A method for vibration-based health monitoring using nonlinearity measurement

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This paper proposes a practical method for input-output vibration-based health monitoring. The method using degree of nonlinearity (DON), which overcomes the limitation of linear models, is proposed. Derived from the Hilbert transform of the frequency response function, the DON can measure how much the nonlinearity is present in the vibration response. The results in this paper show that the DON of a structure has a specific relationship with the magnitude of the excitation. When computed from the structure that is still healthy, this relationship can be used as a baseline for the health monitoring. If the structure is not healthy, the new computed DON will deviate from this baseline.

Keywords: *health monitoring, nonlinearity, frequency response function*

1. INTRODUCTION

During the service life of large civil structures, the safety and the serviceability must be ensured. These terms may be regarded as the *health* of the structures. The health may change gradually by a time-dependent cause such as fatigue, or change abruptly by an extreme event such as strong winds and earthquake. Recently, structural health monitoring (SHM) plays an important role in detecting those changes and assessing the health of large civil structures[1].

Among various kinds of SHM, the vibration-based health monitoring seems the most popular. This kind of monitoring may monitor only ambient vibration response (output-only monitoring) or monitor both vibration response and excitation (input-output monitoring)[2]. Until now, linear system identifications have been the backbone of the vibration-based health monitoring. From the monitored vibration response, the system parameters governing the behaviour of the structure were estimated by a linear model, such as linear state-space model[3,4], linear frequency response function[5], linear statistical time-series model [6], or well-known modal model[7]. A change in these estimated parameters are expected to indicate the change in the health or the damage of the structure.

For large civil structures, however, the above approach is still questionable. This is because the actual complicated vibration behaviour is fitted and described by a simplified linear model. In fact, the purpose of the health monitoring is to monitor the structure as it behaves, and there is no reason to assume that the structure is linear except for the sake of simplicity; moreover, it is widely recognized that damage of structures relates somehow to nonlinear behaviours. For these reasons,

taking account of the nonlinearity should be advantageous to the vibration-based health monitoring.

Although many nonlinear system identifications are available[8], they are too complicated for practical purposes. A practical method that takes account of the nonlinearity and meanwhile identifies the change in the health of structures is then desirable. This method might provide useful information for the vibration-based health monitoring.

This paper is an attempt to propose such a method, which measures the nonlinearity directly from the monitored vibration response, and exploits the nonlinearity for detecting the change in the health of structures. Because the monitored data of a real cable-stayed bridge during earthquakes—i.e. vibration response and ground motion—are available, this paper focuses on the input-output monitoring for detecting the abrupt change in the health caused particularly by the earthquakes. Numerically simulated data and real monitored data are used to test the proposed method.

2. PROPOSED METHOD

This method is for the vibration-based health monitoring of a structure during earthquakes for which the excitation (ground motion) and the vibration response of the structure are monitored. The method can be applied for monitoring data during strong winds with similar manner where the excitation will be wind speed records. Since strong-wind condition data is obtained often more than earthquake data, it may be easier to be an index for the judgement.

Assume that the past monitored data were from the healthy condition, whereas the new monitored data is from an unknown—healthy or unhealthy—condition, which is to be detected. The key step is to measure how

much the nonlinearity is present in the vibration response. In this paper, this quantity is derived and defined by *degree of nonlinearity* (DON). Theoretically, the value of DON should depend on several factors: the characteristics of the excitation (type and magnitude) as well as the characteristics of the nonlinearity (type and severity)[9]. Therefore, the relationship between the DON and these relevant factors should provide information about the nonlinear behaviour for different circumstances. In other words, this relationship can be considered as the *signature* of the structure.

If the past monitored data are analyzed, a healthy signature should be available as a baseline for comparison. If a DON from the new monitored data, during a new striking earthquake, is (statistically) higher than the values in the baseline, the structure will be susceptible to the change caused by that earthquake.

It is important to note that the proposed method assumes that the change in the vibration behaviour—from linear to nonlinear, or from nonlinear to highly nonlinear—indicates anomalies in the structure. The phrase "the change in the health" in this paper, therefore, does not mean concretely how the safety or the serviceability has changed or what kind of damage has occurred. The extension of the method to the damage identification is beyond the current scope of this paper. At this moment, the proposed method gives only the information that the anomalies have occurred, which is enough for some practical purposes.

2.1 Theoretical Background

The degree of nonlinearity (DON) is calculated by the Hilbert transform of the frequency response function (FRF). This section presents a brief theoretical background and the derivation of the DON proposed by the authors.

