Ⅲ－１６ SLIP LINE ANALYSIS OF DILATANT GRANULAR MATERIALS

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ABSTRACT
Two different plane strain problems are considered here,one of these is compression and the other is shear,between rigid,parallel plates of a long slab of compressible cohesionless granular medium. Closed form solutions for the stress and velocity fields are presented which comprise the stress equilibrium equations,the Coulomb yield criterion,and the kinematic equations proposed by Mehrabadi and Cowin (1978). To clarify the effect of the dilatancy concept on the mechanical behavior of the grnular materials,the results of the velocity field in the case of considering the granular material is incompressible are compared with the results obtained from the present analysis(compressible material) .

1. INTRODUCTION

In the theoretical study of the planar deformation of granular materials,a simplifying assumption often made, is the assumption of non-dilatancy or incompressibility of these materials. This assumption, however,is not consistant with the observations and experiments.Experiments to verify the dilatancy property of granular material were conducted as early as 1885 by Reynolds. Spencer(1964) constructed a properly,planar, invariant,general theory for the deformation of non-dilatant granular materials which coincides with the original idea of Coulomb postulated that deformation occurs by shear along the stress characteristics. Spencer proposed two velocity equations,the first one is the non-dilatancy(incompressibility) and the second,is the non-coaxiality conditions.Spencer's velocity equations are

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0$$
$$\sin 2\psi\left(\frac{\partial v_x}{\partial x}-\frac{\partial v_y}{\partial y}\right)-\cos 2\psi\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)+\sin\phi\left(\frac{\partial v_y}{\partial x}-\frac{\partial v_x}{\partial y}-2\dot{\psi}\right)=0 \qquad (1)$$

where ϕ is the angle of internal friction , v_x,v_y are the velocity components,and the superimposed dot is the material time rate.

An extension to dilatant granular materials of Spencer's(1964)was suggested by Mehrabadi-Cowin(1978) based on a kinematic proposal made by Butterfield and Harkness(1972) characterized by a single parameter ν,namely,the angle of dilatancy. Mehrabadi-Cowin's velocity equations are

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=\frac{\sin\nu}{\cos(\phi-\nu)}\left[\left(\frac{\partial v_x}{\partial x}-\frac{\partial v_y}{\partial y}\right)\cos 2\psi+\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)\sin 2\psi\right]$$
$$\frac{\partial v_x}{\partial y}-\frac{\partial v_y}{\partial x}+2\dot{\psi}=\frac{\cos\nu}{\sin(\phi-\nu)}\left[\left(\frac{\partial v_x}{\partial x}-\frac{\partial v_y}{\partial y}\right)\sin 2\psi-\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right)\cos 2\psi\right] \qquad (2)$$

Spencer's theory has been applied by him to the indentation of a half-space by a rigid punch,by Marshall(1967)to compression between parallel plates,and by Pemberton to flow in wedge-shaped channels.Morrison and Richmond(1976)extended the theory by retaining the inertia terms and solved some one-dimensional problems.

It appears that no attempt has been made to apply the equations proposed by Mehrabadi-Cowin(1978) to any soil mechanics problems to determine the velocity field and to study the effect of the dilatancy concept on the mechanical behavior of granular materials .

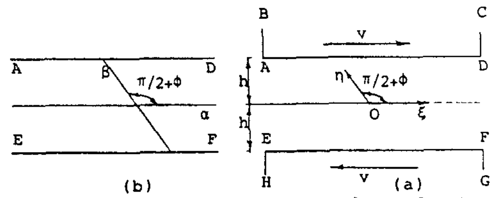

Fig.1. (a)Rock mass geometry and coordinate system,(b)presentation of α,β-lines.

