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DYNAMIC PLASTIC BEHAVIOR OF CIRCULAR PLATE IN VIEW OF UNIFIED YIELD CRITERION

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1. INTRODUCTION

The investigation on the dynamic plastic behavior of structures is getting more importance in the field of civil engineering after the Kobe earthquake. Almost all the previous studies[1][2] on the shear and bending response of a rigid plastic circular plate under transverse impulsive loading are based on the Tresca yield criterion. Little attention has been paid to analyze the influence of yield criterion on the dynamic plastic behavior of circular plates. In fact, it has been proved that the properties of a great number of metal materials agree well with the Mises yield criterion, as well as the properties of some mild steel and aluminum alloy close to twin shear stress yield criterion, a maximum principal deviatoric stress yield criterion (Hill, 1951). This paper applies the unified yield criterion (UYC) to get exact, analytical solutions for a simply supported circular plate under a distributed moderate pulse with rectangular pressure-time history. The previous solution of the Tresca yield criterion is a special case of this paper. The solution by using of the Mises yield criterion, which can not be obtained by analytical manner, is linearly approximated by that of UYC.

2. ANALYTICAL METHOD

The response history is divided into two phases $0 \le t \le \tau$, $\tau \le t \le T$ for the impulsive loading, where τ is the duration of pulse and T is the duration of response. Using non-dimensional variables, the governing equations for circular plates are as follows

$$\partial(rm_r)/\partial r - m_\theta - rq = 0 \qquad (1)$$

$$\partial(rq)/\partial r + rp - \mu r\ddot{w} = 0 \qquad (2)$$

$$\dot{k}_r = -\partial^2\dot{w}/\partial r^2\,,\ \ \dot{k}_\theta = -(\partial\dot{w}/\partial r)/r \qquad (3)$$

where m_r, m_θ, q and w are non-dimensional bending moments, shearing force and transverse deflection respectively, p is partial uniformly distributed rectangular pulse with non-dimensional loading radius as r_p ($0 \le r_p \le 1$). $p = p_0$ in the first phase of motion and $p = 0$ in the second phase of motion. Fig.1 shows unified yield criterion controlled by the generalized stresses in m_θ-m_r space. In plastic limit state, moments of the plate center (r=0) satisfies $m_r = m_\theta = 1$ (point A in Fig.1), the simply supported edge (r=1) satisfies $m_r = 0$ (point C in Fig.1). Bending moments of all the points in the plate are located in the sides AB and BC for the normality requirement of plasticity (Drucker's postulate). Eqs. (1), (2) and (3) derive the differential equations of m_r and $\dot{w}$ with the aid of the linear yield criterion (shown in Fig.1) and associate plastic flow rule as follows,

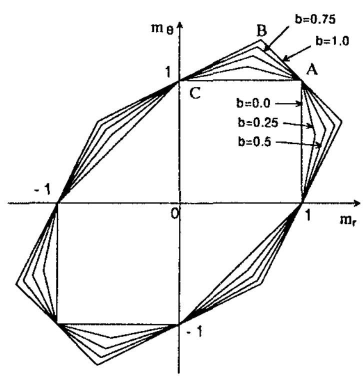

Fig.1 The unified yield criterion

$$\partial^2(rm_r)/\partial r^2 - a_i\,\partial m_r/\partial r = -rp + \mu r\ddot{w} \qquad (4)$$

$$\partial^2\dot{w}/\partial r^2 + a_i\,\partial\dot{w}/(r\partial r) = 0 \qquad (5)$$

where a_i are coefficients corresponding to the unified yield criterion. The subscript i (i=1 or 2) indicates the two different sides AB and BC. Integrating equation (4) and (5) twice respectively, with the aid of continuous and boundary conditions, predicts the distribution of the moments and the transverse velocity for the two phases of motion. The distributions of acceleration of the plate are independent to time during the two phases of motion respectively, thus the moment fields are independent to time, too. While the responses of transverse velocity and deflection are linear and quadratic function of time during the response history. Defining dynamic loading factor $\alpha = p/p_s$, p_s is the static plastic limit loading for the entire plate in plastic state, the moment distributions will violate the yield condition when the impulsive loading p increases to $\alpha \ge \alpha_d$. α_d is defined as a critical dynamic loading factor, above which the moment distribution is not statically admissible and the impulsive loading is called as intense loading. The theoretical analysis of this paper with $1 \le \alpha \le \alpha_d$ is statically admissible while the associated transverse velocity fields are kinematically admissible. Thus, the solution is exact throughout the entire response of a rigid, perfectly plastic circular plate.