(1) Frequency response function (FRF)

If the time histories of the excitation and the vibration response are measured, whether the structure is linear or nonlinear, the frequency response function (FRF), which is denoted by $H(\omega)$, can be estimated by

$$H(\omega) = \frac{Y(\omega)U^*(\omega)}{U(\omega)U^*(\omega)} \tag{1}$$

where Y is the Fourier transform of the vibration response; U is the Fourier transform of the excitation; * denotes the complex conjugate operator; and ω is the frequency variable.

(2) Hilbert transform of FRF

The Hilbert transform of a FRF is defined by a convolution integral

$$\tilde{H}(\omega) = -\frac{1}{\pi i} \int_{-\infty}^{\infty} \frac{H(\Omega)}{\Omega - \omega} d\Omega \tag{2}$$

For a linear structure, it has been proved that the real part and the imaginary part of the FRF in Eq.(1) are dependent on each other[10]; that is, the real part can be computed from the imaginary part, and vice versa.

Therefore, the FRF measured from the linear structure will be invariant under the Hilbert transform[8-10]:

$$\tilde{H}(\omega) = H(\omega) \tag{3}$$

If Eq.(3) is not valid, the structure is nonlinear; in other words, the FRF is different from its Hilbert transform:

$$\tilde{H}(\omega) \neq H(\omega) \tag{4}$$

The difference between both sides in Eq.(4) are the results of the nonlinearity. The numerical value of the difference indicates how much the nonlinearity is present in the vibration. The computation of the Hilbert transform in Eq.(2) is not tedious since it is a convolution integral, and the required information is only the FRF. Without nonlinear system identifications or prior knowledge of structural parameters, the nonlinearity is conveniently computed by Eq.(2), which is used to derive the DON, the parameter proposed in this paper.

(3) Degree of nonlinearity (DON)

The difference or the nonlinearity in Eq.(4), as mentioned, depends on several factors: the characteristics of the excitation (type and magnitude) as well as the characteristics of the nonlinearity (type and severity). The difficulty is that this difference depends on factors that are unknown or unmeasurable; thus, it will be convenient if those factors could be simplified somehow. For large civil structures, the type of the nonlinearity is mathematically indescribable; however, it should exist in a frequency-dependent function. The type of the excitation, which is from far-field or near-field earthquakes, may be written in a frequency-dependent function as well. For these reasons, the difference in Eq.(4), which itself is also a frequency-dependent function, can be approximated by

$$\tilde{H}(\omega) - H(\omega) \approx \delta(\omega) H(\omega) \tag{5}$$

where an unknown frequency-dependent complex function $\delta(\omega)$, multiplied by the known function $H(\omega)$, is expected to represent all frequency-dependent characteristics of the type of the nonlinearity and excitation. If the modulus of each side in Eq.(5) is integrated over the interesting frequency range $[\omega_1, \omega_2]$, then

$$\int_{\omega_1}^{\omega_2} \left| \tilde{H}(\omega) - H(\omega) \right| d\omega = \int_{\omega_1}^{\omega_2} \left| \delta(\omega) \right| \left| H(\omega) \right| d\omega \tag{6}$$

According to the mean value theorem, there exists a mean value δ of the unknown function $\left| \delta(\omega) \right|$ such that

$$\delta = \frac{\displaystyle\int_{\omega_1}^{\omega_2} \left| \tilde{H}(\omega) - H(\omega) \right| d\omega}{\displaystyle\int_{\omega_1}^{\omega_2} \left| H(\omega) \right| d\omega} \tag{7}$$

The mean value δ is the *degree of nonlinearity* (DON), which measures the nonlinearity in the vibration response and simplifies the other relevant factors by the mean value theorem. Because of this mathematical simplification, the DON becomes dependent on only one factor, the magnitude of the excitation. In other words,

