II－２３　流速分布と内部摩擦に関する一提案

PROPOSALS ON VELOCITY DISTRIBUTION AND INTERNAL FRICTION

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ABSTRACT

In this paper, we describe the following matters: New equations of velocity distribution and internal friction are obtained from the general equation of average velocity by the method of differential calculus. These new equations agree comparatively well with the existent ones. Especially, in laminar flow they agree completely with the old ones, and the Newton's law of viscosity also can be drawn from the new general equation of internal friction.

NEW FORMULAS OF INTERNAL FRICTION

The general equation of average velocity was offered as follows from the fact that the relation between the Reynolds numbers and the resistance coefficients based on the existent definition are distributed in a straight line on the log-log graph paper by Yasuda(1973):

$$V=\frac{\sqrt{2}}{\nu^{2\alpha-1}}\left(\frac{g}{\lambda'}\right)^{\alpha}R^{3\alpha-1}\ I^{\alpha} \qquad (1)$$

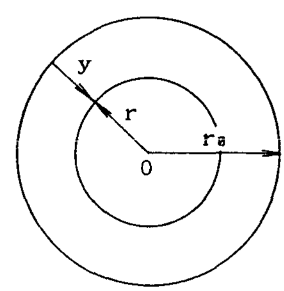
where I is the slope of the total energy line, R is the hydraulic radius and ν is the kinematic viscosity. Equation 1 is also derived from the following equation of wall shear stress(Yasuda, 1988):

$$\tau_0=\lambda'\left(\frac{\rho\nu^2}{R^2}\right)^{1-m}\left(\frac{\rho V^2}{2}\right)^{m} \qquad (2)$$

where $m=1/(2\alpha)$ and ρ is the density of fluid.

As mean velocity of flow in pipes is directly proportional to the βth power of R or the pipe diameter D as Eq.1 show, the formula of velocity distribution is a function of the βth power of the r which is a distance from the center of pipe, where $\beta=3\alpha-1$.

Therefore, velocity distribution is shown by the following formula:



Circular pipe.

$$u=u_{min}+(u_{max}-u_{min})\left\{1-\left(\frac{r}{r_0}\right)^{\beta}\right\}$$

Putting $a=u_{min}/V$ and $b=(u_{max}-u_{min})/V$,

$$u=aV+bV\left\{1-\left(\frac{r}{r_0}\right)^{\beta}\right\} \qquad (3)$$

$$\therefore\ \frac{du}{dr}=-\frac{\beta bV}{(2R)^{\beta}}r^{\beta-1}$$

Substituting this equation into Eq.1 and solving for I, it becomes the following equation:

$$I=\left(\frac{2^{\beta-0.5}}{\beta b}\right)^{1/\alpha}\lambda'\frac{\nu^{(2\alpha-1)/\alpha}}{g}r^{(1-\beta)/\alpha}\left(-\frac{du}{dr}\right)^{1/\alpha} \qquad (4)$$

The internal friction τ at a point of a radius of r from the center of the pipe is given from the equilibrium condition in the flow direction as following formula:

$$\tau=\frac{1}{2}\rho g r I \qquad (5)$$

Substituting Eq.4 into Eq.5, we can obtain the following general formula of τ :

$$\tau = \frac{\lambda'}{2}\left(\frac{2^{\beta-0.5}}{\beta b}\right)^{1/\alpha} \rho \{\nu^{2\alpha-1} \quad r^{2(1-\alpha)}\}^{1/\alpha}\left(-\frac{du}{dr}\right)^{1/\alpha} \tag{6}$$

Substituting the laminar flow condition: $\alpha = 1.0$, $\beta = 2.0$, $\lambda' = 2\sqrt{2}$ and b=2 into Eq.6, we can obtain the following Newton's law of viscosity:

$$\tau = -\mu \frac{du}{dr} \tag{7}$$

In the wholly rough turbulent flow with high Reynolds number, substituting $\alpha = 0.5$ and $\beta = 0.5$ into Eq.6, we can obtain the following equation:

$$\tau = \sigma \rho r^2 \left(\frac{du}{dr}\right)^2 \tag{8}$$

where $\sigma = 2\lambda'/b^2$

Comparing Eq.8 with the formula due to Prandtl-mixing length theory:

$$\tau = \rho l^2 \left(\frac{du}{dy}\right)^2$$

we can obtain the following equation:

$$l = \sqrt{\sigma}(r_0 - y)$$

but this is different from the Prandtl assumption of mixing length: $l = \kappa y$. As this difference between the two is under investigation, we propose Eq.8 as tentative formula of internal friction in the wholly rough turbulent flow with the high Reynolds number.

The general formula of the velocity distribution becomes the following equation from Eqs.1 and 3:

$$u = \frac{\sqrt{2}}{\nu^{2\alpha-1}}\left(\frac{g}{\lambda'}\right)^{\alpha} R^{3\alpha-1} \; I^{\alpha} [a + b\{1 - \left(\frac{r}{r_0}\right)^{\beta}\}] \tag{9}$$

Substituting the laminar flow conditions: $\alpha = 1.0$, $\lambda' = 2\sqrt{2}$, a=0 and b=2.0, it becomes the following equation:

$$u = \frac{\rho g I}{4\mu}(r_0^2 - r^2) \tag{10}$$

This equation agrees completely to the existent formula in the laminar flow.

CONCLUSION

Many facts which have been already well known in the laminar flow can be explained completely by the new Eqs.6 and 9. Viz. from the facts described above, we can conclude as follows:

(a) The general equations of internal friction and velocity distribution have been obtained from the new general equation of average velocity. (Yasuda, 1973)

(b) The Hagen-Poiseuille formula, the Newton's law of viscosity, etc can be drawn from the new Eq.6.

(c) The facts in turbulent flow regions are under investigation, so we propose Eqs.6 and 9 as tentative equations, here.

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