II －56

Development of Coupled Water Quality-Hydrological Model in the Mekong River: A Sediment Transport Case

Tohoku University Student member ○Pichet CHAIWIWATWORAKUL
Tohoku University Member Masaki SAWAMOTO
Tohoku University Member So KAZAMA

1. Introduction

The Mekong River is the major river feeding life to millions of people along its route from Southern China to Vietnam. Since the Mekong River is vital on international scale, any development projects done on the river are needed to be carefully assessed. However, some development projects by local authorities have caused many ecological problems especially to the downstream. Hence, there have to be superior management tools to resolve or minimize such problems and grant sustainability to imminent projects.

Water quality is another topic which directly concern to river ecosystem and water consumption. Since human activities nearby the river have increased, more water is required and more pollution problems are concerned. On the contrary, proper water quality management scheme is not yet actually realized. Hydrological model is used to represent the processes occur in the real catchment. A number of hydrological models are continually developed and already applied to the watershed. However, to ensure the sustainability of any development projects, it is necessary to figure out both hydrodynamic, water quality and the relationship between them. Therefore, water quality/hydrological model then comes to crucial.

Sediment transport is one of the essential keys required for other more advance parameters such as bacteria and viruses. Transportation of such pathogens has close relationship to the suspended solid content in water body (Sakoda et. Al, 1997, Schernewski & Jülich, 2001, Skraber et. Al, 2004).

This study focuses on the development of the sediment transport right from the basic and the numerical methods to solve numerical problems.

2. Advection-Dispersion Equation and Solution Schemes

The equation for the mass transport for one-dimensional unsteady flow reads:

$$\frac{\partial AC}{\partial t}+\frac{\partial QC}{\partial x}-\frac{\partial}{\partial x}\left(A\cdot D\frac{\partial C}{\partial x}\right)=-A\cdot S \qquad (1)$$

where A is the cross-sectional area, Q is the flow, D is the dispersion coefficient, x is the space coordinate and t is the time coordinate.

The numerical solution for the equation can be developed by substituting finite difference approximations for the derivatives. The explicit finite difference is used in the study due to its appropriateness to both linear and non-linear problems which are typical in water quality modeling although its stability has to be considered.

$$\frac{\partial C}{\partial t}\cong\frac{C_i^{t+1}-C_i^t}{\Delta t} \qquad (2)$$

$$\frac{\partial^2 C}{\partial x^2}\cong\frac{C_{i+1}^t-2C_i^t+C_{i-1}^t}{\Delta x^2} \qquad (3)$$

Forward Time Back Space (FTBS)

$$\frac{\partial C}{\partial x}\cong\frac{C_i^t-C_{i-1}^t}{\Delta x} \qquad (4)$$

Forward Time Center Space (FTCS)

$$\frac{\partial C}{\partial x}\cong\frac{C_{i+1}^t-C_{i-1}^t}{2\Delta x} \qquad (5)$$

The straightforward methods, FTBS and FTCS schemes are plagued by numerical dispersion as shown in Fig1.

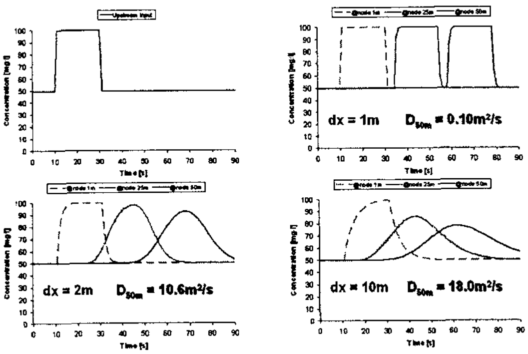

Fig.1. Effect of numerical dispersion of FTBS

McCormack Method has been introduced in order to enhance the calculation stability and eliminate the effect of numerical dispersion. It consists of two steps of calculation, predictor by forward difference (Eqn.6, 7) and corrector by backward difference (Eqn.8, 9).

$$S_{1,i}=-U\frac{C_{i+1}^t-C_i^t}{\Delta x}+D\left(\frac{C_{i+1}^t-2C_i^t+C_{i-1}^t}{\Delta x^2}\right) \qquad (6)$$

$$C_i^{t+1}=C_i^t+S_{1,i}\Delta t \qquad (7)$$

$$S_{2,i}=-U\frac{C_i^{t+1}-C_{i-1}^{t+1}}{\Delta x}+D\left(\frac{C_{i+1}^{t+1}-2C_i^{t+1}+C_{i-1}^{t+1}}{\Delta x^2}\right) \qquad (8)$$

$$C_i^{t+1}=C_i^t+\left(\frac{S_{1,i}+S_{2,i}}{2}\right)\Delta t \qquad (9)$$

However, the McCormack method cannot provide good prediction for the sharp front either (Fig 2).

