EFFECT OF SHEAR STRESSES ON THE ULTIMATE LOAD OF THIN-WALLED MEMBERS WITH OPEN CROSS SECTION

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1. Introduction

Limit state design of thin-walled members with open cross section has been in recent years the subject of many experimental and numerical investigations. Usually, this type of structures before collapse experiences large deformations accompanied by appearance of some yielded zones. Despite of this fact, only few analyses that take into account both geometrical and physical nonlinearities have been reported.

Several constitutive equations have been proposed in the literature. Among these Prandlt-Reuss flow theory[2] (abbreviated J2F) and tangent modulus approach[1,3] have been the most adopted ones. Neither of them takes account of shear stresses due to warping and non-uniform bending which practically play a primary role in the yield criterion of Von Mises.

In a recent work by Maier[4] Vlazov 's assumption which states that the shear in the middle line of the wall can be taken zero was dropped and all the shear effect was taken into account. As a result it has been shown through a comparaison study that their effect is significant especially in the plastic range.

In the present work, an inelastic finite displacement numerical analysis of thin-walled members with open cross section, is developed using the F.E.M. The J2F theory is taken as the basis for the development of the stress-strain relationship which includes the contribution of warping and non-uniform bending shear stress to yielding. The large displacement theory adopted in this analysis is realized by the use of a moving coordinate system.

2. METHOD OF ANALYSIS

Because of the forementioned nonlinearities imposed in this analysis, an incremental form of stiffness equation rather than a total one is adopted using the principle of virtual work. In this formulation the shear stresses of warping and non-uniform bending are assumed to have no work because their corresponding strains are zero according to beam theory. But their contribution to yielding which is usually neglected too by almost all the investigators in this field, is considered here. In developing the stress-strain relationship, it is assumed that a plastic shear strain due to warping and bending can take place.

Using the flow rule of Prandlt-Reuss and Von Mises yield criterion, the stress-strain relationship can be obtained and written as follows,

$$\begin{Bmatrix} \bar{\sigma} \\ \bar{\tau} \end{Bmatrix} = E \begin{bmatrix} C_1 & C_2 \\ C_2 & \frac{G}{E} C_3 \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon} \\ \bar{\gamma} \end{Bmatrix} \quad \cdots\cdots\cdots\cdots (1)$$

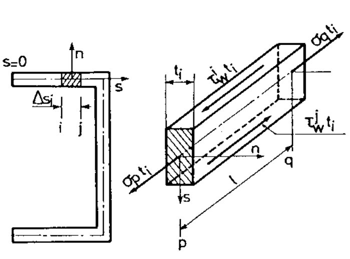

Fig.1 Determination of Shear Stress τ_w.

where $\bar{\sigma}$, $\bar{\tau}$ and $\bar{\varepsilon}$, $\bar{\gamma}$ represent the axial and St. Venant incremental stresses and their corresponding strains, respectively. C_1, C_2 and C_3 are identified to be $(1-D_1)$, $-D_2$ and $(1-D_3)$ in Ref.(2) but by replacing τ by $(\tau + \tau_w)$ with τ_w being the total warping and bending shear stress. Unlike σ and τ, τ_w is determined from equilibrium with σ. This can be achieved easily since each cross section is divided into a convenient number of small segments along the profile and across the thickness inside each σ is known. Referring to Fig.1 , an appropriate contour coordinate s is set up with origin located at an open end and the distribution of τ_w is given by

$$\tau_w^j = \tau_w^i - (\sigma_q - \sigma_p)\Delta s_i/l \qquad ....(2)$$

where σ_p and σ_q represent the total average normal stress acting at the middle line of an interval i of the profile at the extremities p and q of an arbitrary element of length l, respectively. These stresses σ,τ and τ_w will be used in the numerical integration required for the evaluation of the stress resultants needed to form the internal forces and the tangent stiffness equation which will be solved using an iterative procedure to remove the error caused by its linearization (work due to the nonlinear part of the strain is neglected).

3.RESULTS AND DISCUSSIONS

In order to show the effect of the shear stresses caused by non-uniform bending and torsion on the ultimate load ,a variety of problems that includes torsion , bending and compression is examined numerically. Fig.2 shows two members with wide flange cross section dedicated to fit two specimen HT-2 and HT-5 of Ref.(5). The examination of this figure reveals that consideration of warping shear stress in the yield condition leads to a lower value of the ultimate strength that is found to be in good agreement with especially the experimental one of specimen HT-5. The ultimate strength of beams under a simultaneous action of flexure and torsion is examined. The theoretical model is shown in the inset of Fig.3. The effect of load application point is taken into account.The solution of the analysis which takes into account the contribution of shear stresses to yielding seems to be more close to the experimental one and that of Maier(4).

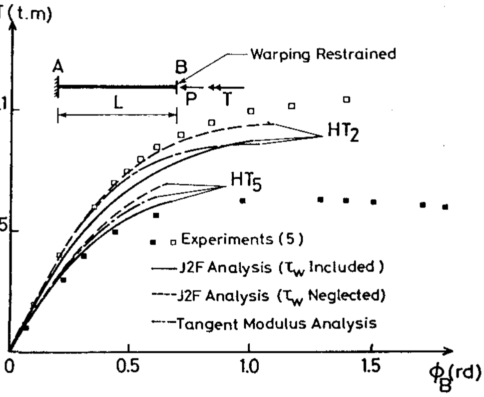

Fig.2 Members Subjected to Torsion

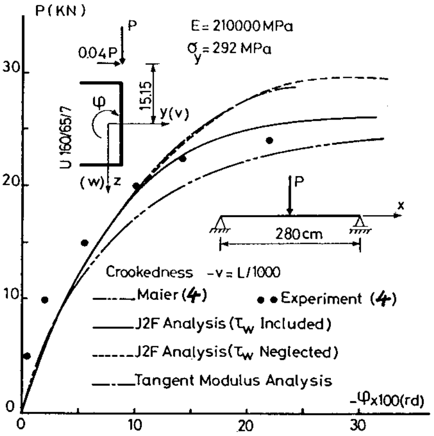

Fig.3 Channel under Flexure and Torsion .

4.CONCLUSION

An inelastic F.E.M formulation of arbitrary thin-walled members with open cross section which includes the contribution of shear stresses to yielding ,is developed. The numerical results confirm its validity and wide applicability and show that the effect of shear in the plastic range ,generally ,can not be neglected.

5.REFERENCES

1)Rajasekaran,S. and Murray,D.W.: ASCE, Vol.99, No.ST6, pp.1025~1041, June 1973
2)Sakimoto,T.,Yamao,T.,Kikuchi,R.and Sakata,T.:Proc.of JSCE,No 362/I-4,October,1985
3)Epstein,M.,Nixon,D.and Murray,D.W.:ASCE,Vol.104,No ST5,pp.841~853,May,1978
4)Maier,D.H.:Heft 7,Institut fur Baustatik und Meßtechnik,Universitat Friedericiana zu Karlsruhe,1986
5)Yamao,T.,Sakimoto,T.:Proc.of JSCE,Vol.32A,pp.207~214,March,1986(in Japanese)