2. THE SYSTEM of EQUATIONS

We denote by σ_x,σ_y and τ_{xy} the components of stress referred to a system of fixed rectangular coordinates(x,y)and v_x ,v_y the components of velocity in the coordinate directions.The equations of equilibrium are

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}=0\ ,\ \frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}=0 \qquad (3)$$

The Coulomb yield criterion for cohesionless soil is

$$(\sigma_x+\sigma_y)\sin\phi+\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}=0 \qquad (4)$$

A complete theory is formed by the system of equations consisting of the stress equations(3),Coulomb yield criterion(4),and one of the equations(1) and (2).It is well known that the stress equations(3)and(4) are hyperbolic,and the system of velocity equations(1)or(2) are also hyperbolic,and for quasistatic deformations the characteristics of the stress and velocity coincide and given by

$$\frac{dy}{dx}=\tan(\psi-\pi/4-\phi/2),\ \frac{dy}{dx}=\tan(\psi+\pi/4+\phi/2) \qquad (5)$$

where ψ is the angle of inclination of the algebrically greater principal stress to the x axis.Theses characteristics are termed α- and β-lines respectively .

3. APPLICATIONS

3.1. Shearin of Granular Material

Rock strata in the earth contain joint and fractures which become filled with broken material and detritus.If such a jointed rock mass be subjected to stress other than hydrostatic pressure,then relative motion of the strata on either side of the discontinuity may occur.The presence of granular material in the joint or fault etc. will influence the mechanical response of the jointed rock mass as a whole.In Fig.(1-a)the rock masses represented by ABCD,EFGH and the granular material filled joint by ABFE.AD is parallel to EF and the motion is assumed to take place in the plane shown.Let the height AE of the joint be 2h,then we shall take the width AD to be large enough in comparison with h.Suppose that, a lateral stress across EH,AB causes the mass ABCD to move to the right,relative to the mass EFGH.Introducing oblique coordinate axes

$o\xi\eta$(see ref.4.for more details),with o midway between AD,EF,such that ABCD,EFGH have velocities(v,0),(-v,0)respectively.If the Coulomb law is assumed to hold on AD and EF,ther,the conditions

$$\tau_{\xi\eta} = 0 \ , \psi = \pi/4+\phi/2 \qquad (6)$$

must be fulfilled,where ψ is the angle of inclination of the direction of the algebrically principal stress to the ξ-axis.Eq.(6) means that,the α-characteristics curves meet AD,EF tangentially.A solution of the stress equilibrium equations for constant ψ is

$$p = p_o \ , \psi = \pi/4+\phi/2 \qquad (7)$$

everywhere in the region ADFE,where p_o is an arbitrary positive constant(hydrostatic pressure).Therefore,the stress characteristics compose of two families of straight lines as shown in Fig.(1-b).After expressing Mehrabadi and Cowin's velocity equations(2)and using Eq.(7),we get a solution in the form

$$v_\xi = \frac{v}{h}\frac{\cos(\phi-\nu)}{\cos\nu} \ , \ v_\eta = \frac{v}{h}\eta\tan\nu \qquad (8)$$

The fact that v_ξ,v_η are independent of ξ,is consistent with the assumption that the sample is long in comparison to its height.It should be noted that,when ν is zero,Eq.(8);indicates the vertical velocity will be zero which is consistent with the incomperssibility codition.

3.2 Compression of a Slab of Granular Material

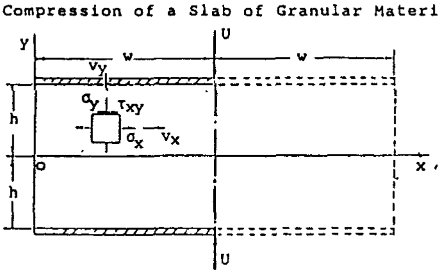

Fig.2. Presentation of the slab geometry and coordinate system

This case is a model for the problem of a raft foundation supporting a building resting on a thin layer of soft soil which covers rigid substrata.The problem is not one of steady motion but is one in which geometrical similarity is preserved.The configuration is illustrated in Fig.2.This solution is due to Marshall(1967),a block of material is compressed between rough plates whose surfaces lie in the planes $y=\pm h$,and move inwards with speed U.Marshall indicated that,the stress characteristics α-and β-lines are two families of curves whose parametric equations

