Ⅱ－45 ESTIMATION OF BOTTOM SHEAR STRESS UNDER SKEW WAVES

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1. INTRODUCTION

Realistic waves in nature often have a shape of sawtooth wave or skew wave when propagating to shallow water, their heights increase and their lengths decrease, furthermore they become remarkably nonlinear waves. Moreover, both wave velocity skewness and asymmetric increase to their maximum value at the onset of breaking. Hence, the simple harmonic variation as sinusoidal wave can not describe the boundary layer behavior occurring in the surf zone in which major part of nearshore sediment are transported.

Bottom shear stress under skew waves can be used in a sediment transport modeling under rapid acceleration occurring in the surf zone. Schäffer and Svendsen (1986) had presented the sawtooth wave as a method expressing the wave motion under broken waves. Suntoyo and Tanaka (2003) had investigated bottom shear stress under sawtooth waves and examined it with two simple calculation method of bottom shear stress based on the consideration of the friction coefficient for sinusoidal wave motion and that of the acceleration effect for sawtooth wave. However, an investigation of a more reliable calculation method to estimate the time-variation of bottom shear stress has not been fully dealt with.

In the present study, we aim to examine the bottom shear stress through experiments in an oscillating wind tunnel over rough bed under sawtooth waves by means of Laser Doppler Velocimeter (LDV) to measure velocity distribution. Furthermore, a new estimation method of the instantaneous bottom shear stress under sawtooth waves based on incorporating both velocity and acceleration terms are proposed.

2. EXPERIMENTAL METHOD

The experiments have been carried out in an oscillating wind tunnel connected with the piston system with air as the working fluid and smoke particle as tracer. Velocity was measured by means of laser Doppler velocimeter (LDV) installed in the middle section of wind tunnel. The wind tunnel is connected to the piston system that has a dimension of 5 m length, 20 cm and 10 cm in height and width, respectively. The bottom roughness elements used in this experiment are the triangular shape elements having a dimension of 5 mm height and 10 mm width, which are pasted over the bottom surface of the wind tunnel with distance of 12 mm along the wind tunnel.

Table 1 Experimental conditions

Exp.	U_c (cm/s)	T(s)	a_m/k_s	α
Case 1	398	4.00	35.1	0.314
Case 2	399	4.00	35.1	0.363
Case 3	400	4.00	35.2	0.406
Case 4	400	4.00	35.2	0.500

Experiments have been carried out for four cases under sawtooth waves. The experimental conditions are given in **Table 1**. Here, a_m/k_s: the roughness parameter, k_s: the Nikuradse's roughness equivalent and a_m=U_c/ω,where, U_c: the velocity at wave crest, ω: the angular frequency, T: wave period, t_p: time interval measured from the zero-up cross point to wave crest in the time variation of free stream velocity, while α shows the wave skewness parameter, as shown in **Fig.1**. The smaller α indicates more remarkable wave skewness, while α=0.5 corresponds to the symmetric wave without skewness.

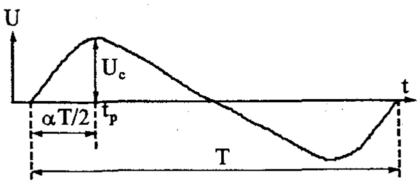

Fig.1 Definition sketch for sawtooth wave

3. RESULTS AND DISCUSSIONS

Bottom shear stress is estimated by fitting the logarithmic velocity distribution to the measured velocity data, which is given in Eq. (1),

$$u = \frac{U_*}{\kappa}\ln\left(\frac{z}{z_o}\right) \tag{1}$$

where, u: the flow velocity in the boundary layer, κ : the von Karman's constant (=0.4), z: the cross-stream distance from theoretical bed level.

A new calculation method of bottom shear stress under sawtooth waves is based on incorporating velocity and acceleration terms all at once that is given through the instantaneous friction velocity, $U_*(t)$ as proposed by authors in Eq. (2). Both velocity and acceleration terms are adopted from a calculation method proposed by Nielsen (2002), but that method could not give a good agreement with experimental data, so in the new calculation method is proposed a new coefficient expressing the wave skewness effect on the bottom shear stress under sawtooth waves. The instantaneous bottom shear stress can be calculated proportional to the square of the proposed instantaneous friction, as shown in Eq. (3),

$$U_*(t)=\sqrt{f_w/2}\left\{U\left(t+\frac{\varphi}{\omega}\right)+\frac{a_c}{\omega}\frac{\partial U(t)}{\partial t}\right\} \tag{2}$$

$$\tau_0(t)=\rho U_*(t)|U_*(t)| \tag{3}$$

The value of a_c is obtained from average value of the time variation of acceleration coefficient, $a_c(t)$ as given in Eq. (4)

$$a_c(t)=\frac{U_*(t)-\sqrt{f_w/2}U\left(t+\frac{\varphi}{\omega}\right)}{\frac{\sqrt{f_w/2}}{\omega}\frac{\partial U(t)}{\partial t}} \tag{4}$$

where, f_w: the wave friction coefficient. The friction coefficient proposed by Tanaka and Thu (1994) can be used for evaluating in Eq. (3). $\tau_0(t)$: the instantaneous bottom shear stress, φ: the phase difference between free stream velocity and bottom shear stress and a_c: the acceleration coefficient.

