粗面上の非定常流に対するk－ε モデルの適用

AN APPLICATION OF k-ε MODEL OF TURBULENCE TO UNSTEADY FLOW OVER ROUGH SURFACES

O Ismail AYDIN*

Nobuo SHUTO**

I- INTRODUCTION

In unsteady flows on the sea bed a correct computation of flow phenomena (such as velocities, bed shear stresses, wave energy dissipation, etc.) requires development of new tools which can predict time dependence of turbulence precisely. The standard k-ε model of turbulence solves partial differential equations (pdes) for turbulence kinetic energy k and turbulence energy dissipation rate ε in the high turbulence-Reynolds number range and incorporates "wall functions" in the wall region. In unsteady flows wall functions fail to represent transitory behaviour of turbulence. Numerical solution of the pdes from wall to outside of the boundary layer is necessary. A full solution is accomplished by adding "Low-Reynolds Number" and "Surface Roughness" effects into the pdes. Method is applied to wave and wave-current motion over flat bed.

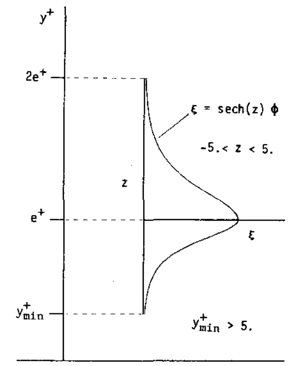

Fig.1 Turbulence Energy source due to surface roughness.

II- THE MODEL OF TURBULENCE AND IT'S SOLUTION

The governing (modeled) equations for turbulent, one-dimensional,unsteady motion can be written as;

Streamwise momentum:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\left[(\nu+\nu_t)\frac{\partial u}{\partial y}\right] - \frac{dp}{\rho dx} \qquad (1)$$

Turbulent viscosity hypothesis:

$$-\overline{u'v'} = \nu_t\frac{\partial u}{\partial y} = (C_\mu\frac{k^2}{\varepsilon})\frac{\partial u}{\partial y} \qquad (2)$$

Turbulence kinetic energy:

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial y}\left[(\nu+\nu_t)\frac{\partial k}{\partial y}\right] + \nu_t(\frac{\partial u}{\partial y})^2 - \varepsilon - \frac{2\nu k}{y^2} + \xi \qquad (3)$$

Turbulence dissipation rate:

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial y}\left[(\nu+\frac{\nu_t}{\sigma})\frac{\partial \varepsilon}{\partial y}\right] + C_1\frac{\varepsilon}{k}\nu_t(\frac{\partial u}{\partial y})^2 - C_2\frac{\varepsilon^2}{k} - \frac{2\nu\varepsilon}{y^2} f \qquad (4)$$

where,

$C_\mu = 0.09\{1.-\exp(-0.0115y^+)\}$

$C_1 = 1.35$

$C_2 = 1.8\{1.-0.22\exp(-(\frac{R_t}{6})^2\}$, $R_t = \frac{k^2}{\nu\varepsilon}$

$\sigma = 1.3$

$f = \exp(-0.5y^+)$

The low-Reynelds number terms ($2\nu k/y^2$ and $2\nu\varepsilon f/y^2$) and related damping functions and constants are proposed by CHIEN [1] and found to be the best fitting among others to available experimental data by PATEL etal.[2] . The last term ξ in the k-equation is included in the present study to model the additional turbulence energy production rate by surface roughness (fig. 1). This energy source is assumed to distribute as

$$\xi = \mathrm{sech}(z)\phi \qquad (5)$$

the origin (z=0) being located at e^+. The peak production rate ϕ is a function of e^+. An implicit relation between ϕ^+ and e^+ is obtained from pipe flow data (Fig.2).

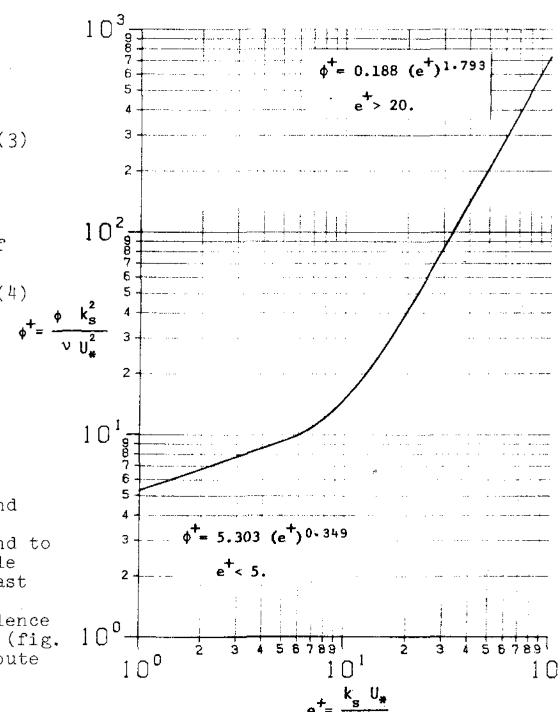

Fig.2 Turbulence energy production rate due to surface roughness

* Graduate student, Civil Engineering Department, Tohoku University
** Professor, Civil Engineering Department, Thohoku Uninersity

