III－6 A STRESS-DILATANCY EQUATION OF GRANULAR MATERIALS BASED ON MICROMECHANICAL CONSIDERATION

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ABSTRACT

A stress-dilatancy equation under cubical triaxial test condition is proposed based on a constitutive model for granular materials following a particulate approach with two assumptions: 1)granular materials is regarded as an assembly of particles which are rigid,unbreakable,and of convex shape,and 2)deformation occurs as a result of sliding between particles.The principle of this approach is that the behavior of a macro-system made of many representitive micro-systems can be established from the behavior of an individual micro-system through ensemble average.The theory is validated by comparing the stress-dilatancy equation with different experimental results.

1. INTRODUCTION

Most currently existing constitutive models are based on continuum mechanics.However,granular material unlike other engineering materials,is an assembly of particles.Therefore,it is desirable to adopt the particulate mechanics approach to develop a simple yet realistic constitutive relationship that is capable of modeling the behavior of granular material.The mechanisms of shear deformation and the constitutive equations for soils have been studied based on a particulate approach by many investigators such as Rowe(1962,1971) , Horne(1965,1969),Mogami(1969),Murayama(1977),Oda (1972a,1972b,1974).However,the difficulties involved in this approach lie in finding an appropriate mathematical description for packing geometry condition which is not only random but also stress dependent.

This paper reports a simplified alternative approach.Using the concept that soil deformation is due to relative movements of rigid particle groups,a constitutive relation is derived based on which a stress-dilatancy equation is deduced under cubical triaxial test condition.

2. BASIC ASSUMPTIONS

The following deformation mechanisms are considered :

1- Soil is treated as an assembly of particles. Deformation of soil due to loading is caused by the relative movement among particles. The elastic deformation of each particle , particle crushing,and particle rolling are neglected.Therefore,it is assumed in this paper that the deformation of the assembly occurs only as a result of sliding between particles ;

2- In a given stress state of soil element,based on Mohr-Coulomb's theory,there exist certain orientations of planes on which the ratios of shear stress to normal stress are maximum (mobilized planes)along which sliding of particles is most likely to occur (see Fig.1) ;

3- The overall deformation of the soil element can be determined by summing up all the relative movements on the sliding planes occurred in the soil element ;

4- Since the orientations of the mobilized planes are stress dependent when subjected to a stress increment,then the deformation pattern is dependent on the stress condition.

3. INTERPARTICLE FORCE AND EXTERNAL STRESS

The sliding(mobilized) plane in Fig.2 is presented only in the first octant of the xyz space.Since $\sigma_1,\sigma_2,\sigma_3$ are coincided with the x, y,z axes,there exist 8 such sliding planes,one in each octant.The directional cosines of the normal vector $\underset{\sim}{n}$ can be expressed as

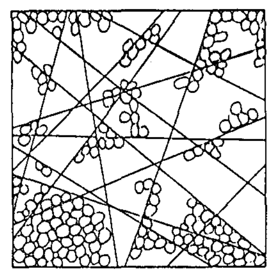

Fig.1.Chosen micro-system (mobilized plane)

$$\underset{\sim}{n} = (n_x,\ n_y,\ n_z) = \sqrt{J_3/J_2}\ (1/\sqrt{\sigma_x},\ 1/\sqrt{\sigma_y},\ 1/\sqrt{\sigma_z}) \quad (1)$$

where J_1, J_2, J_3 are the stress invariants defined as:

$$J_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3 \qquad J_3 = \sigma_1\sigma_2\sigma_3 \quad (2)$$

