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LAMINAR BOUNDARY LAYER CHARACTERISTICS UNDER IRREGULAR WAVES

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1 Introduction

Boundary layer under waves govern different key parameters near the bed those control several physical processes like sediment transport. Under non-breaking waves the turbulent energy is contained in this thin layer bringing about the changes in and near the bed. As ocean waves are random in nature, it is, therefore, necessary that the near bottom flow dynamics are investigated along with the effects of the randomness of sea waves. When practical application is concerned, this might provide improved description of near bed turbulence generation and dissipation processes. Present paper describes the effects of random or irregular waves on bottom boundary layer under laminar flow conditions analyzed through numerical computation. Here, firstly, the irregular wave is generated and it is then applied to compute boundary layer properties through low Reynolds number k-ε model.

2 Governing Equations

2.1 k-ε Model

Boundary layer turbulent structure has been investigated applying Jones and Launder [1] original low Reynolds number k-ε model. Details of the modeling system can be found in [2, 3]. In the model at the free stream boundary, generated irregular wave velocity time series has been applied.

2.2 Irregular Wave

The spectral density function for irregular water surface elevation can be computed using Bretschneider-Mitsuyasu [4] spectral density formulation as given by:

$$S_\eta(f) = 0.257 H_{1/3}{}^2 T_{1/3}\left(T_{1/3} f\right)^{-5} \exp\left[-1.03\left(T_{1/3} f\right)^{-4}\right] \quad (1)$$

where, $H_{1/3}$ and $T_{1/3}$ are significant wave height and period respectively, and f is the wave frequency. Applying small amplitude wave theory, following relationship can be obtained for spectral densities of water surface elevation and free stream velocity:

$$S_u(f) = H_u{}^2(f) S_\eta(f) = \left(\frac{\omega}{\sinh kh}\right)^2 S_\eta(f) \quad (2)$$

where, $S_u(f)$ and $S_\eta(f)$ are spectral densities for velocity and surface elevation respectively, $H_u(f)$ is velocity transfer function, k is wave number, h water depth, ω $(=2\pi f)$ angular frequency and μ $(=\nu\rho)$ is the dynamic viscosity.

Obtained velocity spectral density has then been used to generate velocity time variation considering that irregular waves can be resolved as a sum of infinite number of wavelets with small amplitudes and random phases [5]. The summation equation stands as:

$$u(t) = \lim_{k\to\infty} \sum_{i=1}^{k} A_{ui} \cos(2\pi f_i t + \phi_i) \quad (3)$$

$$A_{ui} = 2\sqrt{S_u(f_i)\Delta f_i}$$

where, $u(t)$ is instantaneous velocity at a point, A_{ui} velocity amplitudes of component waves, f_i component frequencies, ϕ_i component phases and Δf_i is frequency increment between successive wave components.

3 Results and Discussions

3.1 Irregular Wave Simulation

Simulation of irregular wave has been performed to generate sufficiently long time series to use in the k-ε model. The input wave parameters specified for computation have been the significant wave height, $H_{1/3}$=0.5cm and significant wave period, $T_{1/3}$=2.0sec with a water depth (z_h) of 10 cm. The conditions were selected to produce a flow in the laminar region.

The accuracy of generated date has been checked against the input spectrum and the result is quite satisfactory. Figure 1 shows the comparison of input (eq.2) and generated velocity spectrums. As can be seen from the figure that the generated velocity spectrum matches very well with input spectrum. The wave energy is spreaded in the frequency range of 0.25 to 1.5 or equivalently within T = 0.7 to 4.0sec.

3.2 k-ε Model Simulation

Bed shear stress spectrum from k-ε model result has been compared with that obtained analytically for laminar boundary layer flows (Fig.2). The later is given by:

$$S_\tau(f) = H_\tau{}^2(f) S_u(f) = \left(\sqrt{2}\beta\mu\right)^2 H_u{}^2(f) S_\eta(f) \quad (4)$$

$$H_\tau = \sqrt{2}\beta\mu\frac{\omega}{\sinh kh} \quad \text{with} \quad \beta = \sqrt{\frac{\omega}{2\nu}} \quad (5)$$

where, $H_\tau(f)$ is bed shear stress transfer function. For k-ε model the transfer function can be computed as, $H_\tau(f) = S_\tau(f)/S_u(f)$. Figure 3 shows the comparison of shear stress transfer function from k-ε model with that obtained analytically (eq.4). The spectral densities in Fig.2 show a very good agreement. Here larger spectral density concentration can be seed at higher frequency region than that observed for velocity spectrum. This indicates that high frequency components dominate more for bed shear stress. This can also be seen in Fig.3 where the peak of transfer function appears at a higher frequency than that

