Experimental and analytical study on shear capacity in steel fiber and stirrup RC beam

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Abstract

In this study, strength and deformation behavior in steel fiber reinforced concrete (SFRC) and stirrup RC beams, under bending-shear load, is investigated by analytical and experimental methods. In the experimental test, optical measurement method was used. Analytically, a theoretical model for SFRC beams developed by the authors and FEM code was used. The structural response throughout the loading is captured and presented. Strength comparison among SFRC, stirrup and control beam showed that SFRC beam has a better performance. A parametric study on fiber content and shear span to depth ratio variation characterized the strength enhancement in the fibrous beams. Analytical results obtained using the theoretical model and FEM were in accord with the experimental results.

Keywords: SFRC beam, Theoretical model, optical measurement method, shear capacity

1. INTRODUCTION

Shear failure in reinforced concrete structural members is a major concern to civil and structural engineers, for it is known to be sudden and catastrophic. It occurs when the principal tensile stress limit in the shear region is exceeded. This leads to the development of shear cracks. Once tensile cracking occurs, the tensile stress at the crack rapidly softens, which significantly reduces the shear strength of the beam¹. Conventionally, engineers reduce and control this problem in reinforced concrete structural systems through design by providing stirrup reinforcements. However, use of steel fibers in reinforced concrete beams is expected to soften the shear brittle failure and prolong the deformation thus allowing for improvement of the ultimate flexural capacity.

Improvements in ductility, crack control, earthquake resistance, relieve of stirrup reinforcement in congestion areas are identified as possible merits of utilizing SFRC in structural systems¹⁻⁵⁾. The unknown as stated by Pascal et al ²⁾ is whether steel fibers can replace transverse reinforcements in reinforced concrete beams. The answer is still a research subject, since there is no known application of steel fibers independently as shear reinforcements. Dupont and Vandawalle⁶, Brandt⁷⁾ have mentioned the lack of design guidelines and transparency in existing information, coupled with limited understating of SFRC

material as the barrier to wider application of SFRC such as in shear strengthening. For clarity on the shear strength merit offered by steel fibers, it is imperative to have comparative fiber and stirrup reinforced evaluations. Although testing can undoubtedly give fundamental information on the structural behavior, it can be a very costly and time consuming procedure. Therefore, analytical methods are useful in predicting the structural response of the SFRC beams cost-effectively, provided these methods can simulate the test behavior well.

The existing shear capacity analytical relations are basically empirical and ultimate based models, which have also been found not to be inaccurate⁸. In the present study, a non empirical (not experimentally derived) theoretical model derived by the authors, whose details are reported in reference⁹⁾ is applied to predict the complete shear strength response behavior until failure in SFRC beams. Further ultimate strength prediction comparisons are made between the results from JSCE recommended formula¹⁰⁾ and the proposed theoretical model in its ultimate form. From the numerical point of view, literature survey shows that few attempts have also been made in the past to numerically simulate the response of steel fiber reinforced members^{11,12}. This has been attributed to the lack of suitable constitutive material models for SFRC composite¹¹⁾. In the present study, an experimental SFRC stress strain material model was used. Strain based models are attractive, since there

is no need for abstract sophisticated crack laws, furthermore, stress strain relations can be input directly¹³⁾.

Therefore, the purpose of this study is to investigate shear strength-bending deformation response in SFRC and stirrup reinforced beams using analytical and experimental methods. Because generations of cracks occur momentarily and their positions are uncertain, accuracy of the changes in strains can not be assured by using strain gauges only. Therefore occurrence and progress of the cracks and the strain changes can be measured by implementing the full field displacement optical digital image correlation method (DICM). Therefore shear deformation measurement clarity was indicted by comparisons with conventional strain gauge (strain rosette) method. The specific objectives are to (a) compare the performance of SFRC and the stirrup beams under bending shear loading (b) evaluate the response of the proposed theoretical model and (c) evaluate the numerical response.

