I −20 BUCKLING ANALYSIS OF SANDWICH BEAM WITH CONSIDERATION OF SHEAR EFFECT

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1. Introduction

Sandwich structure is a stiff, light-weight and economical structure. This is acomplished by using thin, high strength, high modulus material for the faces and low strength, low density material for the core. Light-weight core permits unusually large shearing deformations. This paper covers the derivation of buckling theory on sandwich beam, using finite displacement analysis with consideration of shear effect within the core.

2. Displacements

It is considered that sandwich beam has an axial directional force. Figure 1 shows the kinematics of sandwich element with equal thicknessess of outer layers. v and w are the y and z components of the displacement. α and λ are the angles of surface rotation and shear deformation. Equations(1), (2) and (3) are the displacement equations of the layer-1, 3 and core, respectively.

$$v_I = v_o - (h_1 - y)(\cos\alpha_u - 1) - h_2(\cos\lambda - 1)$$
$$w_I = w_o + (h_1 - y)\sin\alpha_u - h_2\sin\lambda \qquad (1.a,b)$$

$$v_3 = v_o + (h_1 + y)(\cos\alpha_l - 1) + h_2(\cos\lambda - 1)$$
$$w_3 = w_o - (h_1 + y)\sin\alpha_l + h_2\sin\lambda \qquad (2.a,b)$$

$$v_2 = v_o + y\,(\cos\lambda - 1)$$
$$w_2 = w_o + y\sin\lambda \qquad (3.a,b)$$

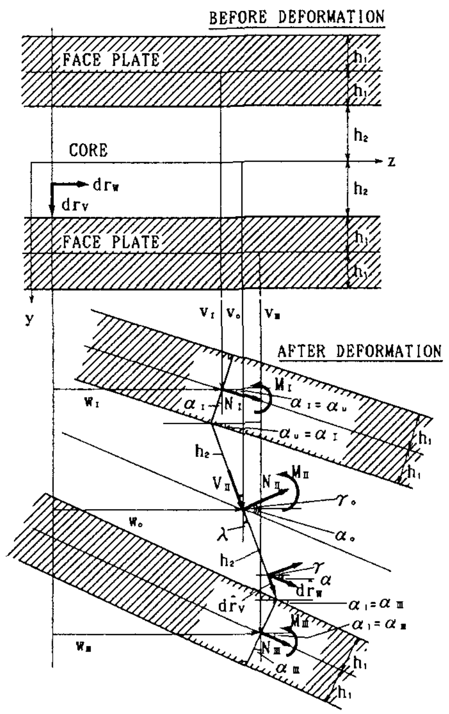

FIG. 1 : BEFORE AND AFTER DEFORMATION

3. Strains

Except the following normal strain e_{zzi} of the face plates and the core and shear strain e_{yz} of the core, all strain tensor components are zero.

$$2e_{zzi} = g_i - 1 \quad (i = 1\sim3) \quad ; \quad 2e_{yz2} = \sqrt{g_o}\sin\gamma_o \qquad (4.a,b)$$

Where, $g_1 = (1 + w_1')^2 + (v_1')^2$; $g_3 = (1 + w_3')^2 + (v_3')^2$;

$$g_2 = (\sqrt{g_o}\cos\gamma_o + y\lambda')^2 + (\sqrt{g_o}\sin\gamma_o)^2 \quad ; \quad \sqrt{g_o} = \frac{1 + w_o'}{\cos\alpha_o} \qquad (5.a\sim d)$$

4. Constitutive Relations

In the linear elastic case, the stresses and strains are given,

$$\sigma_i = E_i e_{zzi} \quad (i = 1\sim3) \quad \text{and} \quad \tau = G\gamma \ \text{(for core only)} \qquad (6.a,b)$$

Linearizes the constitutive relations using small strain assumption[1] and obtain the following equations,

$N_i=E_iA_i\varepsilon_i \quad (i=\mathrm{I}\sim\mathrm{III}) \quad ; \quad M_{\mathrm{I}}=M_{\mathrm{III}}=-E_{\mathrm{I}}I_{\mathrm{I}}\lambda' \quad ; \quad M_{\mathrm{II}}=E_{\mathrm{II}}I_{\mathrm{II}}\lambda' \quad ; \quad V_{\mathrm{II}}=GkA_{\mathrm{II}}\gamma$ (7. a~d)

Where, $\varepsilon_{\mathrm{I}}=w_o'+h_1v_o''-h_2\lambda'+\frac{v_o'^2}{2}$; $\varepsilon_{\mathrm{II}}=w_o'+\frac{v_o'^2}{2}$; $\varepsilon_{\mathrm{III}}=w_o'-h_1v_o''+h_2\lambda'+\frac{v_o'^2}{2}$ (8. a~c)

