress ($\tau_o = \rho U^* |U^*|$) and ρ is the fluid density. By plotting u against $\ln(U^* z / \nu)$, a straight line is drawn through the experimental data, the value of friction velocity, U^* can be obtained from the slope of this line.

Calculation methods of bottom shear stress

Method 1 is based on a basic harmonic wave cycle modified by the phase difference is proposed by Tanaka and Samad (2006). Method 2 is a method for the instantaneous wave friction velocity, $U^*(t)$ incorporating the acceleration effect proposed by Nielsen (2006) as, as follows:

$$U^*(t) = \sqrt{\frac{f_w}{2}} \left\{ \cos \phi U(t) + \frac{\sin \phi}{\sigma} \frac{\partial U(t)}{\partial t} \right\} \quad (2)$$

$$\tau_o(t) = \rho U^*(t) |U^*(t)| \quad (3)$$

Here $\tau_b(t)$, the instantaneous bottom shear stress, t , time, σ , the angular frequency, $U(t)$ is the time history of free stream velocity, φ is phase difference between bottom shear stress and free stream velocity and f_w is the wave friction factor ($f_w=0.041R_E^{-0.16}$).

The new calculation method of bottom shear stress under solitary wave proposed is based on incorporating velocity and acceleration terms provided through the instantaneous wave friction velocity, $U^*(t)$ as given in Eq. (5). The phase difference was determined from an empirical formula for practical purposes (Suntoyo (2006)). The instantaneous friction velocity, can be expressed as:

$$U^*(t) = \sqrt{f_w/2} \left\{ U \left(t + \frac{\varphi}{\sigma} \right)^{0.84} + \frac{a_c}{\sigma} \frac{\partial U(t)}{\partial t} \right\} \quad (4)$$

Here, the value of acceleration coefficient a_c is assumed as 0.45.

5. RESULTS AND DISCUSSION

Experiment data of mean velocity profiles in the smooth turbulent boundary layer for solitary wave at selected phases were compared with the BSL $k-\omega$ model as shown in Fig. 2. An excellent agreement was obtained especially during acceleration phase at phases A, B, C and D. It is found that while the velocity outside the boundary layer remains always positive and move in the same direction of wave propagation, the fluid particle velocity near the bottom inside the boundary layer reverses the direction due to the wave decelerates. Moreover, during acceleration phases at phases A, B, C and D the overshooting velocity cannot be observed. However, the overshooting velocity can be observed at decelerating phases and produces velocity reversal in the boundary layer even the free stream velocity remains always unidirectional particularly at phases F and G.

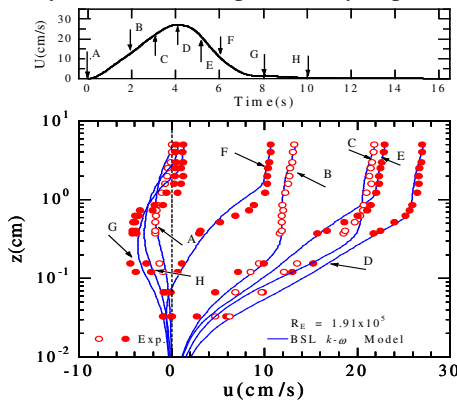


Fig. 2 Mean velocity profiles comparison of the BSL $k-\omega$ model prediction and experimental data

Fig. 3 shows comparison among the experimental data, laminar solution, BSL $k-\omega$ model and calculation methods for bottom shear stress under solitary wave with Reynolds number, $R_E = 1.91 \times 10^5$. An excellent agreement was obtained among the experimental data, present method and BSL $k-\omega$ model. Due to the turbulence the laminar solution result can not predict well the bottom shear stress under solitary wave, especially in the positive part of the bottom shear stress. It is found that while the velocity outside the boundary layer remains always positive and move in the same direction of wave propagation, the fluid particle velocity near the bottom inside the boundary layer reverses the direction due to the wave decelerates, as a consequence the bottom shear stress also changes the sign during the

deceleration phase. Therefore, this phenomenon becomes an important aspect on the sediment transport calculation at near-shore. While, Method 1 and Nielsen (2006) can not predict the negative value of bottom shear stress under solitary wave.

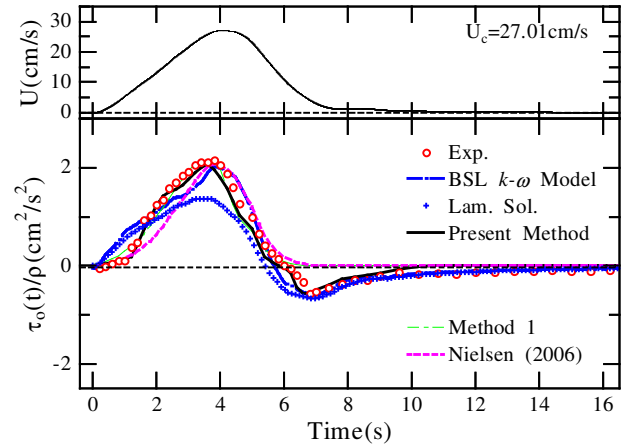


Fig. 3 Comparison of the bottom shear stress

6. CONCLUSIONS

Various methods for estimating bottom shear stress have been studied. The new calculation method for calculation bottom shear stress under solitary wave has been proposed. This method has shown the best agreement with the experimental data and BSL $k-\omega$ model along a wave cycle for solitary wave. While, Method 1 and Nielsen (2006) can not predict the negative value of bottom shear stress under solitary wave. Therefore the present method may be used to improve the accuracy of the net sediment transport rate prediction under rapid acceleration induced by solitary wave in practical applications.

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