2. EXPERIMENTAL PROGRAM

2.1 Materials and specimen manufacture

In the manufacture of specimens, concrete with an average strength of 38MPa was used in which fine and course aggregates meeting JSCE guidelines¹⁴⁾ was used. SFRC was made by incorporating 1.0% high strength steel fibers (1000MPa, d_fl_f of 48.4) in the concrete when in the fresh state. Workability of the mix was improved by using a water reducing admixture (0.8% content by volume) without increasing the water content.

In order to equivalently match the stirrups content and allow for qualitative test results comparisons, the steel fiber content used was determined using eq.(1a)

$$v_f = \frac{N_s a_s l_s}{l_k A_b} \tag{1a}$$

Where, v_f is the equivalent fiber fraction, N_s is the number of stirrups required, and it corresponds to an equivalent fiber content, , l_b is the beam length, A_b is the cross sectional area of the beam, a_s and l_s are the stirrup cross-sectional area and length, respectively. The beams were reinforced in flexure with 13mm diameter deformed re-bars with yield strength of 345MPa. The flexural and stirrup reinforcement capacity was determined by design and the flexural reinforcements increased appropriately using the relative flexural capacity analysis relations given by Russo and Puleri¹⁵. Tailor-made timber moulds were used in



Fig. 1 Casting of RC beam

the casting of the beam specimens as shown in Fig.1. Cylinder specimens were made concurrently with the beams for strength control and evaluation. All the specimens were cured for a period of 28 days before testing.

2.2 Testing procedure

In the experimental program, bending shear tests were conducted on the three 1800mm×230mm×150mm consisting of the control, fiber and stirrup RC beams (see Figs 3, 4 and 5). Tests on the beam specimens were done using a 2000kN capacity universal testing machine. Deformations were measured by conventional (use of strain gauges and LVDTs) and optical Digital image correlation method (DICM) which consists of two sets of high resolution cameras and data acquisition and correlation analysis Laptop/PC as shown in Fig.2 (see also Fig 3). For shear strain determination, strain gauges were applied in the form of a strain rosette in the shear region at the back side of the beam at a depth of 115mm and mid shear span length (i.e. 250mm), while DICM strain measurements, were determined from DICM measurements obtained from the front side, shear region of beam as shown in Fig.3. Moreover, accurate identification of the initiation of the shear cracks was being monitored by the DICM method. Fig. 3 illustrates the testing set up, instrumentation and measurement regions on the beam specimen.



Fig.2 Schematic representation of DICM equipment in front of a specimen



Fig. 3 Test set up and instrumentation



3. ANALYTICAL STUDY

In the analytical study, a theoretical model for SFRC beams developed by the authors and SOFISTIK FEM code was applied. In the theoretical model, details of which have been reported in reference⁹; the necessary fundamental relations are reproduced and applied. Analysis was conducted and the structural response throughout the loading was captured in terms of the load-deflection behavior.

3.1 Proposed theoretical model

In this paper, the fundamental relations applied in the model derivation and analysis is summarized. This is because of the lengthy derivation formulas, whose details are given in reference⁹⁾. In the proposed theoretical model⁹⁾, simplified strain ratio based shear strength and defection relations were derived for the prediction of complete evolution of the shear capacity in steel fiber reinforced concrete (SFRC) beams. In the model shear resistance contributions from the steel fiber reinforced concrete and the dowel action of the main reinforcements have all been considered in a unified manner based on equilibrium of forces and the fiber stress transfer mechanisms. Fig.6 shows the simplified model and stress-strain diagram considered 9). In the derivations, SFRC beam with bending reinforcements but without stirrup reinforcement is considered to resist shear through the fiber's stress transfer mechanism, concrete tensile, compressive and shearing action along the shear crack path. Bending reinforcements are considered to offer some contributions through its dowel action. From Fig 6(iii), the relations of the forces that act in resisting shear are given by

$$F_{cc} = \sigma_c b \left(c - \frac{w}{\psi} \right)$$
 (Compressive force) (2a)

$$F_{ct} = \sigma_{ct} b\left(\frac{w}{\psi}\right)$$
 (Tensile Force) (2b)

$$F_{d} = k\varepsilon_{fp} l_{f}^{ef} \frac{\Delta \gamma}{\gamma_{y}} \cos \alpha \sqrt{\frac{A_{s}}{\pi}} \quad \text{(Dowel force)} \tag{2c}$$

$$F_s = 2\pi \tau_b l_a^{ef} \sqrt{\frac{A_s}{\pi}}$$
 (Re bar tensile force) (2d)