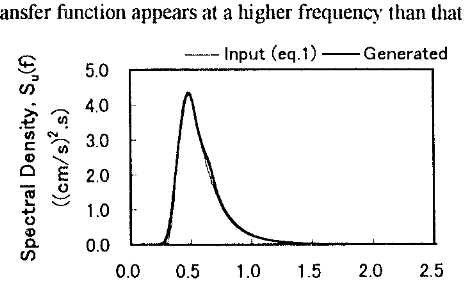


Fig.1: Input and generated wave velocity spectral density

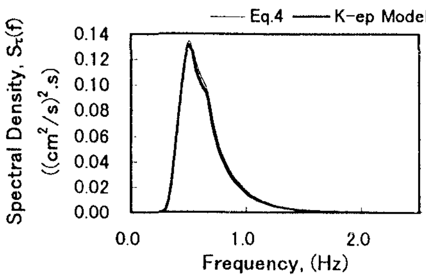


Fig.2: k-ε model and generated shear stress spectrums

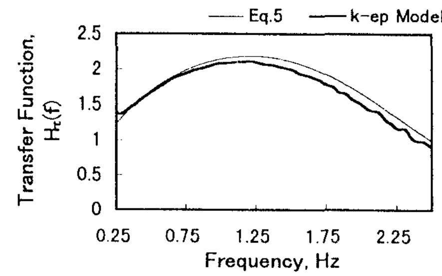
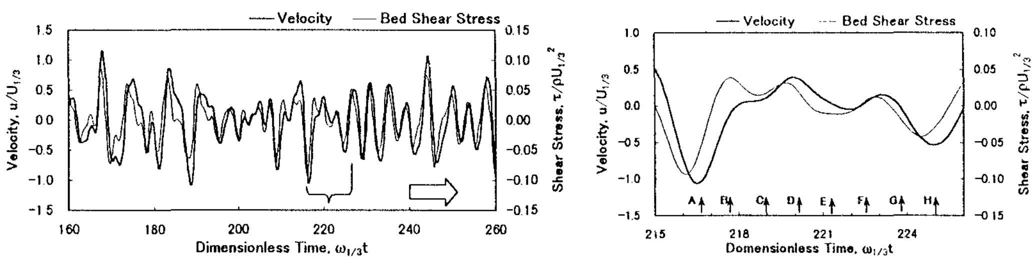


Fig.3: Comparison of bed shear stress transfer functions



(a)

(b)

Fig.4: Velocity and shear stress time series from k-ε model

corresponding to significant wave period.

The *k-ε* model generated free stream velocity time series and corresponding bed shear stress shows some interesting phenomenon (Fig.4). Laminar bed shear stress here can be seen to be much sensitive to free stream velocity variation with a domination of higher frequency components. It can be seen that the bed shear stress does not always respond in the same way to similar velocity conditions. This feature is further illustrated in Fig.5. It shows vertical velocity distribution corresponding to different phases as indicated in Fig.4(b). At phases B and E the velocities are nearly same in accelerating and decelerating phases respectively, but the resulting vertical velocity distribution show marked difference. At B higher acceleration caused a velocity overshooting compared to that at phase E.

4 Conclusions

Laminar boundary layer behavior under irregular wave motion has been investigated through numerical methods. The irregular wave velocity has first been generated and later it has been used for flow computation in the boundary layer through low Reynolds number *k-ε* model. The generation of irregular wave velocity shows a very satisfactory result when compared with input spectral properties.

k-ε model result has been compared with analytical solutions for laminar boundary layer and it shows very good agreement. It has been observed from model result that the bed shear stress is more dominated by high frequency component waves than that for velocity.

It has also been observed from the model result that the bed shear stress is very sensitive to variations in free stream velocity and it does not respond always in the same way to similar velocity conditions.

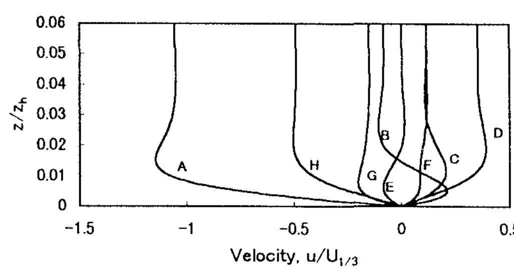


Fig.5: Vertical velocity distribution at different phases

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