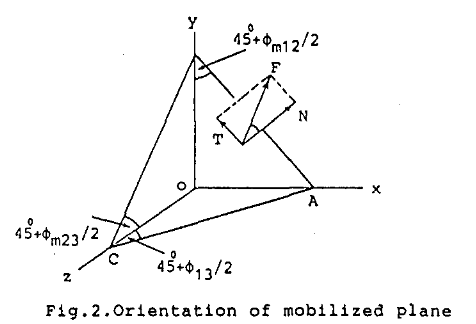

Fig.2.Orientation of mobilized plane

Within a soil element,the relative movements of any two groups,separated by a sliding plane,is caused by the sliding of particle contacts on the plane while both groups act as rigid body.The resulting force $\underset{\sim}{F}$ on the sliding plane is

$$F = (F_x,\ F_y,\ F_z) = (\sigma_x n_x,\ \sigma_y n_y,\ \sigma_z n_z) \quad (3)$$

The magnitude of the force, $|F|$ is

$$|F| = (F_x^2 + F_y^2 + F_z^2)^{\frac{1}{2}} = \sqrt{(J_1 J_3)/J_2} \quad (4)$$

The angle between F and n,denoted as mobilized angle ϕ_m can be expressed as

$$\cos\phi_m = F.n/|F| = 3\sqrt{J_3/(J_1 J_2)} \quad (5)$$

Components of F on this plane are N(normal force) and T(tangential force) and their magnitudes are

$$|N| = \sigma_N = 3\,J_3/J_2 \qquad |T| = \tau_N = \sqrt{(J_1J_2J_3 - 9J_3J_3)}/J_2 \quad (6)$$

Let us take a more microscopic view of the sliding surface of the upper group as shown in Fig.3. The contact force f can be decomposed to two components: the normal force f_η perpendicular to the contact plane and the shear force f_ξ parallel to the contact plane. The two particles begin to slide when the angle between f and f_η equals ϕ_μ. It is noted that, the sliding surfaces has an irregular shape so that the direction of the contact force f on one particle contact is not necessarily the same as that on another particle contact. The summation of all the individual sliding contact forces f should be equal to the resultant force F obtained from Eq.(3). The sum of all the individual contact normal forces and contact shear forces are F_η and F_ξ, respectively. It is noted that, F_η and F_ξ being different from the components N and T, and are not perpendicular or parallel to the sliding plane.

For two rigid groups to slide, the angle between F and F_η should be ϕ_μ, and the direction of sliding s is parallel to the direction of F_ξ. The sliding vector $\underset{\sim}{s}$ is on the same plane of F and N. The unit vector $\underset{\sim}{s}$ can be expressed as

$$\underset{\sim}{s} = (s_x,\ s_y,\ s_z) \qquad (7)$$

where

$$s_x = [\cos(\phi_\mu-\phi_m)\cdot(\sigma_x-\sigma_N)/\tau_N + \sin(\phi_\mu-\phi_m)]\ n_x$$

$$s_y = [\cos(\phi_\mu-\phi_m)\cdot(\sigma_y-\sigma_N)/\tau_N + \sin(\phi_\mu-\phi_m)]\ n_y$$

$$s_z = [\cos(\phi_\mu-\phi_m)\cdot(\sigma_z-\sigma_N)/\tau_N + \sin(\phi_\mu-\phi_m)]\ n_z$$

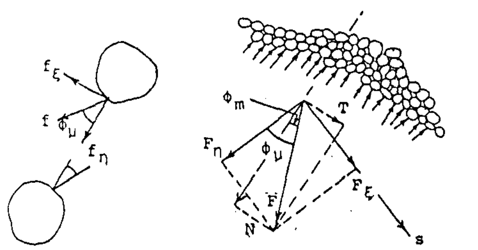

Fig.3. Microscopic illustration of a sliding plane

4. STRESS-STRAIN RELATIONSHIP

There are 8 orientation of a mobilized plane and due to symmetry, there exists only 4 different orientations. Let l_p be the distance AB in the p direction that contains a_j number of sliding planes in the jth orientation as shown in Fig. 4. Consider one of them labelled i, and let Δu^i be the sliding distance, s^j be the direction of sliding. The movement of point B relative to point A can be

$$\delta_p = \sum_{j=1}^{4} \sum_{i=1}^{a_j} \Delta u^i s^j = \sum_{j=1}^{4} \delta_p^j\, s^j \qquad (8)$$

