# II −69 STUDY ON MEAN FLOW UNDER WAVE-CURRENT COMBINED MOTION

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**Abstract**

Modifications of steady current due to surface gravity wave were investigated experimentally and numerically. The experimental results are then used to estimate the form of current profiles in order to calculate the combined flow kinematics. Favorable agreements were found.

**Experiment and Theory**

The experiments were carried out in a wave tank of 14.5 m long, 30 cm wide, and 55 cm deep. The bottom of the flume were covered with triangular strip roughnesses of 4 mm high and 4 cm spacing. The particle velocity was measured in between roughnesses by an LDV.

By assuming the flow to be incompressible,the usual boundary value problem is formulated for a rotational wave-current fields satisfies Eq. 1

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}=\Omega \qquad (1)$$

where (u, w) are $(\partial\psi/\partial z, -\partial\psi/\partial x)$

The problem is solved in a frame of refference moving with the wave celerity (see Fig.1) with the following boundary conditions:

$$\psi(x,z_0)=0,\quad z=z_0 \qquad (2)$$
$$\psi(x,\eta)=Q,\quad z=\eta(x) \qquad (3)$$
$$\frac{1}{2}\left(\frac{\partial\psi}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial\psi}{\partial z}\right)^2+g\eta=R \qquad (4)$$
$$\psi(x+L,z)=\psi(x,z) \qquad (5)$$
$$\int_0^L \eta\, dx=0 \qquad (6)$$

The stream function is written as:

$$\psi=Bo(z-z_0)+\psi_C+\psi_{WIR}+\psi_{WR} \qquad (7)$$

where $\psi_C$, $\psi_{WIR}$, and $\psi_{WR}$ are the stream functions representing the steady current, irrotation and rotational parts of wave respectively.

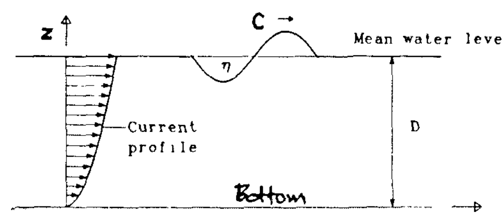
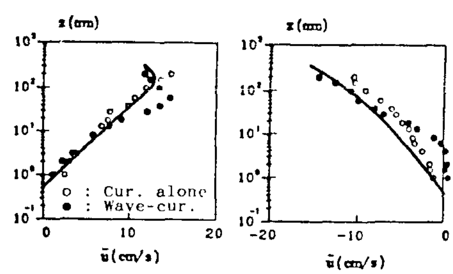



Fig. 1 Definition sketch




(a) Fol. flow (D=0.30 m, H=0.09 m, T=1.3, Uc=0.12 m/s)
(b) Opp. flow (D=0.30 m, H=0.10 m, T=1.3, Uc=0.14 m/s)

Fig. 2 Period-averaged velocity

In order to predict the combined flow kinematics, an accurated form of steady current profiles are neccesary. The present experimental results in term of the period-averaged velocity for following and opposing flows are shown in Figs. 2(a) and 2(b). It can be seen that modifications of the steady current due to the wave are strongly depended on the flow direction, i.e. the mean velocity gradient near the water surface is decreased for the following flow while it is increased in the case of opposing flow.

The momentum equation representing the steady current is written as:

$$-\frac{1}{\rho}\frac{\partial P_c}{\partial x}+\frac{\partial}{\partial z}\frac{\tau_c}{\rho}=0 \qquad (8)$$

Integrating Eq. 8 with the application of linearly time-invarying eddy viscosity approach and the boundary conditions, i.e. $u_c = 0$ and $u_{cd}$ at the bottom and the water surface yields:

$$u_c = \frac{u_{*c}|u_{*c}|}{\kappa u_{*wc}} \ln \frac{z}{z_0} + \left(u_{cd} - \frac{u_{*c}|u_{*c}|}{\kappa u_{*wc}} \ln \frac{D}{z_0}\right) \frac{(z-z_0)}{(D-z_0)} \quad (9)$$

With the modifications at the water surface the following form of steady current are assumed:

$$u_c = \frac{u_{*c}|u_{*c}|}{\kappa u_{*wc}} \ln \frac{z}{z_0} \pm \Delta u_{cd} \frac{(z-z_0)}{(D-z_0)} \quad (10)$$

where ( + : Fol. flow, - : Opp. flow)

$\Delta u_{cd}$ is the velocity reduction and increasing at the water surface for the following and opposing flows respectively. For the first approximation, it is related experimentally to the strength of wave and current (Uo/Uc) as shown in Fig. 3 in which the propotional constant was obtained from the best fit with the experiments.