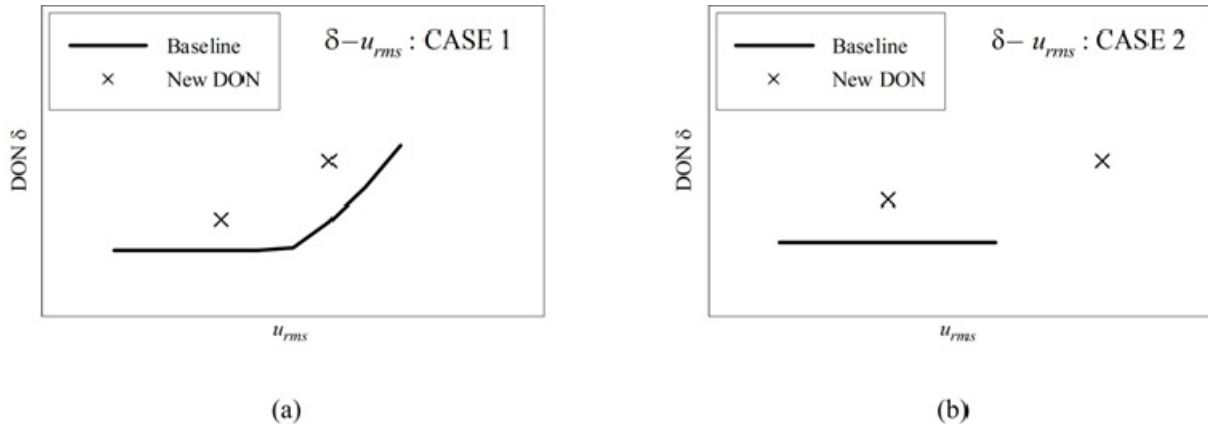$$\delta = F_\delta \left[u_{rms} \right] \tag{8}$$

Fig.1 Possible cases of changes in the health of structures

where u_{rms} is the root mean square average of the magnitude of the excitation (such as ground acceleration or wind speed), and F_δ is the relationship between the DON and the ground acceleration or wind speed. This equation represents the *signature* of the structure.

Note that even if the structure is perfectly linear, the value of the DON may not be exactly zero due to other factors than the nonlinearity such as noise, bias in computation of the FRF, etc. These are difficult to identify practically, but they may not be serious since we need the pattern of the DON with the amplitude of the excitation rather than the exact numerical value of the DON.

2.2 Procedure for Health Monitoring

To better understand the proposed method, the procedure for the vibration-based health monitoring is summarized as follows:

(1) Construct the healthy baseline
   1. From a past monitored data when the structure was still healthy, compute the FRF by Eq.(1).
   2. From the FRF, calculate the Hilbert transform by Eq.(2).
   3. From the Hilbert transform the FRF, extract the DON by Eq.(7).
   4. From the ground motion, calculate the magnitude u_{rms}.
   5. Plot the DON versus u_{rms}.
   6. Repeat step 1 to 5 for all data until the relationship in Eq.(8) is observed. This is the baseline signature for comparison, as shown by solid lines in Fig.1.

(2) Detect the change in the health
   1. From a new monitored data, compute and plot the new DON versus u_{rms}.
   2. If the new DON is higher than the values in the baseline, as shown by cross symbols in Fig.1, the health of structure may be changed by the new striking earthquake.

The test of the method and detailed explanation of Fig.1 are in the next section.

3. TEST OF THE METHOD

The objective of this section is to investigate whether the DON is useful for the health monitoring. That is to show the following statements: 1) the DON can measure the nonlinearity; 2) the DON of a structure has a specific relationship with u_{rms}; 3) the signature can be used to detect the change in the health. Two kinds of data—numerically simulated data from nonlinear systems and monitored data from a real bridge—are used in this test.

3.1 Simulated Data from Nonlinear System

In mathematics, there are many types of nonlinearity that can be simulated, but for large civil structures, the characteristics of nonlinearity may be unknown and then may not be simulated realistically. This difficulty, however, is not a problem since the DON just measures whatever nonlinearity. Therefore, the simulation here is to show whether or not the DON can detect simple nonlinearities, not to simulate the realistic nonlinear behaviour. If the DON can detect these simple nonlinearities, it should do the same for more realistic nonlinearity.

(1) Nonlinear system

Consider a simple nonlinear system described by

$$m\ddot{y} + c\dot{y} + f_s(y) = u(t) \qquad (9)$$

where y is the response; u is the uniform random noise excitation with the rms u_{rms}; the system parameters are $m = 1$, $c = 0.10$, and $k = 25.27$. $f_s(y)$ is a nonlinear term in which two types of nonlinearities are to be simulated: the saturation nonlinearity and the cubic nonlinearity. The saturation nonlinearity is governed by a condition: $f_s(y) = -f_{max} \le ky \le f_{max}$ where the nonlinear parameter $f_{max} = 6.208$ (for low nonlinearity) and $f_{max} = 4.319$ (for high nonlinearity). In this case, the system is linear when the response is small but becomes nonlinear when the response is large enough so that $f_s(y)$ becomes saturated.