Where *b* is the beam width and *w* is the crack width, while ψ , σ_c and σ_{cl} are the crack opening angle, concrete compressive and tensile strengths, respectively, *k* is the concrete bearing strength¹⁵, l_f , l_f^{ef} , l_a and l_a^{ef} are fiber length, fiber effective length, rebar anchorage length and reinforcement bar effective anchorage length, respectively. A_s is the steel reinforcement bar, while τ_b and α are the bond strength and angle of the shear crack path, respectively. The relation for the shearing forces (F_{cv} and F_a) in the compressed and cracked regions (Fig.6iii) is established and considered under equilibrium analysis.



Fig.6 Conceptual model details 9)

(1) Shear strength relation

By applying static equilibrium of the derived forces (eq. 2a to 2d), a relation for evolution of shear strength (ΔV) for SFRC beam was derived, in which incremental shear strength can be predicted by applying the incremental shear strain ratio. The relations simply given by

$$\Delta V = \Delta V_{fc} + V_c + \Delta V_d \tag{3a}$$

In equation (3a) the contribution from the plain concrete V_c is a fixed term because the compressive and tensile concrete strength is considered at yield only (i.e. at cracking, see also Fig.5b) as it is brittle. Eq.(3a) follows the traditional principle of superposition whereby the fiber-concrete composite V_{fc} , concrete tensile V_c and dowels action V_d of the main reinforcements are summed up. Each of the component contributions in eq. (3a) (refer in detail in reference ⁹) are given as follows;

$$\Delta V_{jc} = \frac{dbK_{j}}{3\beta co^{2} \alpha} \left\{ 3 - \left(\frac{\gamma_{y}}{\Delta \gamma}\right)^{2} - \left[\frac{K_{l}}{\sigma_{c}}\left(2 - \frac{\gamma_{y}}{\Delta \gamma}\right) + \frac{\sigma_{ct}}{\sigma_{c}}\right] \left[\frac{3\sigma_{ct}}{2K_{l}} + 3 + \left(\frac{\gamma_{y}}{\Delta \gamma}\right)^{2} - 3\frac{\gamma_{y}}{\Delta \gamma}\right] \right\}$$
(3b)

$$V_c = \frac{db\sigma_{ct}}{2\beta \cos^2 \alpha}$$
(3c)

$$\Delta V_{d} = \frac{1}{\beta^{2}} \sqrt{\frac{A_{c}\rho}{\pi}} \left(k\varepsilon_{jp} I_{f}^{ef} \frac{\Delta \gamma}{\gamma_{y}} \cos\alpha + 2\pi\tau_{b} I_{a}^{ef} \tan\alpha \right)$$
(3d)

Where by

$$K_{I} = \left(E_{f} v_{f} \varepsilon_{fp}\right) / \pi \tag{3e}$$

$$\varepsilon_{fp} = (\tau_b A_f) / E_f$$
 (3f)

$$l_f^{ef} = 0.25 l_f \left(1 - 4\varepsilon_{fp} \frac{\Delta \gamma}{\gamma_y} \right)$$
(3g)

$$l_a^{ef} = 0.3 l_a - \varepsilon_{fp} l_f \frac{\Delta \gamma}{\gamma_v}$$
(3h

Where E_f is the elastic modulus, A_c is the concrete beam cross area, *b* is the beam width, *d* is the beam effective depth, ρ is the reinforcement ratio, v_f is the fiber content, ε_{fp} is the fiber pull out strain, τ_b is the bond strength, A_r is the fiber aspect ratio, σ_{ct} is the split tensile strength, σ_c is the average compressive strength, β is a shear span to depth ratio factor (a/d = 2.38), α is the angle of shear crack inclination and $\Delta \gamma_y / \gamma$ is shear strain ratio increment (which must be applied incrementally i.e. $\gamma_v / \gamma = 0, 1, 2, 3...$).

