Shaking table test of moment-resisting timber joints with pre-tensioned bolts

Ali Awaludin [*], Toshiro HAYASHIKAWA [**], Takuro HIRAI [***], Akio OIKAWA [****], Yoshihisa SASAKI [*****]

[*] Graduate Student, Graduate School of Engineering, Hokkaido University, Kita 13 Nishi 8, Kita-Ku, Sapporo 060-8628
[**] Ph.D., Professor, Lab. of Bridge and Structural Design Eng., Hokkaido University, Kita 13 Nishi 8, Sapporo 060-8628
[***] Ph.D., Professor, Lab. of Timber Eng., Hokkaido University, Kita 9 Nishi 9, Kita-Ku, Sapporo 060-8589
[****] Technician, Lab. of Bridge and Structural Design Eng., Hokkaido University, Kita 13 Nishi 8, Sapporo 060-8628
[*****] Technician, Lab. of Timber Eng., Hokkaido University, Kita 9 Nishi 9, Kita-Ku, Sapporo 060-8589

A relatively good performance of timber joints with pre-tensioned bolts under static-cyclic loadings was previously reported. For a more reliable earthquake-resistance timber structure, however, their performance under seismic forces needs also to be well understood. In this study, seismic performances of pre-tensioned moment-resisting joints are examined through a series of shaking table tests. A part from this study, a single-degree-of-freedom oscillator model is developed to predict their seismic responses in which the joint stiffness is governed by a trilinear-skeleton hysteretic curve. Dynamic equilibrium equation of motion of the model is solved using step-by-step integration method with linear acceleration assumption and constant viscous damping coefficient. The test results show that damping ratio and dynamic stiffness (an equivalent stiffness of moment rotation relationship) of the pre-tensioned joints are much greater than those of the non-prestensioned joints. For small amplitude vibration, average damping ratio is 9.40% and 6.59%, respectively, for the pre-tensioned and non-pretensioned joints. Good agreement is found between the measurement and prediction, though less joint rotation is observed in the measurement after the occurrence of interlayer slip between the joint members. This reduced response is essentially related to additional damping due to clearance around fasteners in pre-drilled holes.

Key Words: moment-resisting joint, pre-tensioned bolt, seismic performance, shaking table test

1. Introduction

For timber joints with dowel-type fasteners, it is common practice to drill bolt holes oversized to facilitate joint assembly and to prevent splitting attributed to moisture related dimensional changes. The presence of clearance around the fastener in a pre-drilled hole however caused very low stiffness at initial loading stage since the joint resistance, which is attained through bearing mechanism, has not yet developed. Efforts to increase the initial stiffness of timber joints consequently give much of benefits such as limiting the deformation of timber structures, which is mostly affected by displacement of their joints rather than by their individual member deformation[1)]. Applying pre-tension force to their fasteners is generally performed as one way to enhance the initial stiffness of timber joints because great frictional resistance between the joint members can be developed.

Pre-stress levels of 1,500 kPa and 3,000 kPa (stress on wood member due to axial pre-tensioning of fasteners) were applied to split-ring connections to improve their structural performances[2)]. Even though those split-ring timber connections had insufficient end distance requirement, significant increase of initial stiffness and ultimate resistance were attained under quasi-static monotonic test. In the authors' first work[3)], great increase of cyclic properties of moment resisting timber joints was obtained when pre-stress level of 1,600 kPa was applied. In contrast to the significant increase of cyclic stiffness and hysteretic damping, only slight increase of ultimate moment resistance was observed because the pre-tensioned bolts were bent with small bending angle at final failure. Secondary frictional force or embedding resistance of steel plate on wood member due to bending deformation of laterally loaded bolts therefore was considerably less in the case of pre-tensioned joints.