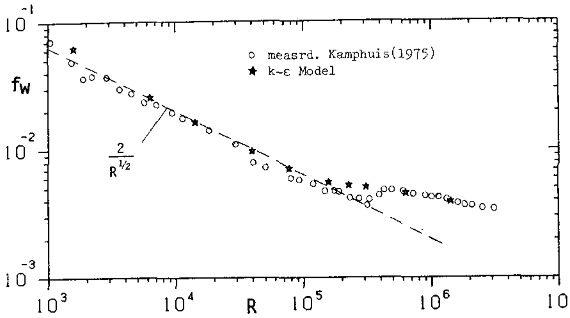

Fig.3 Friction Factor f_w for smooth bed

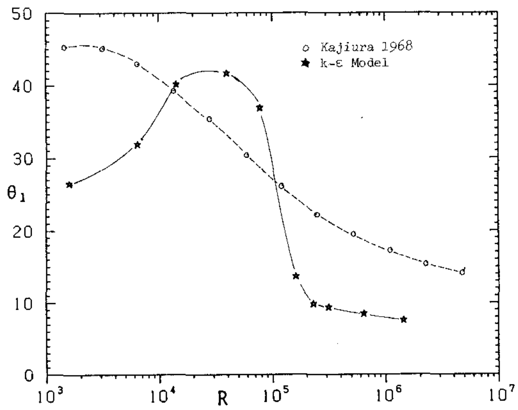

Fig.4 Variation of θ_1 with R

The y-coordinate is transformed by

$$\eta = \mathrm{Log}_{10}\left(\frac{y}{L}+1\right) \quad ; \quad 0 < \eta < 2 \qquad (6)$$

for computational purposes, where L being a length scale. Equations 1~4 are replaced by thier finite difference equivalents (fdes). Each set of fdes are solved by a tridiagonal algorithm. Final solution is achieved by iteration.

III- RESULTS AND DISCUSSIONS

Friction factor, f_w and phase shift between bed shear stress and velocity outside the boundary layer, θ_1 obtained from smooth bed computations are shown in figures 3 and 4. Computed friction factors agree well with maesurements of Kamphuis. On the other hand θ_1 obtained from k-ε model is quite different than that suggested by Kajiura.

In case of rough bed f_w suggested by Kamphuis, Kajiura and Jonsson are compared with predictions of present k-ε model (fig.5). There is a discrepancy between present method and others. Kamphuis and Jonsson obtained their results from experiments performed over artificially roughened surfaces. Kajiura make use of assumptions about the effect of turbulence which have been found to give good results in steady flow. Each method assumes different turbulence structure or different degree of turbulence. It should be noted that the discrepancy decreases as $\hat{U}_w/\omega k_s$ increases; implying that as the flow becomes more turbulent all methods give similar results.

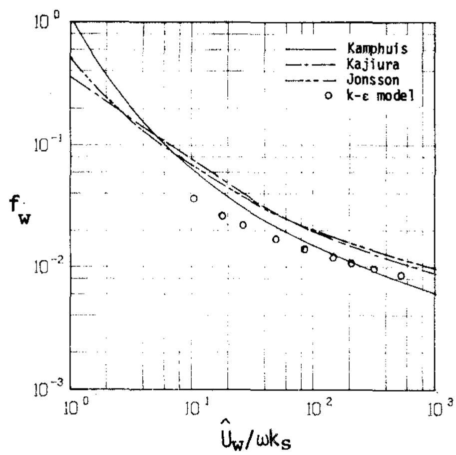

Fig.5 Friction Factor f_w for rough surfaces

In figure-6 wave energy dissipation factors are presented. Curves for rough bed suggested by Sleath seems to agree with k-ε model predictions.

Although computation method becomes more lengty and costy because of the modifications introduced into k-ε model, results are encouraging for more complicated problems such as wave-current combination or flows with recirculation.

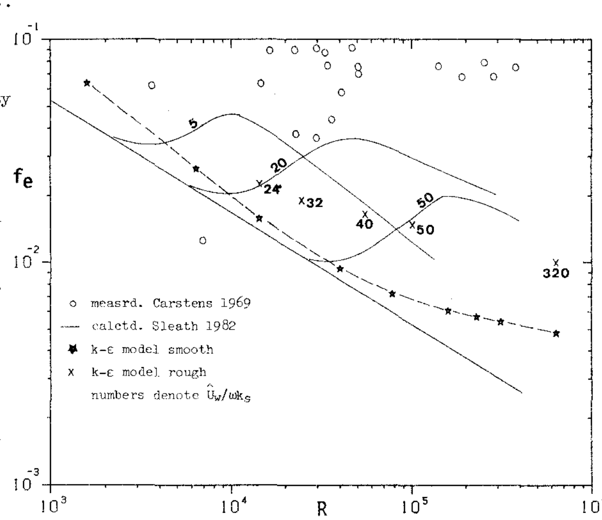

Fig.6 Wave-Energy dissipation factor fe

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