The yield shear strain is determined theoretically based on the relation given by Gere and Timoshenko¹⁶.

(2) Deflections

The curvature ratio relationship in elastic and inelastic bending in beams, given in Gere & Timonshenko¹⁵⁾ and Mosley¹⁶⁾ are applied in the determination of mid span deflections. Through elastic and in-elastic stage the curvature relation in a

beam can be estimated from the relation¹⁵⁾

$$\frac{\Delta\theta}{\theta_{y}} = \frac{1}{\sqrt{3 - \frac{2\Delta M}{M_{y}}}}$$
(4a)

Where, $\Delta \theta = 1/\lambda$ and $\theta_y = 1/\lambda_y$ are the elastic and inelastic curvatures, respectively, while, λ , M and M_y are the radius of the curvature, general and yield moment, respectively. The evolution of the moment M through the incremental moment ΔM is such that $\theta \leq M \leq M_p$, whereby M_p is the plastic moment¹⁶.

In elastic bending, the deflection's increment can be estimated from the relation,

$$\Delta \delta = \xi l_e^{-2} \Delta \theta \tag{4b}$$

Where $\Delta \theta = \Delta M/EI$, while, *E*, *I* and *l_e* are an elastic modulus, moment of inertia, and effective length of the beam, respectively. The factor, ξ is estimated from modification of the relation given for moment deflection and moment curvature diagram for beams with elastic-plastic material^{16, 17)}. In this case, before yielding $\xi = (1-\varphi - 0.083/\varphi)$ and after yielding ξ is multiplied by 0.5 (i.e. $\xi = 0.5(1-\varphi - 0.083/\varphi)$) for a rectangular section¹⁶, where $\varphi = a/l_e$ and *a* is the shear span length. For continuity, it is assumed that at yield (cracking), $\theta \leq \theta_y$ and $\theta_y = M_y/EI$. Applying these relations in eq. (4a) and (4b), the relation for the determination of mid span bending deflections through an inelastic stage is obtained as;

$$\Delta \delta_{b} = \frac{2\xi l^{2} e^{\Delta M}}{EI\left(3 - \left(\frac{\theta_{y}}{\Delta \theta}\right)^{2}\right)} \quad (0 \le M \le M_{p})$$

$$(4c)$$

The relative shear displacement can be estimated from the relation in reference⁹⁾. The proposed relation for shear displacement requires the use of the shear strain ratio, the shear span to depth ratio and modified equations for shear strain given in by Gere and Timoshenko¹⁶⁾. Thus the shear displacements were estimated from the following relation;

$$\Delta \delta_s = a \frac{I.2Q_y}{2GA_c} \left(\frac{\Delta \gamma}{\gamma_y}\right) \tag{4d}$$

From eqs (4c) and (4d), the total incremental deflection at each load step is determined as the sum of the incremental bending and shear displacements respectively and is given by,

$$\Delta \delta_t = \Delta \delta_s + \Delta \delta_b \tag{4e}$$

Where, $\Delta \delta_t$ is the total incremental deflection, while $\Delta \delta_s$ and $\Delta \delta_b$ are the shear and bending incremental deflections, respectively.