On the other hand, the cubic nonlinearity is governed by a condition: $f_s(y) = ky + \alpha y^3$ where the nonlinear parameter $\alpha = 2.527$ (for low nonlinearity) and $\alpha = 12.64$ (for high nonlinearity). In this case, the system is always nonlinear.
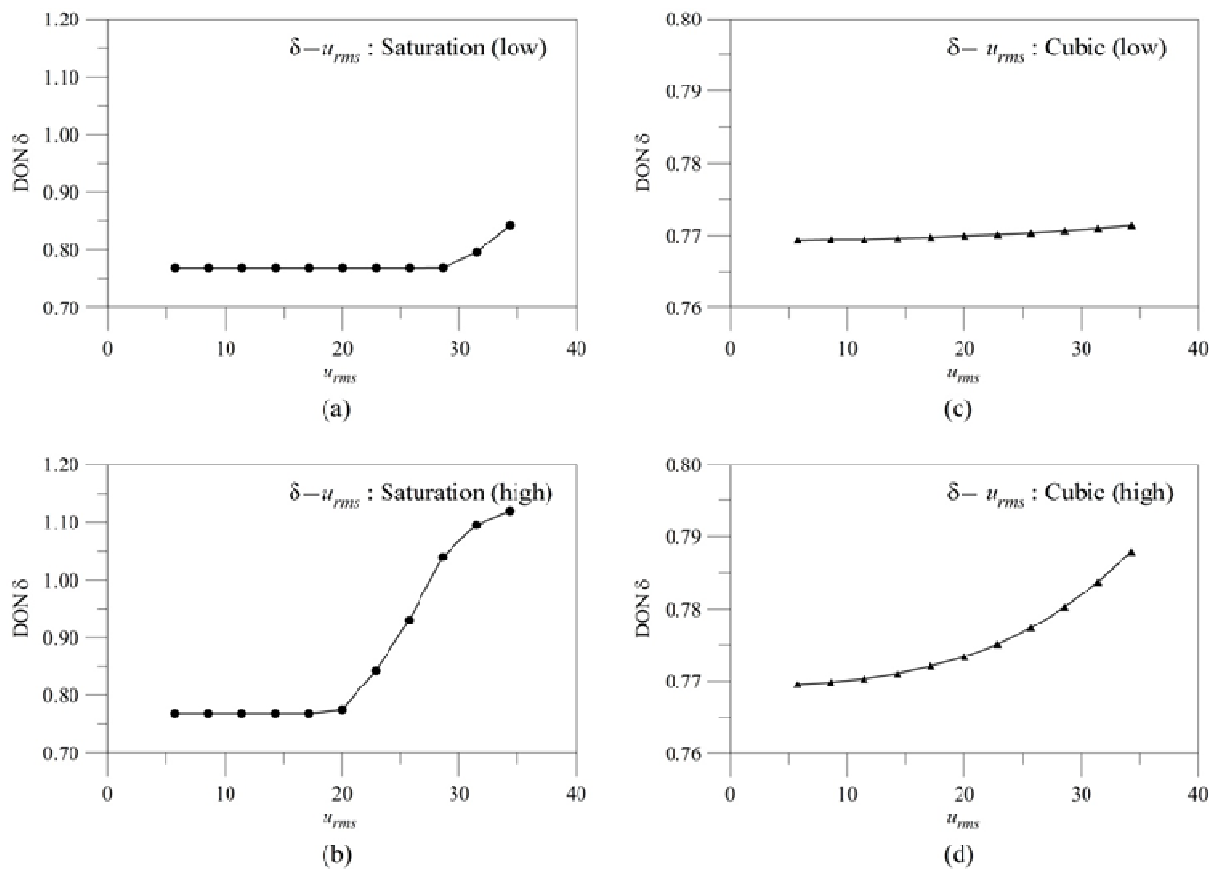
Fig.2 DON versus magnitude of excitation (simulated data)
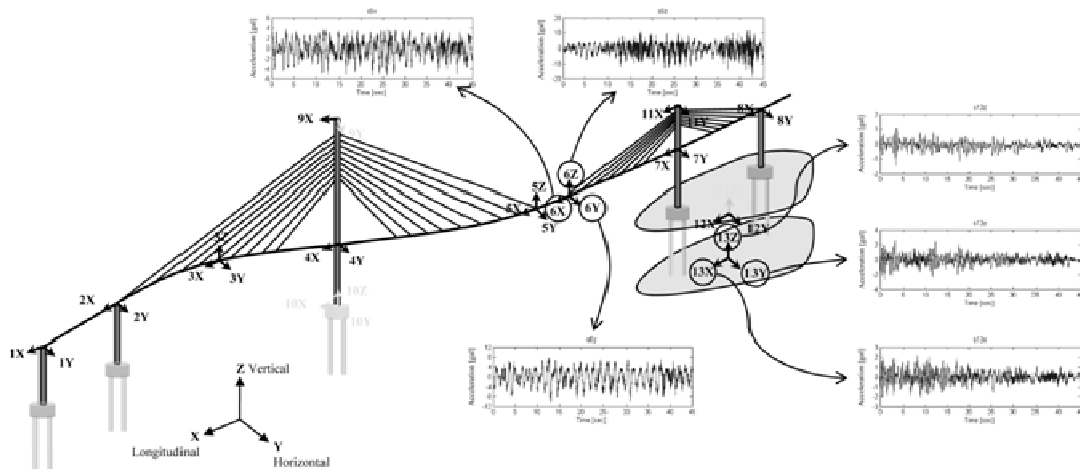

