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BUCKLING ANALYSIS OF SANDWICH BEAM

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1. INTRODUCTION

High bending rigidity and low-weight structures are being used in different fields such as, aeronautics, bridges and high-rise buildings etc. Sandwich structure is a stiff, light-weight and economical structure. This is accomplished by using relatively thin faces covering a thick core. High strength and high modulus material for the faces and low strength and low density material for the core are being used. Light- weight core permits unusually large shearing deformations. This paper covers the extension of buckling study of sandwich beam which was already presented in Symposium of JSCE[1)]. The main coverage of the present work is the consideration of surface distortion effect at interfaces.

2. KINEMATICS AND BOUNDARY CONDITIONS

Consider straight and constant cross-sectional sandwich beam of unit width. Figure 1 shows the kinematics of sandwich element with equal face plates of thickness $2h_1$ and a core of thickness $2h_2$. $(v_1, w_1), (v_o, w_o)$ and (v_2, w_2) are the displacements of centroidal point of upper face plate, core and lower face plate, respectively. Angles of surface rotation of the upper face plate, lower face plate and the centroidal point of the core are λ_1, λ_2 and $(\gamma_o - \lambda)$ respectively. Since face plates have no shear effect and only the core has shear deformation (λ), surface distortion occurs at the interfaces. β_u and β_l are the angles due to surface distortion at upper and lower interfaces, respectively. Equilibrium equations and boundary conditions can be obtained by substituting displacement equations into general virtual work equation. Only the boundary conditions for a sandwich beam are presented below. The last two boundary conditions are due to the surface-distortion-moment at the upper and lower interfaces, respectively. These two boundary conditions were not considered in the previous presentation[1)].

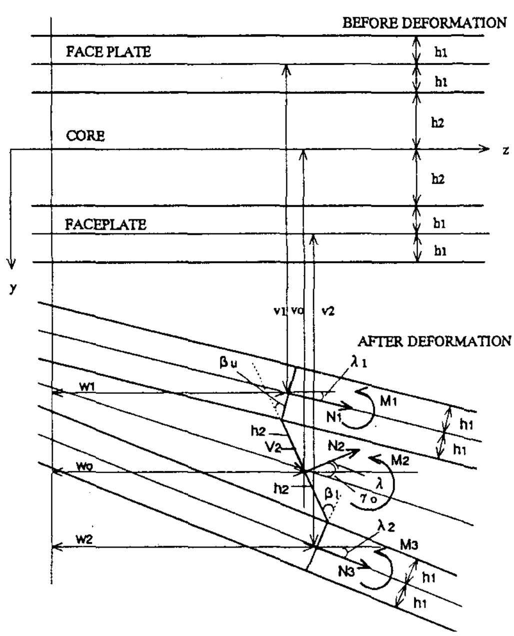

Fig. 1 Before and After Deformation

$$
\begin{aligned}
w &= \text{given or } N_1\cos\lambda_1 - V_1\sin\lambda_1 + N_2\cos\lambda + V_2\sin\lambda + N_3\cos\lambda_2 - V_3\sin\lambda_2 = \overline{N} \\
v &= \text{given or } N_1\sin\lambda_1 + V_1\cos\lambda_1 - N_2\sin\lambda + V_2\cos\lambda + N_3\sin\lambda_2 + V_3\cos\lambda_2 = \overline{V} \\
\lambda &= \text{given or } M_1 - N_1h_1 - N_1h_2\cos\beta_u + V_1h_2\sin\beta_u \\
&\qquad + M_2 + M_3 + N_3h_1 + N_3h_2\cos\beta_l - V_3h_2\sin\beta_l = \overline{M} \quad \cdots\cdots (1) \\
\beta_u &= \text{given or } M_1 - N_1h_1 = \overline{M}_{Fu} \\
\beta_l &= \text{given or } M_3 + N_3h_1 = \overline{M}_{Fl}
\end{aligned}
$$

Where $V_1 = 1/\sqrt{g_u}(M_1 - N_1h1)'$; $V_3 = 1/\sqrt{g_l}(M_3 - N_3h1)'$

$g_o = (1 + w_o')^2 + (v_o')^2$; $g_u = g_l = g_o + (h_2\lambda')^2 - 2\sqrt{g_o}h_2\lambda' cos\gamma_o$ ⋯⋯⋯⋯ (2)

