IV－68

# OPTIMAL TOLL SYSTEM TO MITIGATE LOCATION EXTERNALITY

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## 1. Introduction

Agglomeration economies are caused by various factors; such as communication externalities, complementarity's in labour supply, location externality etc,. To do these externalities, way back 1920's economists like Pigou(1920) and Knight (1924) had recognised that Pigouvian pricing offers the first best solution for optimising such externalities. However, it is difficult to impose in real economy such taxation because of complex nature of firms network, and tax collection and implementation cost are very huge. Hence the other possible way to implement is in the form of second best measures; there fore it is necessary to examine the second best measures to tackle these externalities in real economy.

Though some of the studies (Fujita, 2002 , Kanemoto, 1990) mentioned about the location externality, but none of the study stated so far how to tackle the location externality in real economy in relation to toll pricing /road pricing. To mitigate this problem, the present study is focussed on to explore the toll pricing system in the presence location externality in an urban structure.

This paper is organised as in section 2, we have explained about location externality, in section 3, about the model, in section 4, how to mitigate location externality and discussion about study findings, and finally we have given conclusions in section 5.

## 2. What is location externality?

Location externality is defined, as when a firm moves from one location to other, it will get a benefit from the movement called *internal benefit* to that firm. But by virtue of movement in this firm the other firm also get same amount of benefit is called external benefit to other firm. This phenomenon is called *location externality* in a spatial structure.

## 3. The Model

### 3.1 Frame work

Urban space is divided into three parts. One city centre, area $\bar{A}_c$ ,other two spaces area $\bar{A}_1$ and $\bar{A}_2$ surrounding the city centre as shown in model frame work below.

| Parameter | Zone '1' | City centre | Zone '2' |
|---|---|---|---|
| Area | $\bar{A}_1$ | $\bar{A}_c$ | $\bar{A}_2$ |
| No.of. firms | $n_1$ | $n_c$ | $n_2$ |
| Input from other firms | $S_{c1}, S_{21}, S_{11}$ | $S_{c1}, S_{21}, S_{11}$ | $S_{c1}, S_{21}, S_{11}$ |
| Office rent | $r_1$ | $r_c$ | $r_2$ |
| Land rent | $R_1$ | $R_c$ | $R_2$ |
| Profit | $\pi_1$ | $\pi_c$ | $\pi_2$ |
| Floor space | $F_1$ | $F_c$ | $F_2$ |
| Interaction among zones | 't' between city centre and zone '1' and 2t between zone '2' and zone '1' | $\varphi$ within city centre and 't' between zone '1' and zone '2' | 't' between city centre and zone '2' and 2t between zone '1' and zone '2' |

### 3.2. Assumptions

1. Two spaces are assumed to be symmetric from geographical viewpoint and the equilibrium is assumed to be symmetric for simplicity
2. Economy has goods producers and developers. Every producer is able to migrate to other zone without any cost.
3. Every firm will trade (interact) with all other firms

### 3.3 Market equilibrium

In this section we have discussed the behavour of goods producres and developers, and derived the firm denstiy at market equlibirum by equating the bid floor rent from profit maximization of goods producres with offer floor rent from profit maximization of develoeprs.Further the analysis is as given below.

#### 3.3.1 Goods Producers

Goods producers use one unit of floor space at the prevailing office rent ($r_i$). Profit maximization of firms, Assuming per unit floor space

$\pi_i^f = \max_{X_i, s_i}\{X_i - T_i - r_i + u\}$..(1.1) s.t $X_i = \bar{G} + n_1 f_1(s_{1i}) + n_2 f_2(s_{2i}) + n_c f_c(s_{ci})$....(2)

toll revenues per firm $u = \frac{Q}{n_1+n_2+n_c} = \frac{n_1(n_2 2qS_{21} + n_c qS_{c1}) + n_2(n_1 2qS_{12} + n_c qS_{c2}) + n_c(n_1 qS_{1c} + n_2 qS_{2c})}{n_1+n_2+n_c}$..(3)

Transportation cost

For firms in centre $T_c = n_1 ts_{1c} + n_2 ts_{2c} + n_c \varphi s_{cc}$..(4.1)

For firms in location '1' $T_1 = n_2 2ts_{21} + n_1 \varphi s_{11} + n_c ts_{c1}$..(4.2)

For firms in location '2' $T_2 = n_1 2ts_{12} + n_2 \varphi s_{22} + n_c ts_{c2}$..(4.3)

Where is $\bar{G}_c$ fixed production independent of other firms, q is toll tax.