Table 1 below summarizes the material properties and other structural parameters applied in the theoretical model analysis of equation 3a

Reinforcements	Bar	E _f (MPa)	210000	
		σ_{sy} (MPa) SD345)	345	
	Fibers	A_r	48.4	
		d_f (mm)	0.62	
		l_f (mm)	30	
		E_f (MPa)	210000	
		σ _{fy} (MPa)	1000	
		Fibre content	1.0%	
Beam Concrete/ fiber concrete		σ_c (MPa)	38	
		σ_{ct} (MPa)	3.67 (non fiber_0%)	
		σ_{ct} (MPa)	4.40 (SFRC_1%)	
		E_c (Mpa)	31108 (non fiber)	
		E _{fc,} (Mpa)	54504 (with 1% fiber)	
		V	0.195	
		k (MPa)	6895 (ref. 15)	
		τ_b (MPa)	4.15	
		β (a/d)	2.38	
		<i>d</i> (mm)	210	
		b (mm)	150	
Strain ratio		$\gamma_y / \Delta \gamma$	0, 1, 2, 3, etc.	

Table 1 Values of parameters applied in theoretical analysis

3.2 FE analysis

In the numerical study, modeling and analysis was programmed using SOFISTIK FEM code¹⁸⁾. Incremental loading and modified Newton Raphson method was applied in the analysis. Speed and convergence are increased through Crisfield accelerating algorithm. This method notices the residual forces developing during the iterations and calculates the Crisfield coefficients applied in which convergence is determined. In the iteration steps, new displacements and stresses are determined. It is checked whether cracks or any other non-linear effects have occurred at any element. Cracked elements are considered with a reduced stiffness.

(1) Structural model

The structural model was designed to replicate the test specimen. The FE model analyzed consisted of beams whose overall dimensions were 1800×230×150mm. A minimum mesh size of 16x45 was adopted and applied. The resulting meshed beam model is shown in Fig.7. Four nodes quadrilateral isoparametric plane stress elements were used to model the SFRC/plain concrete elements. In plane elements of SOFISTIK, a general quadrilateral element with four nodes (QUAD) is sufficient, so that the introduction of the six to nine noded isoparametric elements is not necessary¹⁸)



Fig. 7 Meshed Model beam

(2) Material models

The material model applied consisted of a non linear stress-strain relation in tension and compression for 0% (control material) and 1% fiber concrete material. Fig.8 and 9 give these stress strain relations for the SFRC and the control material model (non fibrous concrete). As it was established that there was no significant effect of the fibers on the compressive strength¹⁹, an average compressive stress strain relation was applied as shown in Fig 9. Material behavior for the reinforcing bars was defined with a standard elastic-perfect plastic stress strain relation as shown in Fig. 10.



Fig. 8 Tensile stress strain relation (SFRC & Plain concrete)





Fig. 10 Standard stress strain relation for the main re-bars

4. RESULTS AND DISCUSSION

4.1 Experimental and FEM results

In the results presentation, the following designations have been applied to simplify the description of the specimens:

- CB0% Control beam
- FB1% Fiber beam reinforced with 1% fiber content
- SB1% Stirrup Beam reinforced with stirrups equivalent to 1% of the fiber content.

(1) Comparison of shear load-deflection behavior

Fig. 11 and 12 depicts the performance response in terms of shear load deflection curves from the experimental and the numerical analysis, respectively. It is apparent from these results that the beams reinforced with steel fibers and stirrups ultimately failed at higher loads when compared with the control beams.

Ultimate shear loads were found to be approximately; 67, 64, 55kN and 69, 66, 54kN for FB1%, SB1% and CB0% in the experiment and numerical analysis, respectively. It's also clear in both experimental and FEM results, that the SFRC beam has slightly outperformed the stirrup beam in terms of ultimate strength; however, there was no much difference in the deflections throughout the loading regime (i.e. overall trend), particularly between the stirrup and the control beam. Further, the response is noted to be linear until yielding point (point at the end of the linear stage approximate between 50 and 60kN) beyond which yielding and softening (experimental results) occur. However, the softening trend after ultimate failure could not be achieved in the numerical result.

In the early phases, the numerical and experimental results are linearly in agreement, and although there are minor differences in the non linear phase, generally they are in close accord. The only noticeable difference is that the softening phases in the numerical results have not been predicted. The same phase is characterized in the experimental results by a reduction in strength with an increasing deflection capacity.