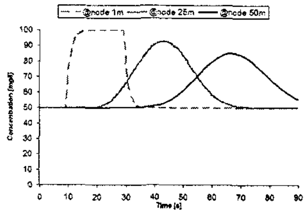

Fig.2. Sharp front smeared out by McCormack scheme

QUICKEST is another explicit finite difference approximation method, developed for unsteady, nonlinear equations. It uses a three-point upstream-weighted quadratic interpolation for the wall values of the independent variables in a control volume.

$$\begin{aligned} C_i^{t+1} = C_i^t + \frac{Cr_l}{2}\Big[\big(C_{i-1}^t + C_i^t\big) - Cr_l \cdot \Delta x \cdot grad_l - \\ \frac{\Delta x^2}{3}\big(1 - Cr_l^2 - 3D\big)curv_l\Big] - \frac{Cr_r}{2}\Big[\big(C_i^t + C_{i+1}^t\big) - \\ Cr_r \cdot \Delta x \cdot grad_r - \frac{\Delta x^2}{3}\big(1 - Cr_r^2 - 3D\big)curv_r\Big] + \\ D\Big[\Big(\Delta x \cdot grad_r - \frac{\Delta x^2}{2} \cdot Cr_r \cdot curv_r\Big) - \\ \Big(\Delta x \cdot grad_l - \frac{\Delta x^2}{2} \cdot Cr_l \cdot curv_l\Big)\Big] \end{aligned} \quad (10)$$

with

$$grad_l = \frac{\big(C_i^t - C_{i-1}^t\big)}{\Delta x}, \quad grad_r = \frac{\big(C_{i+1}^t - C_i^t\big)}{\Delta x},$$

$$curv_l = \frac{\big(grad_r - grad_l\big)}{\Delta x}, \quad curv_r = \frac{\big(grad_{r+1} - grad_l\big)}{\Delta x},$$

and Cr_l, Cr_r = Courant number at the left and right cell walls

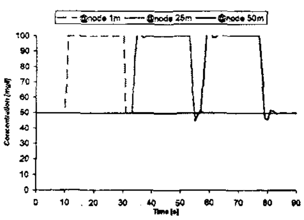

Fig.3. Result by QUICKEST scheme

From Figure 3, the QUICKEST method yields significant better result in term of sharp front than McCormack method. However, as other pure explicit finite difference method, the stability has to be careful.

3. Cohesive Sediment Transport

Cohesive Sediment Transport (CST) is the simplest fashion to model the transportation of sediment, consists of 2 parts; deposition and erosion. The rate of deposition can be expressed by:

$$S = \frac{wC}{h_*}\left(1 - \frac{\tau}{\tau_{cd}}\right) \qquad \text{for}\ (\tau \le \tau_{cd}) \qquad (11)$$

where S is the source/sink term in the advection dispersion equation, C is the concentration of the suspended sediment, w is the mean settling velocity of suspended particles, h_* is the average depth through which the particles settle, τ_{cd} is the critical shear stress for deposition and τ is the bed sheer stress. The rate of erosion has been described by the expression:

$$S = \frac{M_*}{h}\left(1 - \frac{\tau}{\tau_{ce}}\right) \qquad \text{for}\ (\tau \ge \tau_{ce}) \qquad (2)$$

where τ_{ce} is the critical sheer stress for erosion, M_* is the erodibility of the bed and h is the flow depth.

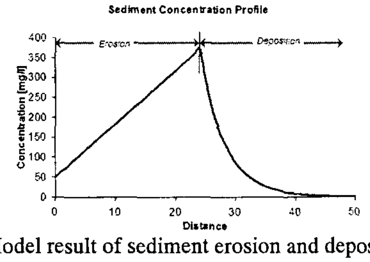

Fig.4. Model result of sediment erosion and deposition

4. Summary

There are many numerical methods available for the approximation of partial differentiate equation nowadays. Each method has its pros and cons. Explicit finite difference is convenient for water-quality modeling since nonlinear terms cannot be directly solved using matrix algebra approaches easily. QUICKEST method is the best explicit finite difference scheme for nonlinear equations, while McCormack method is better for linear system and has ability to enhance itself the calculation stability.

CST is simple but can produce unrealistic results since there is no sediment budget taken to account. The erosion term can cause very high suspended solid in water body for the continually high velocity flow.

Acknowledgements

A part of this study was supported by a Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan.

References

1. Chapra, S. C., 1997. Surface Water-quality Modeling, McGraw-Hill, New York.
2. Chen, Y., Falconer, R.A., 1992. Advection-diffusion modeling using the modified QUICK scheme, International Journal for Numerical Methods in Fluids, 15: 1171-1196
3. DHI Water & Environment, 2003. Mike11: A Modeling System for Rivers and Channels Reference Manual.
4. Sakoda, A., Sakai, Y., Hayakawa, K., Suzuki, M. 1997. Adsorption of viruses in water environment onto solid surfaces. Wat.Sci.Tec. 7:107-114.
5. Schernewski, G., Jülich, W. 2001. Risk assessment of virus infections in the Oder estuary (southern Baltic) on the basis of spatial transport and virus decay simulations. Int.J.Hyg.Environ.Health 4:317-325.