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Representation of Boundary Conditions and Frequency Equations in Timoshenko Beam Theory only in terms of Total Transverse Displacement

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1. Introduction

In Huang's theory, boundary conditions and frequency equations are expressed by the use of total transverse displacement and bending slope. [1]

In this paper, they are given by only total transverse displacement.

2. Equation of motion and Eigenfunction

Governing equation for total transverse displacement y is as follows; [2]

$$\rho A\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}-\left(\rho I+\frac{\rho EI}{kG}\right)\frac{\partial^4 y}{\partial x^2\partial t^2}+\frac{\rho^2 I}{kG}\frac{\partial^4 y}{\partial t^4}=0, \tag{1}$$

where $y(x,t)$= total transverse displacement, x= axial directional coordinate, t= time, ρ= density of bar, A= area of cross-section, E= modulus of longitudinal elasticity, G= modulus of transverse elasticity, I= second moment of area of cross-section, k= coefficient of shear deformation, ω= angular frequency, $i=\sqrt{-1}$.

Let $y(x,t)$ be

$$y(x,t)=Y(x)\exp(i\omega t). \tag{2}$$

Substituting equation (2) into equation (1) yields

$$\frac{d^4 Y}{d\xi^4}+(a+1)\frac{X^4}{H^2}\frac{d^2 Y}{d\xi^2}+X^4\left[a\left(\frac{X}{H}\right)^4-1\right]Y=0, \tag{3}$$

where $X^4=\dfrac{\rho A\omega^2 L^4}{EI}$, $a=\dfrac{E}{kG}$, $H=\dfrac{L}{R}$, $R=\sqrt{\dfrac{I}{A}}$, $\xi=\dfrac{x}{L}$, L =length of bar.

In case of $a\left(\dfrac{X}{H}\right)^4-1<0$;

$$Y=C_1\sin\alpha\xi+C_2\cos\alpha\xi+C_3\sinh\beta\xi+C_4\cosh\beta\xi, \tag{4}$$

where

$$\alpha=\frac{1}{\sqrt{2}}\frac{X^2}{H}\sqrt{(a+1)+\sqrt{(a-1)^2+4\left(\frac{H}{X}\right)^4}}, \tag{5}$$

$$\beta=\frac{1}{\sqrt{2}}\frac{X^2}{H}\sqrt{-(a+1)+\sqrt{(a-1)^2+4\left(\frac{H}{X}\right)^4}}, \tag{6}$$

C_1,C_2,C_3,C_4 = integral constants.

3. Boundary conditions

Boundary conditions in four cases [2] are expressed only in terms of Y as follows;

(1) Supported: $Y=\dfrac{d^2 Y}{d\xi^2}=0$ (7)

(2) Clamped : $Y = \left(\frac{H^2}{a} + \frac{aX^4}{H^2}\right)\frac{dY}{d\xi} = 0$ (8)

(3) Free : $\frac{d^2Y}{d\xi^2} + \frac{aX^4}{H^2}Y = \frac{d^3Y}{d\xi^3} + (a+1)\frac{X^4}{H^2}\frac{dY}{d\xi} = 0$ (9)

(4) Rotation restrained : $\frac{dY}{d\xi} = \frac{d^3Y}{d\xi^3} = 0$ (10)

4. Frequency equations

Frequency equations are classified into ten cases. In case of $a\left(\frac{X}{H}\right)^4 - 1 < 0$,

(1) Supported-Supported : $\sin\alpha = 0$ (11)

(2) Clamped-Supported : $\sin\alpha\cosh\beta + K_3\cos\alpha = 0$ (12)

(3) Rotation Restrained-Supported : $\cos\alpha\cosh\beta = 0$ (13)

(4) Supported-Free : $\mu\delta\sin\alpha\cosh\beta - K_4\cos\alpha\sinh\beta = 0$ (14)

(5) Clamped-Clamped : $2 - 2\cos\alpha\cosh\beta + \left(K_3 - \frac{1}{K_3}\right)\sin\alpha\sinh\beta = 0$ (15)

(6)Clamped-Free :

$K_3\mu - K_4\delta + (\mu\delta K_3 - K_4)\cos\alpha\cosh\beta + (K_3K_4 + \mu\delta)\sin\alpha\sinh\beta = 0$ (16)

(7) Clamped-Rotation Restrained : $\cos\alpha\sinh\beta - K_3\sin\alpha\cosh\beta = 0$ (17)

(8) Free-Free : $2 - 2\cos\alpha\cosh\beta + \left(\frac{\mu\delta}{K_4} - \frac{K_4}{\mu\delta}\right)\sin\alpha\sinh\beta = 0$ (18)

(9) Rotation Restrained-Free : $\mu\delta\cos\alpha\sinh\beta + K_4\sin\alpha\cosh\beta = 0$ (19)

(10) Rotation Restrained- Rotation Restrained : $\sin\alpha = 0$ (20)

where $K_3 = (K_1 - H^2/a)/(\mu(K_2 + H^2/a))$, $K_4 = (K_1 - X^4/H^2)/(K_2 + X^4/H^2)$,
$K_1 = \alpha^2 - aX^4/H^2$, $K_2 = \beta^2 + aX^4/H^2$, $\mu = \beta/\alpha$, $\delta = K_1/K_2$

5. Conclusion

Eigenvalues obtained from frequency equations only in terms of total transverse displacement have completely coincided with those obtained from frequency equations given by Huang.

This implies that both boundary conditions and frequency equations introduced above are exact.

References

[1] T.C. Huang, 1961, The Effect of Rotatory Inertia and of Shear Deformation on the Frequency and Normal Mode Equations of Uniform Beams With Simple End Conditions, Transactions of the ASME. Series E, Applied Mechanics 28, 4, 579-584.

[2] M.Kuroda,1982,Mechanical Vibrations (In Japanese) ,Tokyo,Gakkensya.