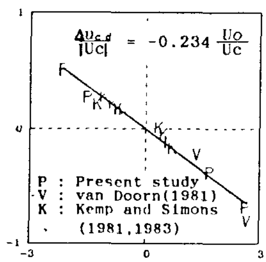



Fig. 3 $\Delta u_{cd}$

The stream function for steady currentis derived from Eq. 10 as:

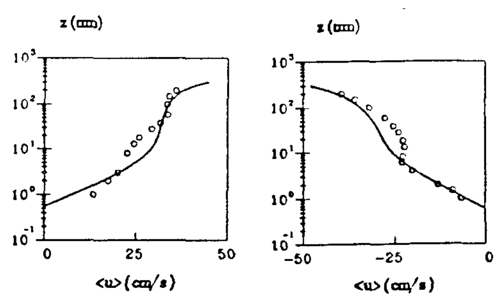
$$\psi_c = \frac{u_{*c}|u_{*c}|}{\kappa u_{*wc}} \left(z \ln \frac{z}{e z_0} + z_0\right) \pm \Delta u_{cd} \frac{(0.5z^2 - z_0 z + 0.5 z_0^2)}{(D-z_0)} \quad (11)$$

The stream function for the wave is represented by truncated fourier series as:

$$\psi_{WIR} = \sum_{j=1}^{n} \frac{B_j \sinh jk(z-z_0) \cos jkx}{\cosh jkD}, \quad \psi_{WR} = \frac{K_0 k}{\omega} \sum_{j=1}^{n} \frac{B_j}{R_{jo} \cosh jkD} F_j \quad (12)$$

**Result discussion**

The solutions are obtained by Newton-Ralpson method. Figs. 2(a) and 2(b) show the period-averaged velocity for following and opposing flows. The corresponding particle velocities under wave crest(fol. flow) and wave trough(opp. flow)are also shown in Figs. 4(a) and 4(b). The predictions by the present models agree fairly well with the experiments. However, a maximum overestimate of 25%(fol. flow) )and of 20% (opp. flow) near the over-shooting zones are found. These are mainly caused by the assumption of linearly time-invarying eddy viscosity.




(a) Fol. flow (crest)　(b) Opp. flow (trough)

Fig. 4 Horizontal particle velocity

**Notations**

$B_o$, $B_1$, ... $B_n$ = Unknown constants for particular wave
$\eta$ = Water surface elevation, $U_c$ = Depth-averaged current velocity
$z_o$ = Roughness height, $U_0$ = Maximum wave velocity from linear theory
$\Omega$ = Vorticity, D = Mean water depth, Q and R = Constants
L = Wave length, k = Wave number, $\omega$ = Angular frequency
$\tau_c$, $u_{*c}$ = shear stress and shear velocity for steady current
$K_o = \kappa u_{*wc}$ ($\kappa$ = 0.4, $u_{*wc}$ = maximum combined shear velocity)
$R_{jo} = 2(\mathrm{Ker}^2 \xi_{jo} + \mathrm{Kei}^2 \xi_{jo})$, $\xi_{jo} = 2(j\omega z_o/K_o)^{1/2}$, $\xi_j = 2(j\omega z/K_o)^{1/2}$
$F_j = \xi_j (\mathrm{Ker}' \xi_j \mathrm{Kei} \xi_{jo} - \mathrm{Kei}' \xi_j \mathrm{Ker} \xi_{jo}) \cos jkx$
$- \xi_j (\mathrm{Ker}' \xi_j \mathrm{Ker} \xi_{jo} + \mathrm{Kei}' \xi_j \mathrm{Kei} \xi_{jo}) \sin jkx$
$+ \xi_{jo} (\mathrm{Kei}' \xi_{jo} \mathrm{Ker} \xi_{jo} - \mathrm{Ker}' \xi_{jo} \mathrm{Kei} \xi_{jo}) \cos jkx$
$+ \xi_{jo} (\mathrm{Kei}' \xi_{jo} \mathrm{Kei} \xi_{jo} + \mathrm{Ker}' \xi_{jo} \mathrm{Ker} \xi_{jo}) \sin jkx$
(Ker and Kei are Kelvin functions)