I－22 FEM SOLUTION FOR THE LATERAL BUCKLING OF CURVED MEMBERS

Tohoku Univ. * K. MAALLA
Tohoku Univ. S. KURANISHI
Tohoku Univ. T. IWAKUMA

1. Introduction

In the analysis of the lateral-torsional buckling of thin-walled members, there exist two different FEM formulations based on the straight beam element, and the researchers are categorized into two groups. One of them may be represented by Murray and Rajasekaran[1], Bazant and Nimeiri[2] and Yang and Kuo[3]. They use the kinematic field in small displacements and incorporate the nonlinear part of the shear strain into the expression of the virtual work in deriving the tangent stiffness equation. On the other hand, the other group uses the second order kinematic field of displacements, and Hasegawa et al.[4] may be included in the latter group. In case of straight members, these two different formulations lead to the similar results which are in agreement with the analytical ones of the pioneering works by Vlasov and Timoshenko and Gere. But when the curved members are concerned, tesults which also disagree with the analytical ones. This problem is still in dispute and has drawn the attention of many researchers especially after the work by Yoo[5], who obtained a closed form solution for the lateral-torsional buckling of circular arches, different from those by Vlasov and Timoshenko, but approved by the numerical solution of Ref.4). In the present study, the straight beam element is used to model the curved member and similar results to those by Vlasov and Timoshenko are obtained.

2. Principle of the virtual work

Choose a known and already defined configuration as the reference state, where the body with its volume V and surface S is in equilibrium under the action of σ^o_{ij} and T^o_i. The virtual work equation in an unknown configuration which is set appropriately close to the reference state can be written in the form[6]

$$\int_V (\sigma_{ij}\delta e^{\ell}_{ij} + \sigma^o_{ij}\delta e^q_{ij})dV - \int_S (T_i\delta u^{\ell}_i + T^o_i\delta u^q_i)dS = 0 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

in which σ_{ij}, e_{ij} and u_i are the incremental components of the 2^{nd} Piola-Kircchoff stress tensor, Green's strain tensor and displacement vector, respectively, while T_i denotes the external incremental surface load components. The superscripts ℓ and q are employed here to distinguish between the linear and quadratic parts of the corresponding quantity, while the superscript o is used to indicate that the quantity is acting in the reference state. Although it seems peculiar to divide the displacement components in such terms, there exist such a nonlinear term in the generalized displacement components; e.g. the rotation in the beam theory.

3. Tangent stiffness equation

Substituting the second order kinematic field (disp. & strains) of the conventional beam theory into Eq.(1) and introducing the well-known displacement functions of Hermite polynomials lead to the general stiffness equation of a thin-walled beam element, as expressed by

$$[\,\mathbf{K}_e + \mathbf{K}_g\,]\,\mathbf{r} = \mathbf{R} + \mathbf{R}^o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

A similar equation has been derived in other references, but the sole difference lies in the fact that in these references $\mathbf{R}^o$ has been missing. This vector is essentially nonlinear term which stems from the quadratic components of the kinematics in the underlined term of Eq.(1), and depends on the incremental rotation components of the displacement vector $\mathbf{r}$. Thus

$$[\,\mathbf{K}_q\,]\,\mathbf{r} = \mathbf{R}^o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-a})$$

The matrix $\mathbf{K}_q$ contains only moments and these moments are found to behave quasitangentially leading to an imbalance of the joint connecting two elements meeting at an angle[6]. Therefore, it seems necessary to make modification on $\mathbf{R}^{O}$ by replacing the quasitangential moment by a semitangential one which ensures the equilibrium of the joint. Thus[6]

$$[\ \mathbf{K}_s\ ]\ \mathbf{r} = \mathbf{R}^{O}\ ;\ \text{i.e.}\ [\ \mathbf{K}_s\ ] = \frac{1}{2}\ [\ \mathbf{K}_q\ ] \qquad \text{................(3-b)}$$

Only linear buckling problem is treated in this study, and the buckling is assumed to occur under a constant load. Therefore, the increments of the applied forces are kept zero ($\mathbf{R} = \mathbf{0}$) and the buckling load is obtained by solving the following eigenvalue problem

$$[\ \mathbf{K}_e + \mathbf{K}_{gs}\ ]\ \mathbf{r} = \mathbf{0}\ ;\ \text{with}\ [\ \mathbf{K}_{gs}\ ] = [\ \mathbf{K}_g\ ] - [\ \mathbf{K}_s\ ] \qquad \text{............(4)}$$

4. Numerical results and discussion

The flexural-torsional buckling of a circular arch, **Fig.1**, under uniform bending is examined using the new geometric stiffness matrix $\mathbf{K}_{gs}$, and the results will be compared with the analytical solution and other numerical solutions to demonstrate its efficiency. The material and cross sectional properties of the arch can be found in Refs.4) or 5). For the sake of comparison with the results of Vlasov and Timoshenko, two models of displacement functions are used. Model 1 uses a cubic variation of the angle of torsion while in Model 2 where the warping effect is neglected and the results must correspond to those of Timoshenko, a linear variation is used. The numerical solutions of the present and other formulations for the critical lateral-torsional buckling moment are shown in the table for several subtended angles $\boldsymbol{\theta}$. On the light of these results, it becomes clear that the results by Vlasov and Timoshenko are correct, and therefore Refs.2) to 5) seem to be in error. This error is probably due to two reasons: 1) Neglect of the nonlinear terms of the beam rotation and 2) Imbalance of the joint connecting two elements meeting at an angle.

Researcher	**Subtended angle in degree**				
	0.05	**10.0**	**30.0**	**50.0**	**90.0**
Vlasov	346.8	590.2	1257.1	1996.3	3519.2
Timoshenko	312.8	561.0	1241.0	1986.0	3513.0
Yoo[5]	345.8	345.9	339.3	323.8	266.1
Yang[3]	347.8	905.7	2343.0	3756.2	6121.5
Bazant[2]	347.8	909.5	2438.5	4101.9	6820.1
Hasegawa[4]	345.8	345.9	339.4	324.8	261.1
Present :					
Model 1	346.8	589.8	1257.3	1998.1	3529.8
Model 2	312.9	561.2	1242.2	1988.1	3520.5

Table: Lateral-torsional buckling moment (in kN.m)

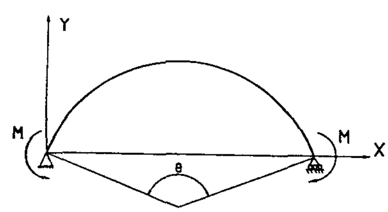

Fig.1 Circular arch under uniform bending

References

1)Murray & Rajasekaran, Jour. Eng. Mech, ASCE, Vol.101, EM5, pp.561, 1975.
2)Bazant & Nimeiri, Jour. Eng. Mech., ASCE, Vol.99, EM6, pp.1259, 1972.
3)Yang & Kuo, Jour. Eng. Mech., ASCE, Vol.112, No 8, pp.821, 1986.
4)Hasegawa et al., Stru. Eng./Earth. Eng., JSCE, Vol.2, No1, pp.57s, 1985.
5)Yoo, Jour. Eng. Mech., ASCE, Vol.108, EM6, pp.1351, 1982.
6)Maalla, Ph. D thesis, Grad. School of Tohoku Univ., January 1989.