Fig.3 Cable-stayed bridge monitored during earthquakes

The simulation has been performed in both cases by varying the magnitude of the excitation from a small to a large value. The corresponding DON for each magnitude of the excitation are computed and plotted in Fig.2

(2) Results and discussions

Fig.2 consists of two parts; the left side is from the saturation nonlinearity, whereas the right side is from the cubic nonlinearity. The upper part is from the result when the level of the nonlinearity is low, whereas the lower part is from the result when the level of the nonlinearity is high

From the saturation nonlinearity in Fig.2(a), when the magnitude of the excitation is small, the DON is constant over that range of the excitation; this is because the system is still linear. However, when the magnitude of the excitation exceeds a certain level at which the system becomes nonlinear, the DON increases. The same tendency can be observed in Fig.2(b). From the cubic nonlinearity in Fig.2(c)-(d), the DON always increases with the amplitude of the excitation; this is because the system is always nonlinear. To conclude, these results are the evidence that *the DON can measure the nonlinearity.*

Now consider the characteristics of the nonlinearity: the type and the severity. It is obvious that each type of the nonlinearity produces its own curve. In addition, the severity of the nonlinearity has an obvious effect on the rate of increase in the DON. Therefore, for a structure which has certain characteristics of the nonlinearity, *the DON versus u_{rms} is the signature of that structure*.

The discussion so far has been on the simulated data. To better confirm the validity of the DON, monitored data from a cable-stayed bridge are discussed in the next section.

3.2 Monitored Data from a Bridge
(1) Bridge monitored during earthquakes

The data used in this section are from a healthy cable-stayed bridge that was monitored during earthquakes, as conceptually shown in Fig.3. The vibration responses are measured in three main directions: X-longitudinal, Y-transversal, and Z-vertical direction. The monitored data from five earthquakes, abbreviated by EQ1 to EQ5, are used. The average values of the DON from all the sensors are computed and plotted versus the magnitude of the ground motion, as shown in Fig.4. These plots can be considered the baseline for the health of the bridge, constructed from five records.

(2) Results and discussions

From Fig.4, the DONs in all plots are almost constant over the range of the magnitude of ground motion (except for EQ5 in Y-direction). This constant tendency is similar to Fig.2(a)-(b) when the magnitude of the excitation is not large enough to induce highly nonlinear behaviour. Thus, Fig.4 implies that the nonlinearity may not play an important role in the vibration behaviour of this bridge.

Now consider the DON of EQ5 in Y-direction, which deviates clearly from its neighbours. In fact, EQ5 has the largest magnitude among the monitored data, and the measured vibration response in Y-direction was highly contaminated by spikes and noise. It is not easy to identify what was the source of this outlier; however, no matter what it was, the DON could indicate a situation that was unusual. Although the damage scenario of this bridge is required to conclude more about the change in the health, it is not in the current scope of this paper. At this moment, the DON seems useful for the health monitoring, and Fig.4 may be the good baseline for the health monitoring.

3.3 DON in Vibration-based Health Monitoring

From the previous results, advantages of using DON have been confirmed. Let consider in details how the signature or the baseline can detect the change in the health of a structure. Consider Fig.1 again, where two possible cases of changes in the health are illustrated. In CASE 1 the baseline has included the range of behaviour from linear to nonlinear. If the new DON is statistically higher than the values in the baseline, the structure may behave more nonlinearly or the characteristics of the nonlinearity have been changed. In CASE 2, the baseline has included only the linear range. If the new DON is statistically higher than the values in the baseline in the linear range, the structure may behave more nonlinearly. If the magnitude of the excitation is beyond the range that

the structure has ever endured, the higher DON may indicate the higher nonlinearity. In both cases, however, an assumption has to be imposed: the change in the vibration behaviour implies some anomalies in the structure. This is how *the signature can be used to detect the change in the health of the structure*.