Under perfect competition $\pi_i^f = 0$, the bid floor rent '$r_i$'from Eq.(1) $r_i = \{X_i - T_i + u\}$..(1.2)

#### 3.3.2 Developers behaviour

Developers supply floor space in an area is $F_i$ the behaviour is formulated as Eq.(5) $\pi^d = \max_{F_i}\left\{\sum_i r_i F_i - C(F_i, \bar{A}_i) - R_i\right\}$(5)

As result of free entry of developers,$\pi^d = 0$ and first order conditions with respect to $F_i$ from equation(5) $r_i = \beta F_i^{\beta-1}$..(6)

#### 3.3.3 At Equilibrium.

At equilibrium, equating bid floor rent from profit maximization of goods producers (Eq.1.2) and offer floor rent from profit maximisation of developers from Eq.(6)

$$\beta F_i^{\beta-1} = \{X_i - T_i + u\}..(7)$$

Assuming market equilibrium conditions as $n_i$=$F_i$, $\beta = 2$ , replacing $n_2$ with $n_1$ due to symmetry of configuration and substituting , $X_i$,$T_i$ values in Eq(7) . The firm density at surrounding zones and city centre is given by eq.(8.1) and (8.2), $F_{cc}(\varphi) = (f_c(s_{cc}(\varphi)) - \varphi s_{cc}(\varphi))$, $F_{c1}(t) = (f_c(s_{c1}(t)) - ts_{c1}(t))$, $F_{1c}(t) = f_1(s_{1c}(t)) - ts_{1c}(t)$, $F_{2c}(t) = f_2(s_{2c}(t)) - ts_{2c}(t)$, $F_{11}(\varphi) = f_1(s_{11}(\varphi)) - \varphi s_{11}(\varphi)$, $F_{21}(2t) = f_2(s_{21}(2t)) - ts_{2c}(2t)$

$$F_{1c} = F_{c1}, \quad F_{2c}(t) = F_{c2}(t), \quad F_{21}(2t) = F_{12}(2t)$$

$$n_1^m = \frac{\bar{G}(F_{cc}(\varphi) - F_{1c}(t) - 2)}{2(F_{1c}(t))^2 - (F_{11}(\varphi) + F_{21}(2t) - 2)(F_{cc}(\varphi) - 2)}..(8.1) \quad n_c^m = \frac{\bar{G}(F_{11}(\varphi) + F_{21}(2t) - 2F_{1c}(t) - 2)}{2(F_{1c}(t))^2 - (F_{11}(\varphi) + F_{21}(2t) - 2)(F_{cc}(\varphi) - 2)}..(8.2)$$

### 3.4 Social optimization under open city model

In the above section we have derived the density of firms under market equilibrium. In the present section we derive the firm density in social optimum conditions considering a open city model, as follows.

Total profit of firms $\Pi = \max_{\{n_1, n_2, n_c;\, s_1, s_2, s_c\}} \{\pi_i^d + R_1 + R_2 + R_c\}$.(9)

where $s.t \quad \pi_1 = \pi_2 = \pi_c = 0$

$\pi_1 = X_1 - T_1 - r_1$, $\pi_c = X_c - T_c - r_c$, $\pi_2 = X_2 - T_2 - r_2$, $\pi^d = \sum_i (r_i F_i - F_i^\beta - R_i)$

where production function $X_i$, transportation cost function $T_i$ are same as market equilibrium. Assuming $n_i$=$F_i$ and rewriting Eq.(9) as $\max_{\{n_1, n_2, n_c;\, s_1, s_2, s_c\}} \Pi = \{r_1 n_1 - n_1^\beta + r_c n_c - n_c^\beta + r_2 n_2 - n_2^\beta\}$..(10)

Substituting $r_i$, in Eq(10) and further simplifying, the firm density at surrounding zones and city centre is given as

$$n_1^s = \frac{\bar{G}(2(F_{11}(\varphi) - F_{1c}(t)) - 2)}{8(F_{1c}(t))^2 - 2(F_{11}(\varphi) + F_{21}(2t) - 2)(2F_{cc}(\varphi) - 2)}..(11.1) \quad n_c^s = \frac{\bar{G}(2(F_{11}(\varphi) + F_{21}(2t) - 2F_{1c}(t)) - 1)}{8(F_{1c}(t))^2 - 2(F_{11}(\varphi) + F_{21}(2t) - 2)(2F_{cc}(\varphi) - 2)}..(11.2)$$

### 3.4 Comparison of market equilibrium and social optimum firm density.

By comparing the market firm density with social optimum firm density, Eq.(8.1), (8.2) with Eq.(11.1) and (11.2), it is difficult to draw a conclusions that, weather social optimum firm density is higher than market firm density or vice versa. However, This can be shown clearly by the graphical representation as follows. Using Eq.(7) and (10) we can rewrite the equations as (12.1), (12.2), (13.1) and (13.2) as follows