3. BUCKLING ANALYSIS

Variational analysis has been performed and linearized incremental equilibrium equations and boundary conditions are obtained in conjunction with the linearized kinematic and constitutive relations. The following governing equation is derived in terms of nondimensional quantities. s is the displacement and the number in superscript parenthesis indicates the order of differentiation.

$$-\frac{4\alpha\xi^2\eta_1}{(1-z\xi^2)^3\zeta}\left(\eta_2+\frac{24\eta_1\zeta^2}{(1-2\zeta)^2}\right)s^{(6)}$$

$$+\frac{1}{1-z\xi^2}\left\{\frac{8\eta_1(1+2\zeta+4\zeta^2)}{1-2\zeta}+\eta_2-\frac{z\alpha\xi^2}{2\zeta(1-z\xi^2)}\left(\eta_2+\frac{96\zeta^2\eta_1}{(1-2\zeta)^2}\right)\right\}s^{(4)}+zs^{(2)}=0 \quad \cdots\cdots\cdots\cdots\cdots (3)$$

$$\text{Where } \eta_1=\frac{(1-\eta_2)(1-2\zeta)^2}{2(1-2\zeta)^2+6(1+2\zeta)^2} \quad ; \quad \eta_2=\frac{E_2I_2}{E_TI_T} \quad ; \quad \zeta=\frac{h_2}{h_t}$$

$$z=\frac{Pl^2}{E_TI_T} \quad ; \quad \alpha=\frac{E_T}{G} \quad ; \quad \xi=\frac{\sqrt{I_T/A_T}}{l} \quad \cdots\cdots\cdots\cdots\cdots (4)$$

Buckling analysis has been performed for different types of sandwich beams using the governing equation and corresponding incremental boundary conditions. In the case of the simply supported sandwich beam, buckling load can be obtained from the governing equation (3) and corresponding boundary conditions. Surface distortion effect at the interfaces is examined by considering two different cases of simply-supported beam as shown in Fig. 2. Boundary condition is prescribed using surface distortion angle β for the first case whereas the moment M_F at interfaces is used for the second one. For both with or without distortion effect, curves of buckling load z versus slenderness-ratio ξ are presented **in Fig. 3.**

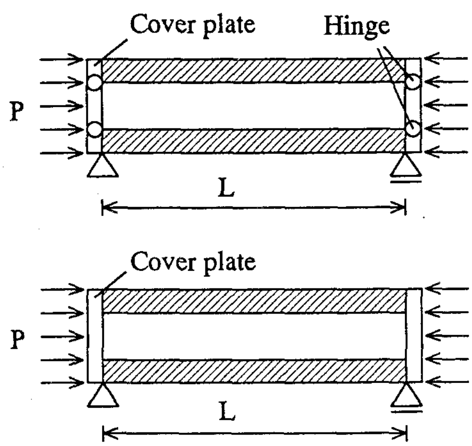

Fig. 2 Simply-supported sandwich beam with hinged end and rigid end

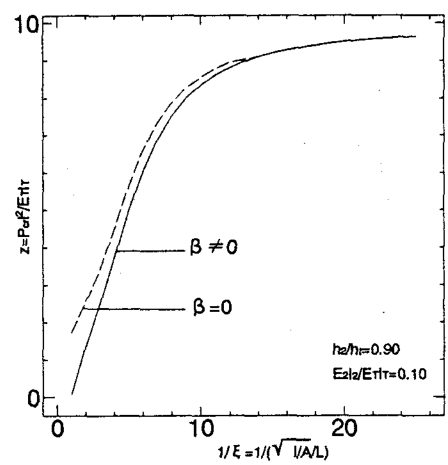

Fig. 3 Curves b/w buckling load z and slenderness ratio ξ

4. CONCLUSION

From this study, it is understood that the beam without surface distortion effect is stronger than the beam which has such effect. It can be concluded that there is less effect of differences of values of Young's modulus when the thickness of the core is relatively very big.

5. REFERENCE

1. M.P. Sharma:*Proceedings of Symposium of JSCE Tohoku branch.* pp. 40,41, March, 1993.