Fig.12 FEM comparative load deflection response

(2) Shear stress strain analysis and response comparisons

Fig.13 shows the experimental relationship between shear stress and shear strains. Fig.14 depicts the FEM non linear shear stress distribution behavior in SFRC (FB1%) beam. In the experimental case the average shear stress is given by

$$\tau_{xy} = \frac{P}{2bh} \tag{5}$$

Where, P is the applied load while b and h are width and depth of the beam, respectively. Shear strains refer to the strains obtained from analysis of the strains measured by the strain rosette and those from the DICM measurement. As for the strain rosette the shear strains are determined by

$$\gamma_{xy} = 2\varepsilon_d - \left(\varepsilon_x + \varepsilon_y\right) \tag{6}$$

Where γ_{xy} is the shear strain, ε_d is the diagonal strain values (measured along a 45° orientation) while ε_x and ε_y are the strain measurements along the Cartesian coordinates x and y respectively of the strain rosette (see Fig 3).

DICM shear strains were obtained by averaging the DICM shear strain values within an area approximately equal to that covered by the strain rosette and within the geometric locations as the strain rosette. The two sets of strain results were made to evaluate the accuracy of the results obtained by the strain rosette and compare the two sets of results.

Since the shear stresses are determined from the load response as discussed in the beginning of this section, a similar performance in the fiber and stirrup beams over the control beam in terms of strength is noted as depicted in the stress strain curves (Fig 13). Fiber concrete ultimately performed better both in terms of shear strength and strain ductility. The influence of the steel fibers in the shear region is clearly illustrated by the high strength-strain capacity in FB1%.

It can also be seen that the shear strain values obtained by means of strain rosette analysis are higher than those obtained by means of DICM method (i.e values in the horizontal axis of Fig. 13a and b). Moreover, shear strains from the stirrups beam SB1 (SB1 %) were curtailed at the ultimate strength. It appears the measurement point coincided with a point of excessive shear deformations at failure, which affected the strain measurements, particularly after ultimate load. In the FEM shear the stress distribution, the region under severe shear loading is found to be in accord with the region in which shear cracks were observed as 1 depicted in Fig.15.





(3) Failure mode

General physical failure modes and the cracking behavior in the shear region as captured by DICM method were as shown in Fig.15 (a), (b) and (c). As depicted in this figure, a trend is noted whereby the control beam, fiber and stirrup-reinforced beams ultimately failed at different load levels. It appears that a combination of diagonal tension failure, flexural cracking near the mid span and concrete crushing in the compression region were responsible for the ultimate failure of the beams. The compression crushing was more pronounced in the control beam which had no any form of shear reinforcements. The failure load for the fiber reinforced beams (FB1) was higher than that of the stirrup beam (SB1). Indeed both fiber and stirrup reinforced beams failed at higher loads than the control beam (CB0). By optical digital correlation image method, the shear cracks which could not be clearly seen by the naked eve could be captured as illustrated in Fig.15. During testing it was observed that fiber beam deformed at a much slower rate than the other the beams.

4.2 Theoretical results and comparisons

Theoretical shear load-deflection prediction results matched alongside experimental and FEM, for comparison purpose are shown in Figs.16 and 17, respectively. As shown in these figures, the theoretical model has predicted well experimental results, while the FEM results were fairly predicted.

It can be observed that the best results were obtained in the FB1% in the model and experiment results. The shear load deformation response as depicted in these figures is such that there is a nearly linear phase in the initial stages up to yielding at approximately 60kN, 50kN for FB1% and CBO% in model and test results, respectively. The strength differences noted in these figures, especially after yielding confirm the strengthening effect of steel fibers and the ability of the proposed model to simulate the same. This strength increase is qualitatively confirmed by the test results from comparative differences as depicted in Fig.16. Further it is apparent from these figures (Figs.16and 17) that the there is no much differences in the deflection results despite of the fiber content.