When this method is applied to practical situations, judgement criteria of DON should be investigated with wide-range and long-time data. The comparison of DON among different locations will also be useful for that purpose.
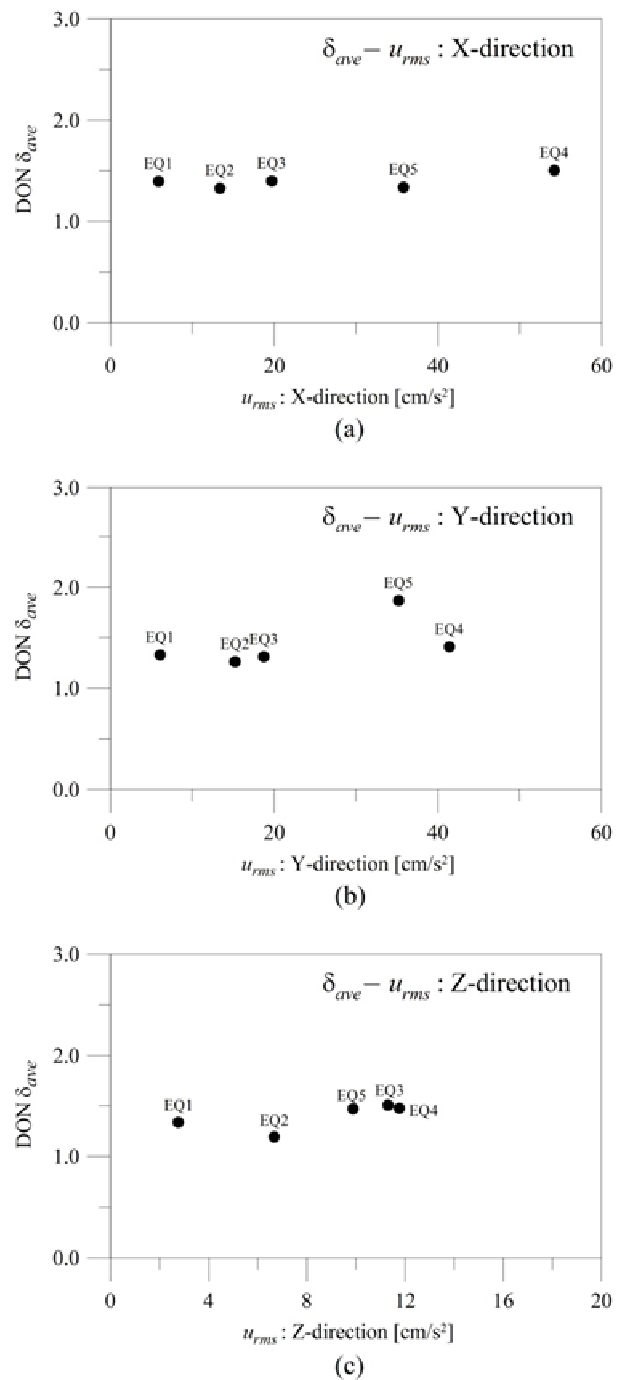

Fig.4 DON versus magnitude of ground motions (monitored data)

4. CONCLUSIONS

This paper proposes and verifies a method for vibration-based health monitoring. The points can be summarized as follows:

- The paper focuses on the input-output vibration-based health monitoring, i.e. the monitoring of the vibration during earthquakes.
- The degree of nonlinearity (DON), which is an idea to overcome the limitation of linear models and the difficulty of nonlinear system identifications, is proposed.
- The DON measures whatever nonlinearities into a single number. The computation of the DON requires only the frequency response function (FRF), thus it is convenient to use in practice.
- From the simulated data, the DON can measure the nonlinearity in the vibration response and can capture the nonlinear behaviour that varies with the magnitude of the excitation.
- For a structure, the DON has a specific relationship with the magnitude of the excitation. This is considered as the signature of the structure, as confirmed by the measured data of a bridge.
- The signature when the structure is still healthy can be the healthy baseline of the structure. The new DON that is statistically higher than the values in the baseline indicates the change in the health of the structure.
- The DON should be accumulated on existing bridges for various conditions. The judgement criteria of DON should be established.

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