At surrounding zones

$$r_i = \underset{\text{offer floor rent}}{2n_1} = \underset{\text{Bid floor rent}}{n_1(\underbrace{F_{11}(\varphi)+F_{21}(2t)}_{A}) + n_c(\underbrace{F_{1c}(t)}_{B})} + \bar{G} + u \ldots (12.1)$$

$$r_i = 2n_1 = n_1 2(F_{11}(\varphi)+F_{21}(2t)) + n_c 2(F_{1c}(t)) + \bar{G} \ldots (13.1)$$

At centre

$$r_i = 2n_c = n_1 \underbrace{2(F_{1c}(t)+F_{2c}(t))}_{D} + n_c \underbrace{2(F_{cc}(\varphi))}_{C} + \bar{G} \ldots (13.2)$$

$$r_i = 2n_c = n_1(F_{1c}(t)+F_{2c}(t)) + n_c(F_{cc}(\varphi)) + \bar{G} + u \ldots (12.2)$$

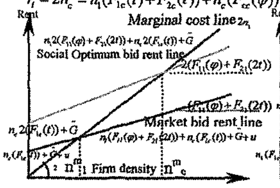

Figure 1. Firm density at surrounding zones

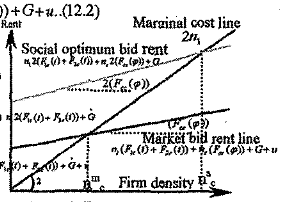

Figure 2. Firm density at city centre

From the above figures 1 &2, it clearly indicates that social optimum firm density is higher than the market firm density at surrounding zones as well as at city centre; in market solution the firms are not optimally located because of existence of location externality.

### 4. Analysis

### 4.1 Exploring the optimal toll system to mitigate the location externality.

As it is explained in the previous section that, the social optimum firm density is higher than the market firm density this is due to the existence of location externality. One way to mitigate is to impose toll tax as explained below. The imposition toll fee create total benefits in the form of land rents as shown in Eq.(14).

$$\frac{\partial w}{\partial q} = \underbrace{\frac{\partial n_1^*}{\partial q}(X_1^* - T_1^*) + \frac{\partial n_2^*}{\partial q}(X_2^* - T_2^*) + \frac{\partial n_c^*}{\partial q}(X_c^* - T_c^*)}_{\text{Indirect effect}} + \underbrace{n_1(-n_2^* 2\dot{s}_{21} - n_c^* \dot{s}_{c1}) + n_2(-n_2^* 2\dot{s}_{12} - n_c^* \dot{s}_{c2}) + n_c(-n_1^* \dot{s}_{1c} - n_2^* \dot{s}_{2c})}_{\text{Direct effect}} \ldots (14)$$

This comprises of (1). Indirect effects due to firms are not located optimally in the presence of location externality, (2). Direct effect due change in external transport cost. From the Eq.(14).Direct effects are negative in nature. However, the nature of indirect effects can be further studied as follows. From the Eq.(12.1) and (13.1), we can rewrite as

$$2n_1 = An_1 + Bn_1 + \bar{G} + u(n_1, n_c, q) \ldots (15.1), \quad 2n_c = Cn_1 + Dn_1 + \bar{G} + u(n_1, n_c, q) \ldots (15.2),$$

Where $A = F_{11}(\varphi) + F_{21}(2t)$, $B = F_{1c}(t)$, $C = F_{cc}(\varphi)$, $D = F_{1c}(t) + F_{2c}(t)$

from above, solving for $n_1$ and $n_c$ on imposition of toll tax

$$\frac{dn_1}{dq} = \frac{\left(n_1\frac{\partial(A)}{\partial q} + n_c\frac{\partial(B)}{\partial q} + \frac{\partial u}{\partial q}\right)\left(2 - C - \frac{\partial u}{\partial n_c}\right) + \left(n_1\frac{\partial(D)}{\partial q} + n_c\frac{\partial(C)}{\partial q} + \frac{\partial u}{\partial q}\right)\left(B + \frac{\partial u}{\partial n_c}\right)}{\left(2 - A - \frac{\partial u}{\partial n_1}\right)\left(2 - C - \frac{\partial u}{\partial n_c}\right) - \left(B + \frac{\partial u}{\partial n_c}\right)\left(D + \frac{\partial u}{\partial n_1}\right)} \ldots (16.1)$$

