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Numerical Shape Coefficient of Shear Deformation for Beam with Circular Cross-Section on Phase Velocity of Transverse Elastic Waves

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1. INTRODUCTION

In this paper, numerical shape coefficient of shear deformation for a beam with circular cross-section is determined based on the first mode curve of the phase velocity for transverse elastic waves obtained from Pochhammer [1] and Chree [2] .

2. PHASE VELOCITY

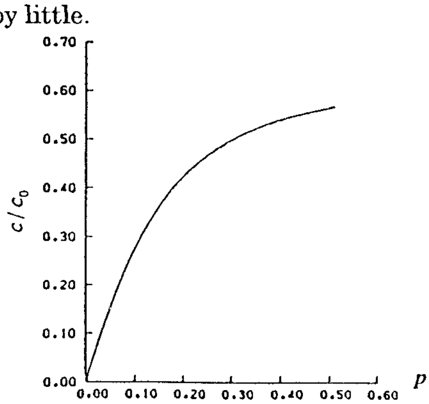
In circular cross-section, the first mode curve of phase velocity obtained from Timoshenko Beam Theory can be found as follows.

$$\frac{c}{c_0}=\frac{\sqrt{2}(\pi p)}{\left[1+(a+1)(\pi p)^2+\left\{1+2(a+1)(\pi p)^2+(a-1)^2(\pi p)^4\right\}^{\frac{1}{2}}\right]^{\frac{1}{2}}} \tag{1}$$

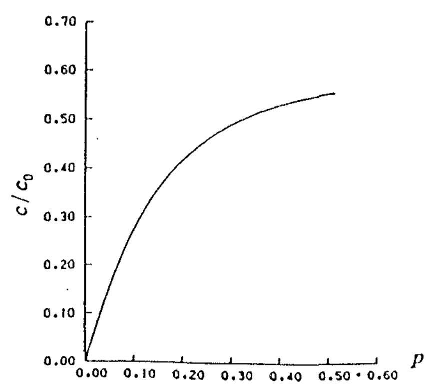
where $c=\dfrac{\omega}{\gamma}$, $c_0=\sqrt{\dfrac{E}{\rho}}$, $\gamma=\dfrac{2\pi}{\lambda}$, $a=\dfrac{E}{kG}=\dfrac{2(1+\nu)}{k}$, $p=\dfrac{\frac{d}{2}}{\lambda}$, c = phase velocity, ω = angular frequency, γ = wave number of transverse elastic waves, λ = wave length of transverse elastic waves, E = modulus of longitudinal elasticity of the beam, G = modulus of shear deformation, ν = Poisson's ratio, ρ = density, k = numerical shape coefficient of shear deformation, d = diameter of circular cross-section.

3. COMPUTATIONS

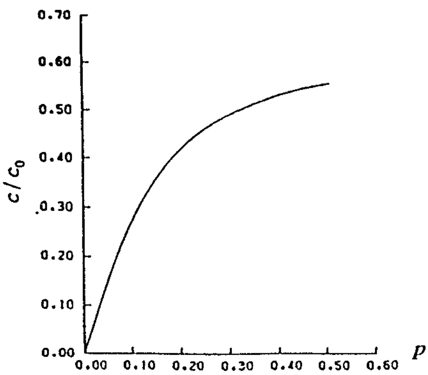
In Figure 1, results of the present authors' computations according to three-dimensional treatment [3] are compared with those according to Timoshenko's theory. The real line is the first mode curve of phase velocity obtained from Pochhammer and Chree, and the dotted line is that obtained from Timoshenko. In each case of (a)-(f), giving the numerical value of ν first, computations have been repeated untill a suitable value of k is found out with which two lines well coincide with each other, by changing the value of k little by little.



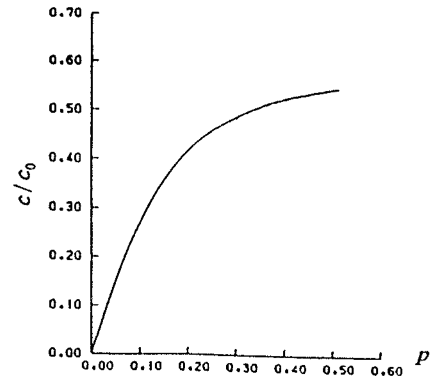
(a) $\nu=0.1, k=0.875$



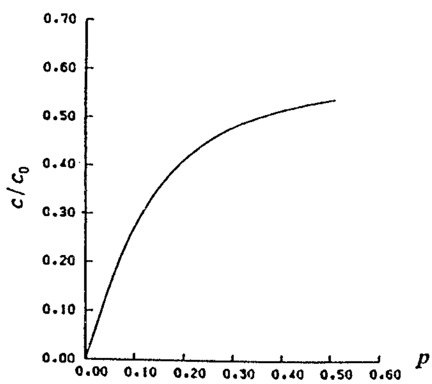
(b) $\nu=0.2, k=0.905$



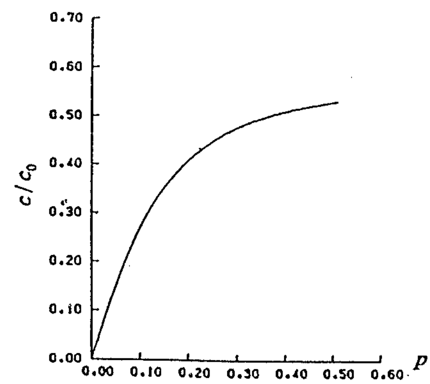
(c) $\nu = 0.25, k = 0.915$



(d) $\nu = 0.30, k = 0.925$



(e) $\nu = 0.35, k = 0.935$



(f) $\nu = 0.40, k = 0.945$

Figure 1. First mode curves of phase velocity computed from three-dimensional treatment and Timoshenko's theory

4. CONCLUSIONS

Numerical shape coefficient of shear deformation for a beam with circular cross-section depends on Poisson's ratio. For the suitable k, the real line and the dotted line well coincide with each other.

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