BED FRICTIONAL AND PHASE DIFFERENCE CHARACTERISTICS IN A ROUGH TURBULENT FLOW UNDER SAWTOOTH WAVES

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1. INTRODUCTION

Bottom shear stress estimation is the most important step, which is required as an input to all the practical sediment transport models. Therefore, a reliable bottom shear calculation methods is needed to overcome the effect of wave non-linearity occurring in the near-shore.

In reality, ocean waves may have a strongly non-linear shape with respect to horizontal, i.e. skew or saw-tooth waves shape. Therefore, it is envisaged that turbulent structure, bottom shear stress and sediment transport behaviors having the effect of acceleration in the skew-ness of the wave are different from those in sinusoidal waves. Moreover, relation among bottom shear stress, friction factor and phase difference are one of important factor for evaluating the sediment transport in the coastal area.

In the present study, the BSL k-ω model is applied to predict the turbulent properties for saw-tooth waves that are validated with the experimental data. The bottom shear stress for saw-tooth waves was proposed. Moreover, the characteristics of bottom shear stress and phase difference between bottom shear stress and the free stream velocity are examined.

2. EXPERIMENTAL STUDY

Rough turbulent flow experiments under saw-tooth waves were carried out in an oscillating tunnel using air as the working fluid for four cases. The velocity was measured in the center part of wind tunnel at 20 points in the vertical direction by means of LDV. Triangular elements of roughness were chosen in order to the roughness elements protrude out of the viscous sub-layer. The definition sketch for saw-tooth wave is shown in **Fig. 1**.

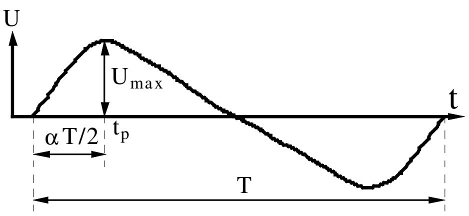

Fig. 1: Definition sketch for saw-tooth wave

Here, U_{max} is the velocity at the crest that was kept almost 400 cm/s for all cases, T is wave period, t_p is time interval measured from the zero-up cross point to wave crest in the time variation of free stream velocity and α is the wave skewness parameter given α=0.314 for Case SK1, α=0.363 for Case SK2, α=0.406 for Case SK3 and α=0.500 for Case SK4, respectively. The smaller α indicate stronger the wave skewness, while the symmetric wave without skewness has α=0.50.

3. TURBULENCE MODEL DESCRIPTION

Turbulence models can be used to predict the turbulent properties under waves motion. The base line (BSL) k-ω model proposed by Menter (1994) is one of two-equation turbulence models. The basic idea of the BSL k-ω model is to retain the robust and accurate formulation of the Wilcox (1988) k-ω model in the near wall region, and to take advantage of the free stream independence of the k-ε model in the outer part of boundary layer. The BSL model gives results similar to the k-ω model of Wilcox (1988) in the inner of boundary layer but changes gradually to the Jones-Launder (1972) k-ε model towards to the outer boundary layer and the free stream velocity. For brevity here, complete description of the turbulence model, governing equations, numerical technique, boundary conditions and model parameters are provided in Suntoyo (2006).

4. BOTTOM SHEAR STRESS ESTIMATION

Bottom shear stress is estimated by fitting the logarithmic velocity distribution to the measured velocity data. Alternatively, the instantaneous bottom shear stress can be calculated proportional to the square of the proposed instantaneous friction velocity, as shown in Eq. (1),

$$U^*(t)=\sqrt{f_w/2}\left\{U\left(t+\frac{\varphi}{\sigma}\right)+\frac{a_c}{\sigma}\frac{\partial U(t)}{\partial t}\right\} \qquad (1)$$

$$\tau_o(t)=\rho U^*(t)\left|U^*(t)\right| \qquad (2)$$

where, $\tau_o(t)$ is the instantaneous bottom shear stress and f_w is the wave friction factor (see Tanaka and Thu (1994)) and φ is the phase difference between bottom shear stress and the free stream velocity. a_c is the value of acceleration coefficient determined empirically from both the experimental and the BSL k-ω model results and the acceleration coefficient as function of α is given as $a_c = -0.36\,Ln\,(\alpha) - 0.249$.

The modified phase difference, φ for saw-tooth waves in Eq. (5) was obtained from that of for sinusoidal wave study proposed by Tanaka and Thu (1994) in Eq (3), are used to evaluate the bottom shear stress under sawtooth waves.

$$\varphi_s = 42.4C^{0.153}\frac{1+0.00279C^{-0.357}}{1+0.127C^{0.563}}\ \text{(degree)} \qquad (3)$$

$$C=\frac{1}{\kappa\sqrt{\dfrac{f_w}{2}\dfrac{a_m}{z_0}}} \qquad (4)$$

$$\varphi = 2\alpha\frac{\varphi_s}{T}\ \text{(degree)} \qquad (5)$$

Where, φ_s is phase difference between free stream velocity and bottom shear stress proposed by Tanaka and Thu (1994) based on sinusoidal wave study and C defined by Eq. (4).

5. RESULTS AND DISCUSSION

Fig. 2 shows the phase difference obtained from measured data under sawtooth waves, as well as from theory proposed by Tanaka and Thu (1994) in Eq. (3) for sinusoidal wave. Eq. (3) was reasonable to estimate the phase difference for sinusoidal wave cases, as shown by Tanaka and Thu (1994).

