NUMERICAL STUDY ON SAND BAR DEVELOPMENT

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1. INTRODUCTION

River mouth topography changes frequently under the combinative actions of wave and current. Low river discharge creates a propitious condition for wave-induced sand transport at the river mouth seasonally. In the shallow water area near river mouth, the height and length of wave change strongly. Wave shoaling on the sloping bottom first decreases in height, and then increases gradually. In the shallower region, wave height increases rapidly to produce asymmetric wave profile and breaks. When the bed shear stress exceeds a critical value, sand at the bottom will be transported and causing erosion and accumulation areas. Sediment transported to the river mouth and deposited there forms sand bar. A numerical model was applied to simulate the transformation of wave and sand bar formation at a river mouth.

2. NUMERICAL MODEL

2.1. Wave model

Incident wave propagates to shallow water area in front of the mouth under the influence of the sloping bottom and the roughness of sand ripples, sand particles on the bed. Wave shoaling over a sloping bottom is calculated by perturbation method. The nonlinear shoaling laws proposed by Shuto (1974) were used in an effort to provide an improved description of the waves increasing nonlinearity as they approach breaking (shown in Equation (1)).

$$\left.\begin{array}{ll} \dfrac{H}{H_0}=\sqrt{\dfrac{1}{2n}\dfrac{1}{\tanh kh}} & \dfrac{gHT^2}{h^2}\le 30 \\ Hh^{2/7}=const & 30\le\dfrac{gHT^2}{h^2}\le 50 \\ Hh^{5/2}\left(\sqrt{\dfrac{gHT^2}{h^2}}-2\sqrt{3}\right)=const & 50\le\dfrac{gHT^2}{h^2} \end{array}\right\} \quad (1)$$

here H and H_0 are the height of wave and deep water wave, respectively, h the water depth, n the ratio between the group velocity and the wave celerity, k the wave number, T the wave period, g the gravity acceleration.

Waves move toward the river mouth with an increase in height while there is a decrease in water depth. When the height of wave reaches a critical height, breaking wave occurs. Breaking wave height was calculated by the criterion proposed by Komar and Gaughan (1972) as follows

$$\frac{H_b}{H_0}=0.563\left(H_0/L_0\right)^{-0.2} \quad (2)$$

where H_b is the breaking wave and L_0 the deep water wave length.

In the surf zone, wave transformation was described by several numerical models. The breaker decay model allows waves reformation occur, that relate closely to modeling profiles with multiple bars (Dally, 1985). Wave height in the surf zone was determined as following equation

$$\frac{\partial(H^2\sqrt{h})}{\partial x}=\frac{-K_1}{h}\left(H^2\sqrt{h}-K^2h^2\sqrt{h}\right) \quad (3)$$

in which x is the distance in cross-shore direction, K = 0.15 the dimensionless decay coefficient, and the dimensionless coefficient K_1 = 0.4. The wave-induced set-up and set-down η was obtained from Eq. (4).

$$\frac{\partial\eta}{\partial x}=-\frac{1}{\rho gh}\frac{\partial S_{xx}}{\partial x} \quad (4)$$

where S_{xx} is the onshore momentum flux in shallow water, ρ the fluid density.

Fig. 1 shows a typical fit between results of the model and measured wave height and wave set-up at the rive mouth, in which h_R is the water depth at the river mouth.

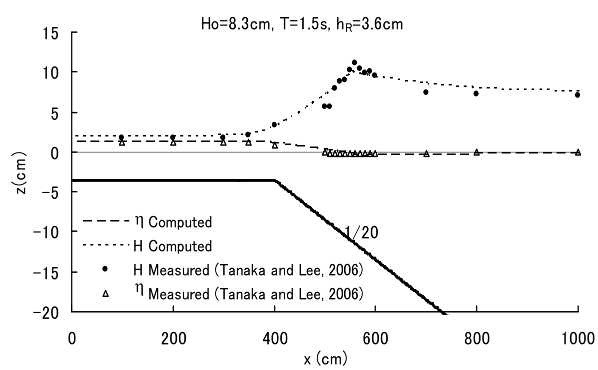

Fig. 1 Measured and calculated wave height and wave set-up at a river mouth

2.2. Sand transport under the action of waves

Under strong vortices created on the onshore side of ripples, suspended sand clouds are formed. When the flow velocity is in the onshore direction, sand is transported as bed load movement on the ripples and then suspended at the crest of the ripples to form a suspended sand cloud. Non-dimensional sediment transport rate Φ is calculated through non-dimensional bed shear stress Ψ (Shields parameter) as follows

$$\Phi=45\psi^3 \quad (5)$$

$$\psi=\frac{\tau_m}{(s-1)\rho gd} \quad (6)$$

where s is the sediment specific gravity, d the grain size. The dimension sediment transport rate q_s at the mouth can be obtained from Eq. (7).

$$\Phi = \frac{q_s}{wd} \tag{7}$$

in which w is the fall velocity of sand particles. The maximum bed shear stress τ_m in Eq. (6) is given by Eq.(8).

$$\tau_m = \frac{1}{2} f_w \rho u_b^2 \tag{8}$$

here, u_b is the maximum value of near bottom velocity, f_w the friction coefficient.

