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MODELING THE WATER TABLE VARIATIONS IN A MOUNTAIN CATCHMENT

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1. INTRODUCTION

The study proposes a new scheme to estimate the distributed phreatic surface fluctuation and soil moisture variation in a mountain catchment, where the number of observed points could be highly limited. The phreatic surface fluctuation is described using a stochastic differential equation (SDE)[1)]. The spatial distribution of water table depth is obtained taking into account the topographic index value, calculated as a function of contributing area and slope on each location.

Using proposed scheme, the daily map of water table depths were predicted in a catchment during the whole year, allow obtaining the full space-time representation of aquifer screen variations over a large area.

2. DATA

The study area is 970 km^2 within the Natori River catchment, Japan. The soil texture data was observed within a catchment. There are three main soil types associated with land use distribution. Coarse textured sandy loam is in the forest area, medium-textured soil is in the paddy field and the fine-textured silt loam is found in the crop fields.

The elevation of the watershed ranges from 0 to 1470 m. The resolution of elevation data is 1km. Water body areas were deleted from the maps and are not considered in this study. The meteorological data was obtained from three observation stations on a daily frequency from the AMEDAS database, which includes rainfall data, air temperature and sunshine duration.

2. METHODOLOGY

3-1 Stochastic Differential equation (SDE)

As a basic idea, the conception of the SDE model proposed by Bierkens[1)] describing the water balance at a single location was used.

The stochastic differential equation is given by:

$$dh = a(h,t)dt + b(h)d\beta_t \qquad (1)$$

with

$$a(h,t) = \left[\frac{P(t) - AE(h,t) + q_v(t) - q_d(t)}{G(h)}\right] \qquad (2)$$

$$b(h) = \left[\frac{\sigma}{G(h)}\right] \qquad (3)$$

where $h(t)$ is the height of the water table at time t with respect to sea level. $d\beta_t$ is a wide band noise process increments described by the Wiener-Levy process (Brownian motion), where $E[d\beta_t] = 0$ and $E[d\beta_t^2] = dt$ [2)]. The variable σ is the magnitude of noise processes. In this study only deterministic part of equation (1) was considered, however, in the mountainous area SDE model gives good results even without stochastic increment. $q_d(t)$ is contribution to the surface water derived by Darcy's law. Obtaining of precipitation $P(t)$, actual evapotranspiration $AE(h,t)$ and regional flux $q_v(t)$ from/to deeper groundwater are described by Bierkens[1)].

The expression for deriving a dynamic storage coefficient is given by[1)]:

$$G(h) = \varepsilon_0 + (\theta_s - \theta_r)(1 - \{1 + [\alpha \cdot (z-h)]^n\}^{-\frac{n+1}{n}}) \qquad (4)$$

where θ_s and θ_r are the saturated and residual moisture contents of soil, respectively and ε_0 is the residual groundwater storage. Parameters α and n are constants, depended on the soil charasteristics[3)].

3-2 Spatial distribution of water table depth

The topographic index I_x, can be calculated for each cell by the following formula[4)]:

$$I_x = \ln\left(\frac{a}{\tan\beta}\right) \qquad (5)$$

where a is the upslope area, per unit contour length, contributing flow to the location. In case study unit area is pixel with size $1 \times 1km$, $\tan\beta$ is local slope between current cell and downstream cell in flow direction.

The simplified expression for water table depth can be written as

$$wd = \overline{wd} - \frac{1}{f}\{I_x - \lambda - \ln(K_0) + \tilde{\gamma}\} \qquad (6$$

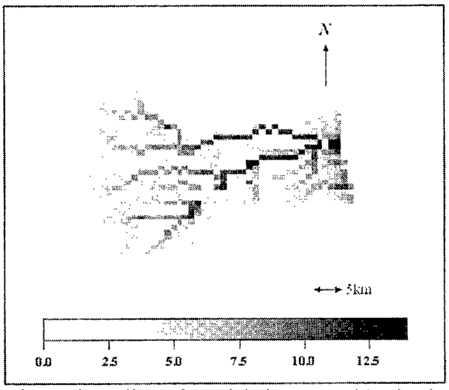

Fig.1 Time-slice of modeled water table depth for DOY 110, April 19,1992.

where $\tilde{\gamma} = \frac{1}{A}\int_A \ln(K_0)dA$ is the averaged catchment values of $\ln(K_0)$. The K_0 value can be derived from standard retention curve using soil texture information (Wosten and Genuchten, 1988) and $\tilde{\gamma}$ can be calculated as a mean value for the catchment[3]. Consequently, in equation (6), only two parameters $\overline{wd}$ and f are unknown. By definition, the parameters $\overline{wd}$ and f are positive constants for the catchment at fixed time and can be easily derived from equation (6) by fitting measured values of the water table depths from two locations and solving obtained set of linear equations in two unknowns.

Finally, the spatial distribution of water table depth for each time step can be modeled by equation (6) from two water table depths at measuring points.

4. RESULTS AND DISCUSSIONS

In the study catchment, the SDE was applied at four locations coded as p1, p2, p3, p4, respectively, with data set in 1992. The location p1 is covered with woods and other locations are in the paddy fields. Fig.1 shows modeled map of water table depths. For validation of results, the general water budget of catchment was used:

$$P - (Q + AE) = \Delta S \tag{7}$$

where Q is measured daily average discharge, ΔS is daily average storage deficit.

From deterministic part of SDE, the quantity of modeled daily average storage deficit ΔS can be calculated from equation (1) as follows:

$$\Delta S = \frac{1}{A}\int_A (-\Delta wd) \cdot G \cdot dA \tag{8}$$

with $\Delta wd = wd - wd_{-1}$, where wd and wd_{-1} are water table depths for present and preceding day, respectively.

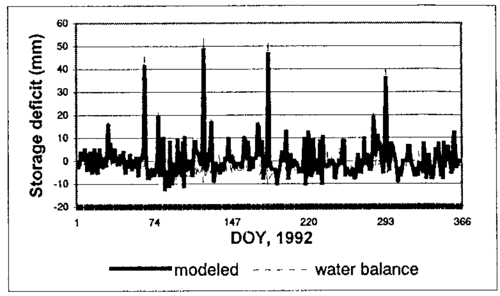

Fig.2. Comparison between modeled and predicted storage deficit values

G is the dynamic storage coefficient is given by equation (4).

Comparison between average storage deficit values, accounted from the general water balance equation and modeled value from equation (1) is shown in Fig.2. Modeling results have acceptable agreement with general water balance. The Kamafusa lake watershed is located in a mountainous area with 565m as the average elevation. The basin area is 197 km^2. This validation result shows that the model is suitable for whole catchment area applications, including both high elevation region and lower lands, although all calibration points are located in lower lands.

5. SUMMARY

This paper proposed a simple scheme for the assessment of spatial and temporal distribution in water table variation in a mountainous catchment. The equation to describe the water table fluctuation process, proposed by Bierkens was developed using the topographic characteristics of the catchment and the distribution of water table depths in each time step was predicted. Modeling results were consistent with observed data and the water budget of the catchment. In the case of scant observed data, described scheme can give a preliminary representation of full space-time distribution of aquifer screen characteristics in the Natori River basin in Japan.

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