$$Cy=h(\sin 2\psi+2\psi\sin\phi),Cx=h(\cos 2\psi\mp 2\psi\cos\phi) \qquad (9)$$

where ψ depends only on y and C is a constant determined by Marshall in two cases (a) a cohesive material compressed between perfectly rough plates,and(b)a cohesionless material with Coulomb sliding friction at the interfaces.He showed that,Spencer's velocity equations(1) have the solution

$$\frac{v_x}{U} = \frac{x}{h} - \frac{2}{C}\cos 2\psi + B \ , \frac{v_y}{U} = -\frac{y}{h} \qquad (10)$$

where 2h denotes the current thickness of the slab and U=-dh/dt.B is a constant of integration which was evaluated by Marshall in the two cases cosidering the material is incompressible. Here,we will concentrate on the second case(cohesionless material)in determining the velocity field for compressible material assuming the angle of internal friction of the material equals the angle of friction between the material and the compression plates.For that case,the constant B in Eq.(10) can be

$$B = -\frac{w}{h} + \frac{\phi+\pi/2 \ \sin\phi\cos\phi}{[\cos\phi+\sin\phi(\phi+\pi/2)]^2} \qquad (11)$$

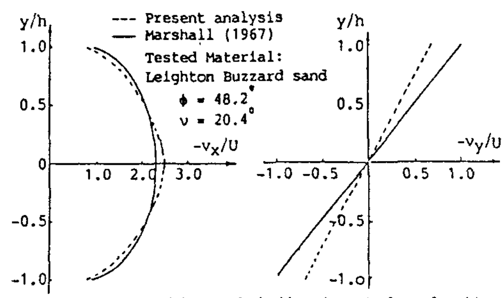

Fig.3. Distribution of (a)horizontal velocity, (b)vertical velocity.

where w is the half width of the slab.In the case of dilatant granular material,we found that Mehrabadi-Cowin velocity equatios have the solution

$$\frac{v_x}{U} = \frac{x}{h} + \Phi_1(y) \ , \frac{v_y}{U} = -\frac{y}{h} + \Phi_2(y) \qquad (12)$$

where

$$\Phi_1(y) = -\frac{2}{G}\cos 2\psi \frac{\sin\nu}{\sin(\phi-\nu)[D\sin 2\psi+2\psi E+\cos 2\psi]} + H$$

$$\Phi_2(y) = \frac{2\tan\nu}{G\cos(\phi-\nu)}[2\psi\sin\nu + \sin(\phi-\nu)\sin 2\psi]$$

$$D = \cos(\phi-2\nu) \ , E = \sin\nu\cos\nu+\sin(\phi-\nu)\cos(\phi-\nu)$$

$$G = \frac{\cos(\phi-2\nu)\sin 2\psi+2\psi E}{\cos\nu\cos(\phi-\nu)}$$

$$H = -\frac{w}{h} - \frac{2}{G}\sin\phi \frac{\sin\nu}{\sin(\phi-\nu)[D\sin\phi+E(\pi/2+\phi)-\sin\phi]}$$
$$+(1\frac{2\tan\nu\sin(\phi-\nu)}{G\cos(\phi-\nu)}[\sin(\phi-\nu)\cos\phi+\sin\nu(\pi/2+\phi)])$$

From Stroud's experimental work,one can find the values of ϕ and ν to be 48.2,20.4° respectively for Leighton Buzzard sand.Fig.3. shows typical horizontal and vertical velocity distributions calculated using Marshall's equation for the case of Leighton Buzzard sand compared with the author's equation(12).The dilatancy cocept has an effect on both verticl and horizontal velocity distributions as can seen from Fig.3.

4. CONCLUSION

Through the application of Spencer's theory(1964) for incopressible material,and Mehrabadi-Cowin's theory for compressible material,it has been shown for two soil mechanics problems that the dilatancy concept has a remarkable effect on the velocity field distribution which must be taken into account in studying the mechanical behavior of granular materials .

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