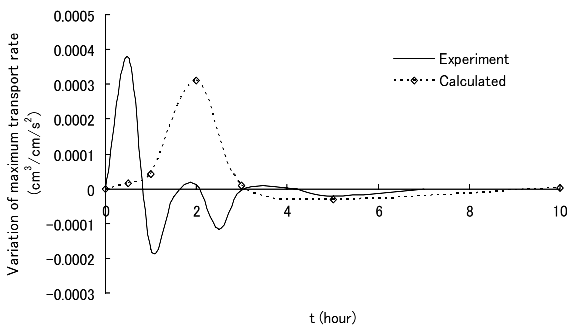

Fig. 2 Variation of maximum sediment transport rate at the river mouth,
Case h_R= 1cm, H_0= 6cm, T=1.44s, slope i= 1/10

2.3. Sand bar development

Energy dissipation of waves at the river mouth causes sand movement toward river mouth and deposits there to form a sand bar. The changes of bottom layer under wave actions for sand bar formation are calculated from the distribution of the cross-shore transport rate and mass conservation of sand (as shown in Eq. (9)). In order of calculation, the elevation of sand bar profile is determined from previous and transport rate of two times after a time step.

$$\frac{\partial h}{\partial t} = \frac{1}{(1-\lambda)} \frac{\partial q}{\partial x} \tag{9}$$

$$q = q_s + \varepsilon_s |q_s| \frac{\partial h}{\partial x} \tag{10}$$

Where λ is the porosity of sediment (λ =0.4), q the net volumetric sediment transport rate, ε_s the positive constant (ε_s= 2).

In order to solve the finite difference scheme, the distance in cross-shore direction was divided with the interval of length Δx. The wave height and near bottom velocity as well as sediment transport rate were computed at each grid point in specific time step of Δt. Then, the bottom elevation changes in previous time step are used and the transport rate is calculated explicitly. The mass conservation can be written as in Eq. (11).

$$\frac{h_{j,k+1} - h_{j,k}}{\Delta t} = \frac{1}{(1-\lambda)} \frac{\left(q_{j+1,k} - q_{j,k}\right)}{\Delta x} \tag{11}$$

in which j is grid number, k the time step number.

There was a agreement in variation trend of sand transport rate at the river mouth, however variation in experiment was more fluctuation as shown in Fig. 2. In which the transport rate changes in the experiment quicker than in the model. Results obtained from the model fit with measured height in sand bar development as in Fig.3. In experiment sand bar developed more gradually compared with the model. Fig. 4 shows the comparison of sand bar height in the simulation and experiment. Results in the model were a little higher than the experiment with large values.

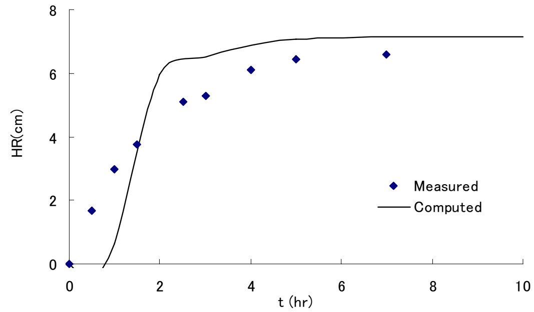

Fig. 3 Development of sand bar height
Case h_R= 1cm, H_0= 6cm, T=1.44s, slope i= 1/10

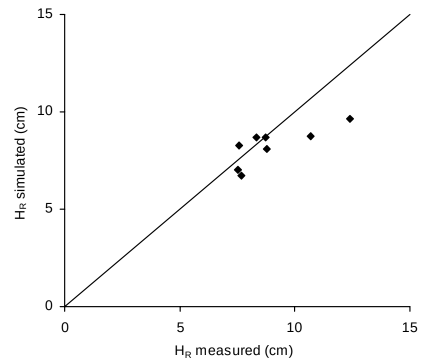

Fig. 4 Comparison of bar height between model and experiment

3. CONCLUSION

The model was applied to simulate sand bar formation at a river mouth due to waves. Results of the model described wave transformation at the river mouth, sediment transport and the evolution of sand bar by the duration of wave action. There was a good agreement between the simulation and experiment.

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