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It was then proposed to modify this theory with consider the wave skewness effect under sawtooth waves as shown in Eq. (5). If we insert the value of $\alpha = 0.500$, Eq. (3) is then equal to Eq. (5). As seen in **Fig. 2** that the phase difference at crest, trough and average between crest and trough for Case SK4 with α = 0.500 is about 19.1^o, this value is attached well with the theory in Eq. (3) as well as Eq. (5) for $\alpha = 0.500$. The increasing of the wave skewness or decreasing α causes the average value of phase difference in experimental results gradually decrease i.e. 15.7^o, 13.9^o and 12.1^o for Case SK3 with α=0.406, Case SK2 with α=0.363 and Case SK1 with α=0.314, respectively. The relation proposed in Eq. (5) has given very good agreement with phase difference of experimental data for saw-tooth waves cases with variation in the value of a as shown in **Fig. 2**. Moreover, many researchers e.g. Jonsson and Carlsen (1976), have shown that the phase difference from laminar flow is 45^o and drop from 45^o to about 10^o in the turbulent flow case.

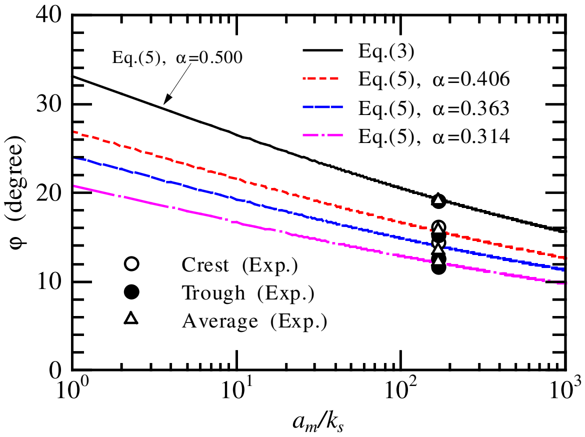

Fig. 2 Phase difference between the bottom shear stress and the free stream velocity

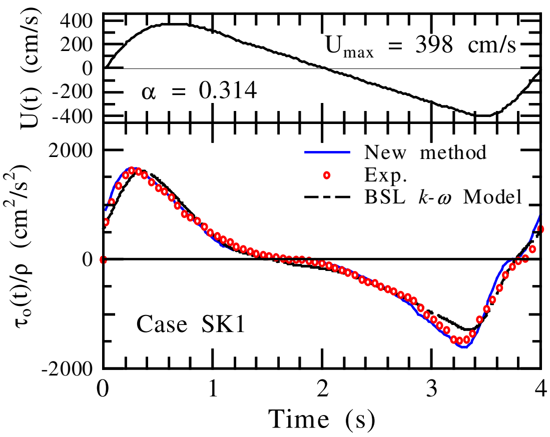

Fig. 3 Comparison among the BSL k-ω model, calculation method and experimental results for Case SK1

Figs 3 and **4** show a comparison among the BSL k-ω model, calculation method and experimental results for bottom shear stress under saw-tooth waves, for Case SK1, Case SK2, respectively. New method has shown the best agreement with the experimental results along a wave cycle for all saw-tooth wave cases. Generally, the BSL k-ω model can predict well along a wave cycle for all cases, however, for the higher wave skewness (smaller α) the BSL k-ω model could not predict well especially on the crest and trough part, but for the higher α a good agreement with experimental data could be obtained, i.e. in Case SK2. Moreover, the BSL k-ω model prediction results showed more close to the experimental and the new method result. New method has shown as a reliable calculation method of bottom shear stress under saw-tooth waves for all cases.

Furthermore, both the phase difference and the acceleration coefficient defined in the new method were sufficient for this calculation. It can be concluded that the new method for calculating the instantaneous bottom shear stress under saw-tooth waves proposed in this present study has a sufficient accuracy. Therefore, this method can be used to an input sediment transport model under rapid acceleration in a practical application.

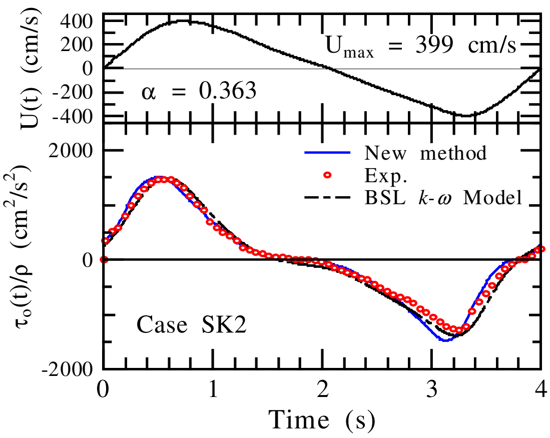

Fig. 4 Comparison among the BSL k-ω model, calculation method and experimental results for Case SK2

6. CONCLUSIONS

The behaviors of the rough turbulent boundary layer under saw-tooth waves has been examined through both experimental and BSL k-ω model. Moreover, the new method of bottom shear stress under saw-tooth waves can be further used to an input sediment transport model under rapid acceleration in practical application on near-shore.

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