Timber joints that assembled with slender dowel-type fasteners had shown relatively good performances under quasi-static monotonic and cyclic tests since the failure modes of these connections are closely related to ductile behavior of their steel fastenings[4-5]. The dynamic behavior of these joints under seismic forces or earthquake loadings, however, has not been thoroughly understood yet. Only limited experimental result from few researchers is available[6-8]. Moreover, available commercial structural analysis packages are not efficient in modeling timber constructions due to some special properties of timber structures (e.g. material properties, and lateral load-resisting system). These conditions are being major obstacle for a more reliable earthquake-resistance timber structure.

This present study is primarily aimed to investigate the dynamic behavior of moment-resisting timber joints that assembled with pre-tensioned bolts by conducting a series of shaking table tests. A single-degree-of-freedom oscillator model is also developed to analytically evaluate the dynamic responses of the joints in which the dynamic equilibrium equation of motion is solved using step-by-step integration method with linear acceleration assumption during time increment. In the analysis, the joint stiffness is defined by a hysteretic model that describes compound behavior of interlayer slip between joint members, bearing action, apparent yielding, and pinching mechanism or less resistance at small range of joint rotation. This hysteretic model was obtained from the test results of similar moment resisting joints under static and cyclic loadings.

2. Shaking Table Test

The test structure illustrated in Photo 1 is secured by a stiff steel base plate that is anchored to the top plate of a shaking table (2.5m x 2.5m). This test structure is a glued laminated column of spruce-pine-fir species that has a concentrated weight of 700 N at the top, and is connected at the bottom to the base plate through a steel gusset plate of 4 mm thickness and 12 mm bolts. The geometry of the moment-resisting joint (at the bottom of the test column) is shown in Fig. 1 where the distance from the concentrated weight to the centroid of the joint is 2,100 mm. An axial bolt pre-tension of 20 kN is applied to each fastener using calibrated torque wrench device. By considering the contact area between the steel plate and glued laminated member, the pre-stress level caused by this pre-tension force is approximately equal to 1,600 kPa or about 36% to 46% of the compressive strength perpendicular to the grain of spruce species[9]. Pre-drilled hole of 1 mm larger than the bolt diameter is made in the glued laminated member and steel gusset plates to accommodate the actual practice of joint assembly.

Displacements at the top and at the bottom of the test column are measured using two high-speed cameras incorporated with MATLAB binary image processing[10]. HAS-200R high-speed camera is used and it can take two hundred pictures within one second. The method of displacement measurement using high-speed camera was previously discussed in detail by the authors[8]. In this