5. Incremental Variational Principle

Variational analysis have been performed and obtained incremental equilibrium equations and boundry conditions and in conjuction with the constitutive relations, we have following defelection equation with axial force only,

$$F_1\Delta v_o^{\mathrm{VI}}+F_2\Delta v_o^{\mathrm{IV}}+P\Delta v_o''=0 \tag{9}$$

Where,

$$F_1=\frac{2E_{\mathrm{I}}A_{\mathrm{I}}h_1h_2}{GkA_{\mathrm{II}}}\{E_{\mathrm{II}}I_{\mathrm{II}}+2E_{\mathrm{I}}A_{\mathrm{I}}h_2(h_1+h_2)\}-\frac{2}{GkA_{\mathrm{II}}}\{E_{\mathrm{I}}I_{\mathrm{I}}+E_{\mathrm{I}}A_{\mathrm{I}}h_1(h_1+h_2)\}(E_{\mathrm{II}}I_{\mathrm{II}}+2E_{\mathrm{I}}A_{\mathrm{I}}h_2^2)$$

$$F_2=\{2E_{\mathrm{I}}I_{\mathrm{I}}+E_{\mathrm{II}}I_{\mathrm{II}}+2E_{\mathrm{I}}A_{\mathrm{I}}h_1(h_1+h_2)^2\}-\frac{1}{GkA_{\mathrm{II}}}(E_{\mathrm{II}}I_{\mathrm{II}}+2E_{\mathrm{I}}A_{\mathrm{I}}h_2^2)P \tag{10. a, b}$$

Buckling analysis have been performed for different types of sandwich beams based on above sixth order differential equation. In the case of the simply supperted sandwich beam, buckling load is presented below,

$$z=\frac{\pi^2m^3}{m^3+6m^2n+12mn+8n}\left[\frac{8n+6mn}{m^3}+\left(1+\frac{6mn+6n}{m^2}\right)\left\{\frac{1-6n\pi^2\beta^2\alpha/m^2}{1+(1+6n/m)\pi^2\beta^2\alpha}\right\}\right] \tag{11}$$

Where, $z=\frac{P_{cr}l^2}{E_TI_T}$; $\alpha=\frac{E_{\mathrm{II}}}{Gk}$; $\beta=\sqrt{I_{\mathrm{I}}/AI_{\mathrm{I}}}/l$; $m=\frac{h_2}{h_1}$; $n=\frac{E_{\mathrm{I}}}{E_{\mathrm{II}}}$ (12)

Using above equation, graphs between z and $1/\beta$ with different values of $E_{\mathrm{I}}/E_{\mathrm{II}}$ are presented below,

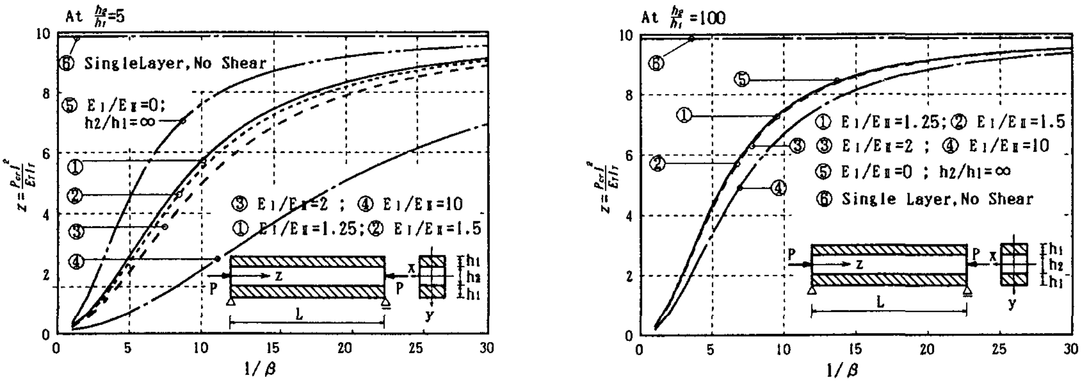

Fig. 2. a, b: Graphs between z and $1/\beta=l/\sqrt{I_{\mathrm{II}}/A_{\mathrm{II}}}$

6. Conclusion

Buckling equations developed for sandwich beams can be used for the single layered beam by eliminating values of face plates of eq. (11). It can also be concluded from fig. 2 that closer the values of elastic modulus or larger the differences of thicknesses of faces and core, the buckling characteristics of sandwich beams are closer to characteristics of single layered beam.

7. Reference

(1). T. Iwakuma: Timoshenko Beam Theory with Extention Effect and its Stiffness Equation for Finite Rotation. Computers & Strs. Vol. 34, 1990.