$$\frac{dn_c}{dq} = \frac{\left(n_1\frac{\partial(D)}{\partial q} + n_c\frac{\partial(C)}{\partial q} + \frac{\partial u}{\partial q}\right)\left(2 - A - \frac{\partial u}{\partial n_1}\right) + \left(n_1\frac{\partial(A)}{\partial q} + n_c\frac{\partial(B)}{\partial q} + \frac{\partial u}{\partial q}\right)\left(D + \frac{\partial u}{\partial n_1}\right)}{\left(2 - A - \frac{\partial u}{\partial n_1}\right)\left(2 - C - \frac{\partial u}{\partial n_c}\right) - \left(B + \frac{\partial u}{\partial n_c}\right)\left(D + \frac{\partial u}{\partial n_1}\right)} \ldots (16.2)$$

From the above two equations Eq(16.1) and (16.2) the following conclusions can be drawn.

| Point of comparison | Change in firms density with imposition of toll | |
|---|---|---|
| | $\frac{\partial n_1}{\partial q}$ | $\frac{\partial n_c}{\partial q}$ |
| Price elasticity $\varepsilon > 1$ | <0 | <0 |
| $\varepsilon < 1$ and condition 'E' holds | >0 | >0 |
| $\varepsilon < 1$ and condition 'F' holds | <0 | <0 |

$$E = \left(\left(2 - A - \frac{\partial u}{\partial n_1}\right) + \left(D + \frac{\partial u}{\partial n_1}\right)\right)\frac{\partial u}{\partial q} > \left(\left(2 - A - \frac{\partial u}{\partial n_1}\right)\left(n_1\frac{\partial(D)}{\partial q} + n_c\frac{\partial(C)}{\partial q}\right) + \left(D + \frac{\partial u}{\partial n_1}\right)\left(n_1\frac{\partial(A)}{\partial q} + n_c\frac{\partial(B)}{\partial q}\right)\right)$$

$$F = \left(\left(2 - C - \frac{\partial u}{\partial n_c}\right) + \left(B + \frac{\partial u}{\partial n_c}\right)\right)\frac{\partial u}{\partial q} < \left(\left(2 - C - \frac{\partial u}{\partial n_c}\right)\left(n_1\frac{\partial(A)}{\partial q} + n_c\frac{\partial(B)}{\partial q}\right) + \left(B + \frac{\partial u}{\partial n_c}\right)\left(n_1\frac{\partial(D)}{\partial q} + n_c\frac{\partial(C)}{\partial q}\right)\right)$$

After getting the changes in $n_1$ and $n_c$ with respect to toll pricing, we can arrive the optimal toll pricing by the solving the following Eq.(17) for optimal toll tax 'q',

$$\frac{\partial w}{\partial q} = 0 \ldots (17)$$

### 4.2 Study findings

The study finding are summarised as follows

1. From section 3.4 ,market solution firm density is less than the social optimum firm density because in market solution firms are not located optimally due to presence of location externality, where location of firms does not consider the other firms location. The firm density at city centre as well as surrounding areas are increased in this open city model
2. Location externality can create agglomeration economies.
3. Decrease in $\frac{dn_1}{dq}$, $\frac{dn_c}{dq}$ when price elasticity is $\varepsilon > 1$
4. Increase and decrease in $\frac{dn_1}{dq}$, $\frac{dn_c}{dq}$ when price elasticity is $\varepsilon < 1$ and depending on conditions as given in section 4.1.
5. Fujita paper discussed only for the case when price elasticity is $\varepsilon$ zero. This can be explained using this model with the above finding.
6. $\frac{dn_1}{dq} > \frac{dn_c}{dq}$ and $\frac{dn_1}{dq} < \frac{dn_c}{dq}$ are possible depending up on economic conditions
7. The above analysis is carried out assuming the open city model, where as in case of closed city model the firm density at city centre may increase; decreasing the firm density at the surrounding areas this can be examined further.

### 5. Conclusions

This paper focused on exploring the second best measures in the form of toll tax to mitigate location eternality. As the implementation of first best measures is difficult in real economy due to the complex nature of firms network and huge tax collection and implementation cost. We have developed model showing the market solution firm density is lesser than social optimum firm density in city centre as well as at surrounding zones, because the firms are not optimally located in market solution in the presence of location externality. To mitigate this problem we have explored a optimal toll system assuming a open city model. With respect to present model we can work out impact of toll tax on firm density with different price elasticity options. However, this can be applied to closed city model also with appropriate changes in the analysis further. The analysis is based on generalised production function. To get exact values of $n_1$ and $n_c$ and optimal pricing, we can carry out the analysis with a specific production function.

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