Fig.15 Failure pattern/shear cracking visualization by DICM



Fig. 16 Model and test load deflection response comparisons



Fig. 17 Model and FE load deflection response comparisons

4.3 Parametric evaluation of the theoretical model

Theoretical parametric analysis is applied in this section to evaluate the prediction ability of the derived model. Influence of the steel fibers and the shear span to the depth ratio on the shear capacity of SFRC beams is checked with a view of evaluating the model response to the variation of these parameters.

In the parametric analysis, evaluation of the prediction ability of the model is carried by applying parameters given in Table 1 related to equation 3a. However to obtain the influence of the steel fibers, the steel fiber contents is applied variably (e.g. 0.5%, 1.0% etc) in equation 3e, and in the case of the shear span to depth ratio (a/d), variation is made by a applying a variable a/d (i.e. β =0.5, 1, 1.5, 2, etc in Eqs 3b, 3c and 3d).

(1) Influence of fiber content variation

As earlier shown, the influence of the steel fibers is to increase post yielding shear strength. This is because the fibers are effective in bridging the cracks and thereby enhancing the stress distribution. This should be reflected by the model when subjected to a variation of fiber content. To evaluate this, the derived model is tested for a fiber content variation of 0.5% 1.0% and 1.5% in which the additional material parameters given in reference ¹⁹ were applied. The results of this analysis are as shown in Fig.18. It is evident in this figure that evolution of the load deflection response is predicted, in which there is increase in the shear strength commensurate with the fiber content. It can be noted that increment is also consistent

throughout the load regime. Prior to the yielding, which is noted to occur at approximately at 50, 60 and 70kN for FB0.5, FB1.0 and FB1.5%, respectively, the initial phase is linear and almost same for all the fiber contents considered. Beyond this range non linear behavior is occurring in which the strength increase is clearly visible depending on the fiber content used. Softening behavior beyond the ultimate capacity is noted to occur.

(2) Influence of shear span to depth ratio (a/d) variation

To illustrate the influence of shear span to depth ratio variation, an increase in the shear span to the depth ratio (up to a/d=3.5) parametric analyses are made while keeping the same fiber content as previously used (i.e.0.5%, 1.0% and 1.5%). In this case, ultimate shear strength is drawn against the shear span to the depth ratio (a/d). As shown in Fig.18, a further reduction in shear strength is noted to occur with the overall response similar to what has been found experimentally to occur in conventional RC beams.

The high shear strength in the initial stages in which the shear span to the depth ratio ranges from 0.5 to 1 is often attributed to the reserve strength due to compression shear failure. However, as the shear span to depth ratio increases, this influence is reduced as the failure mode changes to shear flexure and ultimately flexural mode when a/d becomes large. Despite this observation, it is apparent that increase in strength when steel fibers are present is observed to be effective within the range of a/d = 0.5 and 2.



Fig.18 Response prediction with increase in fiber



4.4 Comparison with JSCE recommended formula

Japanese society of Civil Engineers (JSCE) SFRC concrete column design guidelines¹⁰⁾ recommend fundamental relation for the prediction of the ultimate shear strength of steel fiber reinforced concrete members. In this section, a comparative evaluation of the ultimate strength prediction of the proposed model is evaluated against the JSCE formula.

(1) Proposed ultimate strength formula

The proposed shear strength predictive formula for SFRC beams (eq.3a) is applied here in its ultimate form. It was established from shear strength evolution analysis that the ultimate shear strain ratio γ_y/γ in eq.3a is approximately 0.00833. Thus equation (3a) can be re-written in its ultimate form as,

$$V_{u} = V_{fc}^{u} + V_{c}^{u} + V_{d}^{u}$$
(7a)

$$V_{fc}^{u} = \frac{2dbK_{l}}{3\beta\cos\alpha} \left\{ 2.99 - \left[\frac{\sigma_{cl}}{\sigma_{c}} + \frac{1.99K_{l}}{\sigma_{c}} \right] \left[\frac{3\sigma_{cl}}{2k_{l}} + 2.98 \right] \right\}$$
(7b)

$$V_c^u = \frac{db\sigma_{ct}}{2\beta\cos^2\alpha} \tag{7c}$$

$$V_d^u = \frac{1}{\beta^2} \sqrt{\frac{A_c \rho}{\pi}} \left(k \varepsilon_{fp} l_f^{ef} \, 120 \cos \alpha + 2\pi \tau_b l_a^{ef} \, \tan \alpha \right) \tag{7d}$$

Where V_u is the total ultimate shear capacity, while $V_{fc}{}^{u}$, $V_{u}{}^{c}$ and $V_{d}{}^{u}$ are a contribution from fiber concrete, plain concrete and dowel action, respectively. The designations of the other terms in these equations are same as those given for eq. (3a) in section 3.1(1).

