CONSIDERATION ON A FAILURE CRITERION FOR GRANULAR MATERIALS

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ABSTRACT

A failure criterion for granular materials is proposed,in which the yield stress ratio reffers to the stress acting on a plane of maximum shear stress which makes an angle ν/2 (ν is the angle of dilatation) with the direction of the slip line (zero-extension line). The stress on a plane perpendicular to a slip line is considered as a constant for the material,whatever the stress level or the instantaneous void ratio.

1. INTRODUCTION

In the case of Coulomb's hypothesis, the failure of granular materials is governed by the equation

$$\tau = \sigma \tan \phi \qquad (1)$$

where, τ is the shear stress along,and σ is the effective normal stress across the failure plane,while φ is the angle of internal friction of the soil. This equation is usually taken to signify that the failure plane is a plane of maximum obliquity of stress and that the soil does not yield until the stress ratio τ/σ,at all points along this plane has attained the limiting value of tan φ. If φ is assumed to be a constant,as it is generally taken to be for a given initial voids ratio of the soil,then equation (1) may be represented by the straight lines OT and OD in Figure (1). These lines form the envelope of the Mohr's stress circles which give the limiting stress ratio that is required for yield to start or failure to take place.

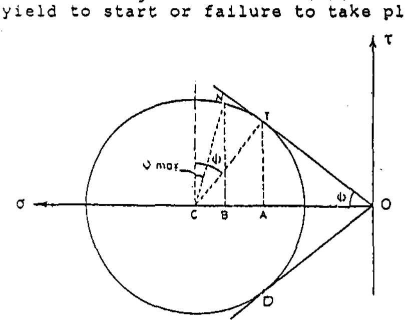

Fig.1. Stress ratios on planes of maximum obliquity,maximum shear stress,and parallel to a line of zero extension.

This criterion of yield has been adopted by many researchers in attempts to extend the assumption of considering the soil behaves like a perfect rigid-plastic material to an idealized rigid plastic material possessing a constant internal friction. For any given rate of dilatation,the value of ν is fixed (coincidence of axes of stress and strain rate) as shown in Figure (1),this will correspond to point N on the circle,and the stress ratio on the slip line will be NB/OB. Initially ν may be negative if the initial dilatation is negative,but as this rate increases ν will increase through zero to a maximum positive value at which NB/OB attains the greatest value. Consequently,the assumed ratio TA/OA according to the Mohr-Coulomb criterion considerably overestimates the real stress ratio on the slip plane.

In conventional methods of analysis earth pressure problems,the shape of a rupture surface is assumed on which the limiting maximum value of the stress ratios acts. now instead of

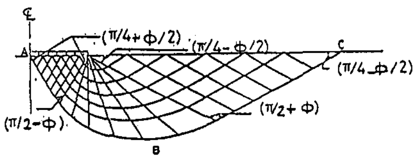

Fig.2. Typical assumed failure surface ABC for footing.

assuming that φ is constant at all points along the rupture surface ABC (and sometimes at all points within the area ABC) as shown in Figure (2). It is only necessary to assume that ν is constant at all such points. As can be seen in Figure (3-a),the assumption that φ is constant,for example in dense sand is equivalent to assume that all the sand has been subjected to the shear strain γ corresponding to point G. Any variation in γ will cause large reduction of φ ,on the other hand ν is given by the slope of the curve OJ in Figure(3-b) this slope is approximately constant over large range of shear strain γ . Hence the assumption that ν is constant at all points in the deforming sand is likely to be much more relevant than that φ is constant throughout,(K.H.Roscoe,1970).

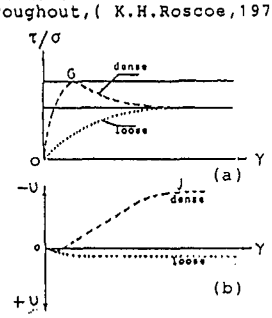

Fig.3. Relatioships between,(a)shear strain γ and stress ratio τ/σ ,(b)volumetric strain υ for sand on a slip line ABC.

2. THEORY

Coulomb's original concept is a simple development of the equation of solid body sliding,i.e. if a block of material of weight W is placed on rough horizontal table,then it will not slide until the applied horizontal force F is equal to μW,while μ is the coefficient of friction between the block and the table. If this concept is to be applied to a soil then the further idea accompanies in the simple model, is of zero velocity in the direction normal to that of motion of the block, should also be taken into account. These requirements concerning stress ratio and velocity will be combined to give an equation for yielding of cohesionless granular dilating media.

Consider an element of cohesionless medium shown in Figure (4-a), in which the material is yielding according to the strain rate system shown in Figure (4-b). Assuming the direction of principal compressive strain rate is shown in Figure (4-a), then the directions of the zero extension lines CA and CB in this diagram can be obtained by determining the magnitude of the angle θ (θ denotes the angle between the X-axis and the principal strain rate direction) as shown in Figure (4-b). These directions of zero extension can be called slip lines.