3. ANALYSIS RESULTS

The foregoing solutions show maximum transverse deflection response and minimum statically admissible impulsive loading for the case of the Tresca yield criterion (b=0), minimum transverse deflection response and maximum statically admissible impulsive loading for the case of the twin shear stress yield criterion (b=1), respectively. Solutions of the Mises yield criterion are between those obeying the Tresca yield criterion and the twin shear stress yield criterion, and can be approximated by the special case of UYC when b=0.5. Fig.2 and Fig.3 show the moment fields during the first phase of motion and the deflection response of the plate center respectively when the plate is subjected by a uniformly impulsive loading ($r_p = 1$) for the three special cases of yield criterion, with the loading factor $\alpha = \alpha_d$. Fig.4 shows the moment profiles for the plate subjected to a concentrated impulsive loading with $r_p = 0.01$. In this case, the moment fields are singular at the plate center because the shear force at the center is infinite, while the statically admissible impulsive loading p_d is close to the static plastic limit loading p_s. The velocity profiles of the first phase of motion for concentrated impulsive loading is plotted in Fig.5.

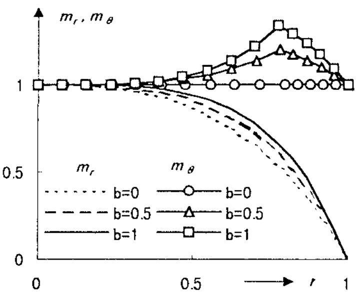

Fig.2 Moment fields during the first phase of motion ($r_p = 1$, $\alpha = \alpha_d$)

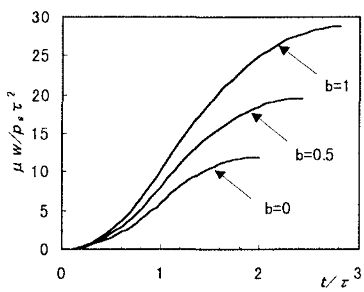

Fig.3 Displacement responses at the plate center ($r_p = 1$, $\alpha = \alpha_d$)

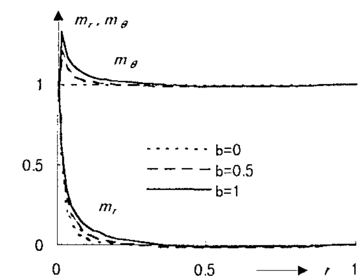

Fig.4 Moment profiles during the first phase of motion ($r_p = 0.01$, $\alpha = \alpha_d$)

Fig.6 illustrates the relation of the maximum statically admissible loading factor α_d to the loading radius r_p which shows clearly the two loading action regions responding to the two cases of statically admissibility. The effect of yield criteria to the dynamic solutions is greater than to static plastic limit state[3]. Fig.7 shows that Tresca yield criterion estimates the maximum permanent transverse deflection, which has been proved greater than the experimental results[4].

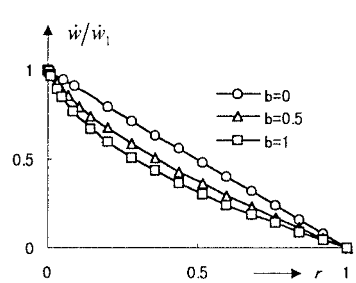

Fig.5 Velocity profiles of the first phase of motion ($r_p = 0.01$, $\alpha = \alpha_d$)

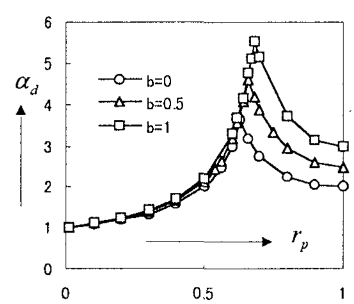

Fig.6 The variation curves of α_d with r_p

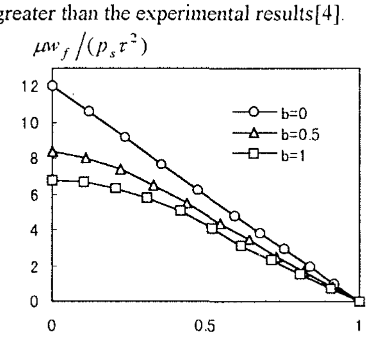

Fig.7 Permanently deformed transverse deflection ($r_p = 1$, $p_0 = 12$)

4. CONCLUSIONS

Unified yield criterion which has piecewise linear mathematical expression is applied successfully to analyze the dynamic plastic response behavior for simply supported circular plate under moderate impulsive loading. Solution of Mises yield criterion is approximated by the linear function of UYC with b=0.5. The analytical solutions of this paper have theoretical important meaning and more wider application range. By choosing a certain material parameter b, the unified yield criterion can be applied to all the isotropic metal materials. The dynamic solution for the plate under intense impulsive loading ($\alpha \geq \alpha_d$)should be analyzed by numerical method, which will be examined in a later study.

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