(2) JSCE recommended formulas

The relation given in the JSCE Guideline¹⁰⁾ are reproduced in this section and applied. The JSCE Guidelines for SFRC concrete column design guidelines¹⁰⁾, the ultimate shear capacity is determined from the following relation,

$$V_{yd} = V_{cd} + V_{sd} + V_{ped}$$
(8a)

In which the first term of equation (8a) accounts for the SFRC contribution while the second and the last term accounts for the shear contribution from stirrups and longitudinal reinforcements. For which there is an axial load. For comparisons with fiber beams, only the middle term is omitted and the relation for SFRC beam can be given by the first term of equation (8a), where by,

$$V_{cd} = \beta_d \beta_p \beta_n f_{vcd} b_w d / \gamma_b$$
(8b)

$$V_{\rm ned} = P_{\rm ed} \sin \alpha_{\rm p} / \gamma_{\rm b} \tag{8c}$$

$$f_{vcd} = 0.20(1+\kappa)\sqrt[3]{f'_{cd}}$$
 (N/mm²) (8d)

where the fiber contribution is accounted for $\kappa = l$

$$\beta_d = \sqrt[4]{1/d} \tag{8e}$$

$$\beta_p = \sqrt[3]{100} \rho_w \tag{8f}$$

 $\beta_p = l$, for the case where no axial force as in this case

 ρ_w and *d* are the rebar ratio and effective depth of the beam, respectively.

(3) Analytical and test comparisons for FB1.0%

In the comparative evaluation, structural and material properties for the FB1.0% beam as evaluated in this study are used (see Table 1 and Fig.4). Table 2 shows the comparative shear strength and correlation results among the proposed models (7a), JSCE recommended formula (8a) and test results. From these results, it is established that the model slightly under predicted the values obtained from JSCE eq.(8a), while over predicted the test results by a very slight margin (approximately 1kN) the test results. These are marked by correlation factors of 1.02 for eq. (7a) and 0.82 for JSCE eq.(8a) results, respectively. The difference in the prediction is small in the case of JSCE eq.8a (about 20%). It is also apparent that the experimental shear capacity was found to be equal to 66kN, which is very close to the proposed model predictions (eq.7a) which was 67kN.

Table 2 Correlation between analytical and test results

	Shea	Correlation				
Model Eq.7a (a)		JSCE Eq.8a (b)		Test (c)	a/c	b/c
V_{fc}	53	V_{cd}	54		-	-
V_c	9	V_{sd}	0		-	-
V_d	5	V_{ped}	0		-	-
V_u	67	V_{cd}	54	66	1.02	0.82

5. CONCLUSION

In this study, an experimental and analytical study was conducted, from which comparative shear strength deformation evolutions and ultimate strengths were made. The key findings are summarized as follows,

- (a) Steel fiber reinforced RC beams showed enhanced shear capacity strength over non fibrous and stirrup beam.
- (b) Proposed theoretical model reproduced the experimental results accurately. A comparison of ultimate strength shear capacity prediction between the proposed model and JSCE recommended formula showed fair agreement.
- (c) Numerical simulation by FEM code reproduced fairly accurately the experiments and the model results. Load deflection curves were in fair agreement with the experimental and model result.
- (d) Failure pattern, linear, non linear stress and strain distributions were obtained in which the failure mechanism is illustrated.
- (e) DCIM was found to be effective in shear strain measurement and visualization of the fracture pattern in an RC structure.

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