The vector δ_p has 3 components in x, y, z directions which can be expressed as

$$\delta_{pq} = \sum_{j=1}^{4} \delta_p^j\, s_q^j \quad ; \quad (q = x,\ y,\ z) \qquad (9)$$

In a volume defined by l_x, l_y, l_z, the average rate of strains are

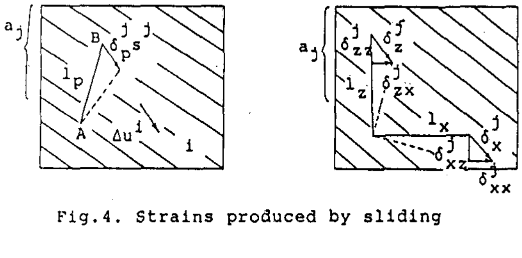

Fig.4. Strains produced by sliding

$$d\varepsilon_x = \delta_{xx}/l_x\ ,\ d\varepsilon_y = \delta_{yy}/l_y\ ,\ d\varepsilon_z = \delta_{zz}/l_z$$

$$d\gamma_{xy} = \delta_{xy}/l_x + \delta_{yx}/l_y \quad d\gamma_{xz} = \delta_{xz}/l_x + \delta_{zx}/l_z$$

$$d\gamma_{yz} = \delta_{yz}/l_y + \delta_{zy}/l_z \qquad (10)$$

Assuming, the magnitude of sliding movement in each orientation are the same ($\delta_p^j = |\delta_p|/4$) for a cubical triaxial test, then Eq.(9) be

$$\delta_{pq} = \left|\delta_p\right| \frac{1}{4} \sum_{j=1}^{4} s_q^j \quad ; \ (p = x,y,z \ ;\ q = x,y,z) \qquad (11)$$

The directional cosines can be related to l_x, l_y, l_z, as $l_x : l_y : l_z = 1/n_x : 1/n_y : 1/n_z$. Substitute this into Eq.(11), we obtain the constitutive relationship as

$$\begin{vmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\varepsilon_z \end{vmatrix} = \lambda . \begin{vmatrix} s_x/\sqrt{\sigma_x} \\ s_y/\sqrt{\sigma_y} \\ s_z/\sqrt{\sigma_z} \end{vmatrix} \qquad (12)$$

where λ is linearly proportion to the magnitude of sliding movement $|\delta_p|$; (p = x, y, z).

5. STRESS-DILATANCY EQUATION

On using Eq.(12), a stress-dilatancy equation in the case of cubical triaxial test can be expressed as

$$-\frac{d\varepsilon_v}{d\varepsilon_x} = \frac{3\sigma_x\ (\tau_N/\sigma_N)\ \tan(\phi_\mu-\phi_m)}{(\sigma_N-\sigma_x) - \tau_N\tan(\phi_\mu-\phi_m)} \qquad (13)$$

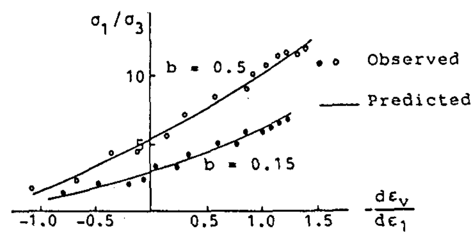

Fig.5. Stress-dilatancy plots in true-triaxial test (after Lade, 1972)

The stress-dilatancy relation of Eq.(13) is compared in Fig.5 with experimental data obtained from a set of cubical triaxial tests performed on Monterey No. 0 sand under the condition of constant mean stresses with different stress parameter $b = (\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$. As can be shown from Fig.5, the comparison shows reasonably good agreement.

6. CONCLUSIONS

A constitutive model based on a particulate approach has been proposed that can be easily applied for sand. The dilatancy equation(13) based on this methodology is noted as being in comparatively good agreement with experimental data which indicates that the particulate approach can be an effective alternative means to model soil behavior and provide some insight into the deformation of a granular assembly.

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