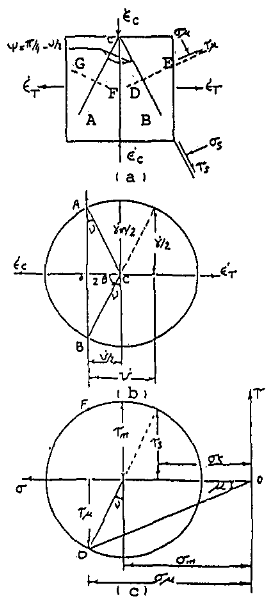

Fig.4. (a)directions&planes in soil element, (b)strain rate circle, (c)stress circle

A plane containing a slip line is not analogous to the table top discussed at the begining of this section, because the soil is dilating in a direction normal to that plane. There are however two planes within the yielding element of Figure (4-a), which are analogous to that table, that the strain rate is zero across them. These planes are DE and FG which are normal to the slip lines CA and CB respectively. It is now assumed that conditions of solid body sliding can be applied to these planes and that when the element of material is yielding, the shear stress(τ_μ) to the normal stress(σ_μ) on these planes is a constant value μ, which is a constant for the material irrespective of its instantaneous void ratio. This criterion for yielding of cohesionless media may be stated like " For an element of cohesionless dilating(increase or decrease of volume) granular material to be yielded, the stress ratio on the planes across which the strain rate is zero must have a maximum limiting value μ ".

In the case of coincidence of the principal axes of stress and of strain rate(as in our case), there is a direct relatioship between Mohr's circle of strain rate and of stress. By applying simple trigonometry to these circles, it can be shown that

$$\frac{\tau_m}{\sigma_m} = \frac{\mu}{\cos \nu - \mu \sin \nu} \qquad (2)$$

$$\frac{\tau_s}{\sigma_s} = \frac{\mu}{1 - 2\mu \tan \nu} \qquad (3)$$

where ν is the angle of dilatation and is given by either

$$\sin \nu = \frac{1/2(\dot{\epsilon}_T + \dot{\epsilon}_C)}{1/2\ \dot{\gamma}_m} = \frac{\dot{\upsilon}}{\dot{\gamma}_m} = \frac{d\upsilon}{d\gamma_m} \qquad (4)$$

or

$$\tan \nu = \frac{1/2(\dot{\epsilon}_T + \dot{\epsilon}_C)}{1/2\ \dot{\gamma}} = \frac{\dot{\upsilon}}{\dot{\gamma}} = \frac{d\upsilon}{d\gamma} \qquad (5)$$

and

$$\mu = \frac{\tau_\mu}{\sigma_\mu} \qquad (6)$$

Either equation (2) or equation (3) can regarded as the basic equation of the proposed criterion of yielding. It is preferable equation (2) since τ_m and σ_m refer to planes which are more directly related to the principal axes of stress.

To express that failure criterion in terms of the stress invariants, first, the principal stresses can be expressed as

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{pmatrix} = I_1/3\begin{pmatrix}1\\ 1\\ 1\end{pmatrix} + \frac{2}{\sqrt{3}}(J_2')^{1/2}\begin{pmatrix}\sin(\theta+2\pi/3)\\ \sin\theta\\ \sin(\theta+4\pi/3)\end{pmatrix} \qquad (7)$$

where

I_1 = The first invariant of stress tensor

J_2' = The second invariant of stress deviator tensor

J_3' = The third invariant of stress deviator tensor

$$\sin 3\theta = \frac{-3\sqrt{3}}{2}\ \frac{J_3'}{(J_2')^{3/2}} \qquad (-\pi/6 \le \theta \le \pi/6)$$

The angle θ has a distict physical meaning as it is similar to the Lode parameter Γ defined by $\Gamma = -\sqrt{3}\tan\theta$.

Then, the failure criterion can be expressed as

$$-\mu/3\ I_1 + (J_2')^{1/2}\ [(\cos\nu - \mu\sin\nu)\sin(\theta+\pi/3) - \frac{\mu}{\sqrt{3}}\cos(\theta+\pi/3)] = 0 \qquad (8)$$

3. CONCLUSION

The idea of stress characteristics in soil mechanics appears to be based upon the assumption that soils fail according to the Mohr-Coulomb criterion, but it seems that the concept of velocity characteristics is of greater practical value. In that theory no direct use has been made of the Mohr-Coulomb envelope. Yield can occur in a soil element at lower stress ratio than those defined by the Mohr-Coulomb hypothesis, and the yield stress ratio is a unique function of the dilatation angle(i.e. τ_m/σ_m is a unique fuction of ν)

REFERENCES

1. Roscoe, K.H. (1970). The influence of strains in soil mechanics. Geotechnique 20, no. 2, 129-170.