ULTIMATE STRENGTH ANALYSIS OF THIN-WALLED OPEN CROSS SECTION MEMBERS

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1. INTRODUCTION

The common use of the thin-walled member, especially in bridges, buildings, ships and aircraft, has made the prediction of the ultimate strength of such structures a problem of great importance. Recently, a considerable amount of theoretical studies has been carried out on the post-buckling elastic behaviour and the spatial instability of these structures. In most of these studies, the updated Lagrangian approach has been used, since, as it is well known, it is more expedient than the total one in the case when displacements and rotations are large while the strains remain small.

A finite element large deflection elasto-plastic analysis has been the subject of many investigations. In order to form the tangent stiffness matrix, in Ref.3) 12 degrees of freedom and the tangent modulus approach were used while in Ref.4) only the usual 7 degrees of freedom were used and yielding was considered to be a function of the normal and shear stresses. However, in this study, a moving element coordinate system(similar to Ref.2) and a "transformed area concept" based upon the tangent modulus approach (including the effect of strain reversal) are the two fundamentals in order to form the tangent stiffness matrix(14*14).

2. BASIC EQUATIONS

1) Stress-strain Relationship

$$\dot{\sigma} = E_t(\dot{\varepsilon}, \varepsilon)\,\dot{\varepsilon} \qquad (1)$$

where, Fig.(1)

$$E_t = \begin{cases} E & \text{if } \dot{\varepsilon} \le \varepsilon_y \text{ or } \dot{\varepsilon} > \varepsilon_y \text{ and } \dot{\varepsilon}\varepsilon < 0 \\ E_p & \text{if } \varepsilon_y < \dot{\varepsilon} \le \varepsilon_s \text{ and } \dot{\varepsilon}\varepsilon > 0 \\ E_s & \text{if } \dot{\varepsilon} > \varepsilon_s \text{ and } \dot{\varepsilon}\varepsilon > 0 \end{cases}$$

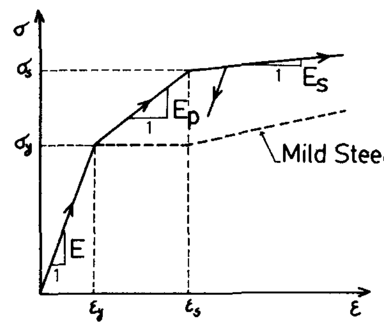

Fig.1 Stress-Strain Diagram

2) Incremental Stiffness Equation

The incremental stiffness equation for the whole structure under consideration obtained after introducing the selected displacement functions into the expression of the virtual work, may be written in matrix from, for the (i+1)-th step

$$[K_T(E_t, P_i(x))]\,\{\Delta r\} = \{\Delta R_i\} \qquad (2)$$

and

$$\{R_i\} - \{F_i\} = 0 \qquad (3)$$

where

$[K_T]$: tangent stiff matrix after i steps
$P_i(x)$: internal forces which are assumed to vary linearly through the element
R_i, F_i : total external and internal force vectors
$\Delta R, \Delta r$: incremental load and displacement vectors

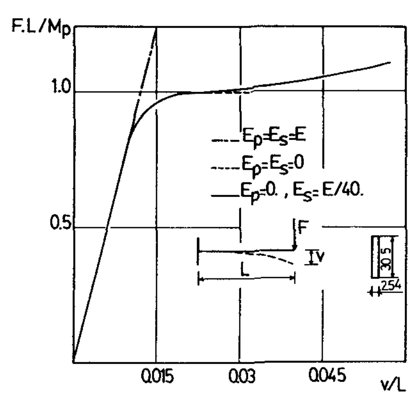

Fig.2 Load versus Relative End Deflection

3) Solution Procedure:

In dealing with elastic materials, Eq(3) needs not to be checked(in the condition that small increments are used), but when elasto-plastic material is concernd it has been found better to check it otherwise $[K_T]$ will deviate from its correct path and then there will be possibility of error.

3. RESULTS AND DISCUSSIONS

Some simple examples were examined numerically in order to justify the applicability of the proposed analysis to a variety of problems.

An inelastic bending analysis of a cantilever beam with thin rectangular section is carried out mainly to show the effect of the strain hardening. Good results can be observed in Fig.2.

As a second example, the ultimate strength of a cantilever of wide-flange cross section subjected mainly to a torque T applied at the free end, is determined for different initial axial forces. Here the ultimate load is judged to be the load at which the tangent stiffness matrix becomes no longer definite positive. No available results for comparison. In Fig.4, plastic buckling analysis is shown. The model and the cross sectional dimensions are shown in the inset of the figure. The plastic buckling load obtained from the present method is found to be 5.3% smaller than the one given by Perry Robertson formula. This can be explained since it is widely adopted that the tangent modulus approach leads to a reasonable lower bound estimates of the ultimate strength.

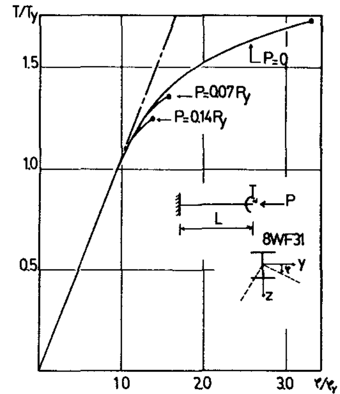

Fig.3 Torque versus End Twist

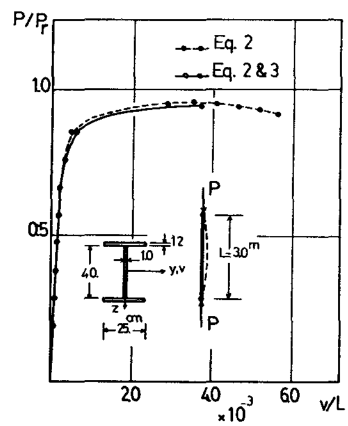

Fig.4 Thrust versus Relative Mid-Span Deflection

4. CONCLUSION

In determining the ultimate or even the elastic critical load which both are fundamental for safe and economical design, the present method seems to be powerful and leads to satisfactory results.

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