Furthermore, there are three approximation of phase difference that are used to obtain the value of a_c, namely (i) phase difference based on an equation proposed by Tanaka and Thu (1994), as given in Eq. (5); (ii) phase difference obtained from average value between crest and trough part of measured data under sawtooth waves and (iii) phase difference calculated from Eq. (7) based on Eq. (5) with consider the wave skewness effect under sawtooth waves.

$$\varphi = 42.4C^{0.153} \frac{1+0.00279C^{-0.357}}{1+0.127C^{0.563}} \text{ (degree)} \qquad (5)$$

$$C = \frac{1}{\kappa\sqrt{\frac{f_w}{2}\frac{a_m}{z_0}}} \qquad (6)$$

$$\varphi' = \frac{\alpha T}{2}\frac{4\varphi}{T} \text{ (degree)} \qquad (7)$$

Hereafter, the values of a_c as function of α from three approximation of phase difference are plotted in **Fig. 2**. The value of a_c based on phase difference from Eq. (7) gives a better result than others as shown in the correlation result of friction velocity, U_* between experiment and calculation results as shown in **Fig. 3**. Furthermore, an equation based on regression line to estimate the acceleration coefficient, a_c is proposed as given in Eq. (8),

$$a_c = -1.945\alpha + 0.975 \qquad (8)$$

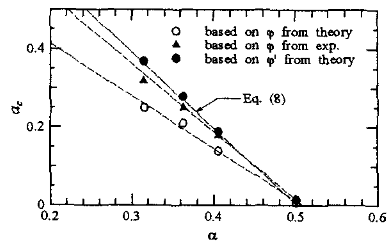

Fig. 2 Acceleration coefficient, a_c as function of α

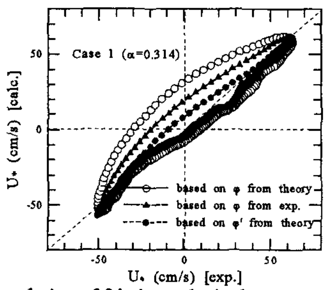

Fig. 3 Correlation of friction velocity between experiment and calculation result, Case 1

To examine a new calculation method of bottom shear stress under sawtooth waves is used the existing calculation method, namely Method 1 and Method 2, as shown by Suntoyo and Tanaka (2003). Method 1 is proportional to the square of the time variation of $U(t)$, in Eq. (9)

$$\tau_0\left(t-\frac{\varphi}{\omega}\right) = \frac{1}{2}\rho f_w U(t)|U(t)| \qquad (9)$$

Method 2 is proportional to the square of the instantaneous friction velocity, $U_*(t)$ incorporating the acceleration effect under a bit of sawtooth asymmetric wave as proposed by Nielsen 2002 in Eq. (10) and (11), as follows

$$U_*(t) = \sqrt{\frac{f_w}{2}}\left\{\cos\varphi U(t) + \sin\varphi \frac{\partial U(t)}{\partial t}\right\} \qquad (10)$$

$$\tau_0(t) = \rho U_*(t)|U_*(t)| \qquad (11)$$

Phase difference equation given in Eq. (7) is used for calculating in Method 1 and Method 2.

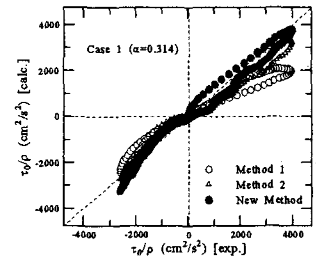

Fig. 4 Correlation between experimental and calculation results of bottom shear stress, for Case 1

Correlation between the experimental and calculation results of bottom shear stress from three methods are shown in **Fig. 4**. Method 1 and Method 2 give underestimated value at crest part, while at trough part from three methods give almost the same estimated value, as shown in **Fig. 4** that new method give the best agreement with the bottom shear stress under sawtooth waves from experimental results.

4. SUMMARY

A new method gave the best agreement with bottom shear stress under sawtooth waves from experimental result than others methods. Furthermore, the phase difference and average acceleration coefficient defined in the new method was sufficient for this calculation. It can be concluded that the new method of instantaneous bottom shear stress under sawtooth waves proposed in this study has a sufficient accuracy. Therefore, this method can be used to an input sediment transport model under rapid acceleration in a practical application.

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