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NUMERICAL ANALYSIS OF TWO PHASE FLOW BY UPSTREAM FINITE DIFFERENCE METHOD

Yukio Kono　Tohoku Gakuin University
Angus Simpson　University of Adelaide
Masaji Watanabe　Okayama University

1. Introduction

The one phase fluid flow of waterhammer is analized by upstream finite difference method and compared with the result by the method of charactericties. Nonlinear Partial differential equations of the vapuor mixture two phase flow are analized and compared with the experimental results.

2. Governing Equations

One phase fluid flow in water hammer analysis is usually dealt as one dimensional unsteady flow. The equation of motion and the continuity equation are given as follows.

$$V_t + V V_x + g H_x + \frac{f V|V|}{2D} = 0$$
$$H_t + V H_x - V \sin\theta + \frac{a^2}{g} V_x = 0 \qquad (1)$$

The governing equations in two phase flow will be explained as follows The equation of motion is shown as equation (2a). Where W is velocity V in previous equation. The continuity equation is shown as the equation (2b). Where U is the void ration α in general equation.

$$W_t + W W_x + g \sin\theta + \frac{f W|W|}{2D} = 0 \qquad (2a)$$

$$U_t + W U_x - W_x = 0 \qquad (2b)$$

3. Numerical analysis by upstream finite difference method

(1) One phase fluid flow

By setting as follows,

$$Y = \frac{aV - gH}{\sqrt{a^2 + g^2}}, \qquad Z = \frac{aV + gH}{\sqrt{a^2 + g^2}} \qquad (3)$$

The equation for upstream finite difference (4) is given.

$$Y_t + (V - a) Y_x = Q^-(V), \qquad Z_t + (V + a) Z_x = Q^+(V) \qquad (4)$$

The function Q (V) is given by equation as follows.

$$Q^{\mp}(V) = \frac{-1}{\sqrt{a^2+g^2}}\left(\frac{a\, f\, V|V|}{2D} \pm g\, V \sin\theta\right)$$

a is wave speed of fluid then V- a is always negative、 V+a is always positive. The solution of equations in upstream finite difference method is $V_{i,j}$, $H_{i,j}$ and $Y_{i,j}$, $Z_{i,j}$ .The equation (5) is given by substituting forward difference and backward difference into euation (4) .

$$\frac{Y_{i,j+1}-Y_{i,j}}{\Delta t} + \left(V_{i,j}-a\right)\frac{Y_{i+1,j}-Y_{i,j}}{\Delta x} = Q^{-}(V_{i,j}) \ ,$$

$$i=0,1,2,\ldots,n-1, \quad j=0,1,2,\ldots, \qquad (5)$$

$$\frac{Z_{i,j+1}-Z_{i,j}}{\Delta t} + \left(V_{i,j}+a\right)\frac{Z_{i,j}-Y_{i-1,j}}{\Delta x} = Q^{+}(V_{i,j}) \ ,$$

$$i=1,2,3,\ldots,n \ , \quad j=0,1,2,\ldots$$

(2) Two phase flow

Governing equations are given previously as equations (2).
The equation of motion (2a) becomes the finite difference form as equation (6).

$$\frac{W_{i,j+1}-W_{i,j}}{\Delta t} + \frac{W_{i+1,j}^2-W_{i-1,j}^2}{4\Delta x} = R(W_{i,j})$$

$$i=1,2,3,\ldots,n \ , \quad j=0,1,2,\ldots \qquad (6)$$

where、 R(W) is the function given by next equation.

$$R(W_{i,j}) = -\left(\frac{f\,W|W|}{2D} + g\sin\theta\right)$$

The non-linear term becomes as equation (7) by transform.

$$W\,W_x = \left(\frac{W^2}{2}\right)_x \qquad (7)$$

The continuity equation (2b) becomes the finite difference form as equation (8).

$W_{i,j} \geqq 0,$

$$U_{i,j+1} = U_{i,j} + \frac{\Delta t}{\Delta x}\left[W_{i,j} - W_{i,j-1} - W_{i,j+1}\left(U_{i,j}-U_{i,j-1}\right)\right] \ ,$$

$$i=1,2,3,\ldots,n \ , \quad j=0,1,2,\ldots,$$

$W_{i,j} < 0,$ (8)

$$U_{i,j+1} = U_{i,j} + \frac{\Delta t}{\Delta x}\left[W_{i+1,j} - W_{i,j} - W_{i,j+1}\left(U_{i+1,j}-U_{i,j}\right)\right] \ ,$$

$$i=0,1,2,\ldots,n-1, \quad j=0,1,2,\ldots$$

4. Conslusion

The upstream finite difference method is successfully applied to solve the quasi-linear one phase fluid flow of waterhammer and the nonlinear partial equations of fluid gas mixture two phase flow.