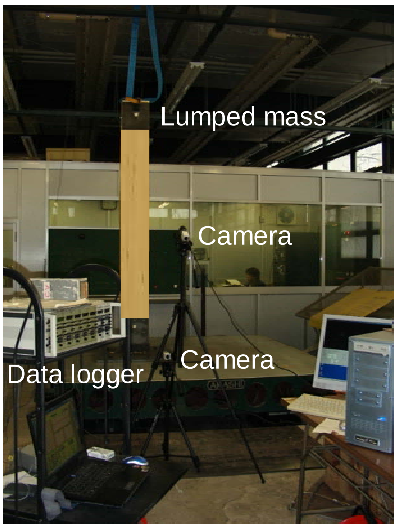

Photo.1 Shaking table test

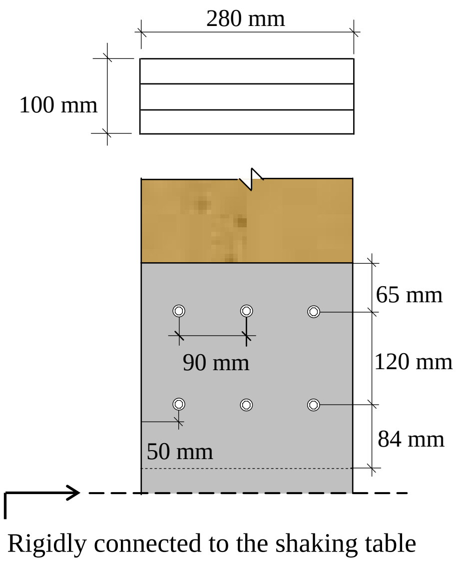

Fig.1 Geometry of moment-resisting joint

shaking table test, only the first mode shape of column vibration is considered and displacement of the column is attained by subtracting the displacement recorded at the top from the displacement measured at the bottom. In addition, three accelerometers that can record the acceleration up to 20*g* are deployed in this test program to measure the acceleration of the shaking table, the middle and the top of the test column. The signs of these three accelerations are monitored continuously and used to analyze the current vibration mode of the test column. A ten-second ramped input acceleration is chosen as the input excitation because this kind of loading protocol is widely used in static-cyclic tests[4-5, 11]. This input excitation is performed at a frequency close to the natural frequency obtained from damping test and the magnitude of peak ground acceleration of this input excitation, which increases linearly, is arbitrarily selected to produce both linear and nonlinear responses.

Damping, which is responsible for eventual decay of free-vibration motion, is a result from various components under complex mechanisms. For a mathematically convenient approach, viscous damping model is adopted in the dynamic equilibrium equation of motion. The behavior of the oscillator model will be sufficiently close to that of the actual system for most purposes when the viscous damping property is determined at the natural frequency of the actual system[12]. The viscous damping property is experimentally evaluated using Half-Power method[13] under two kinds of excitation function since the natural frequency of the test structure is not known in advance. First, a sine swept function (sinusoidal function with forcing frequency linearly increases from 0 Hz to 20 Hz) is generated by the shaking table within ten seconds. Second, a sinusoidal function with constant-amplitude is generated several times at different frequency values; ranging from 3 Hz to 15 Hz with an increment of 0.5 Hz. Through this test, the fundamental natural frequency can also be examined and it provides important information about pre-stressing effect on the joint stiffness. Ten replicates are prepared in this damping measurement, and only five of them are tested under the ten-second ramped excitation.

3. Dynamic Equilibrium Equation of Motion

Dynamic equilibrium equation of motion based on a single-degree-of-freedom oscillator model shown in Fig. 2 is derived by equating to zero the sum of inertial moment force (M_I), the damping moment force (M_D), the spring moment force (M_S), and the external moment force (M). Lateral displacement of the model at the top (v) is assumed to be only affected by the joint rotation. Therefore, the equilibrium equation at time t_i is expressed as

$$mL^2\ddot{\theta}_i + c\dot{\theta}_i + (k_i - mgL)\theta_i = M(t_i) \qquad (1)$$

and at short time latter, $t_{i+1} = t_i + \Delta t$

$$mL^2\ddot{\theta}_{i+1} + c\dot{\theta}_{i+1} + (k_{i+1} - mgL)\theta_{i+1} = M(t_{i+1}) \qquad (2)$$

Subtracting Eq. 1 from Eq. 2 results in the different equation of motion in terms of increments, namely

$$mL^2\Delta\ddot{\theta}_i + c\Delta\dot{\theta}_i + (k_i - mgL)\Delta\theta_i = \Delta M_i \qquad (3)$$

In which,

$$\Delta\ddot{\theta}_i = \ddot{\theta}(t_i + \Delta t) - \ddot{\theta}(t_i) \;\; ; \;\; \Delta\dot{\theta}_i = \dot{\theta}(t_i + \Delta t) - \dot{\theta}(t_i) \;\; ; \;\; \text{and}$$

$$\Delta\theta_i = \theta(t_i + \Delta t) - \theta(t_i) \qquad (4a\text{-}c)$$

When the acceleration is assumed to be expressed by a linear function of time during the time interval Δt, the equation of motion in term of unknown incremental joint rotation ($\Delta\theta_i$) can finally be written as

$$\bar{k}_i\,\Delta\theta_i = \Delta\bar{M}_i \qquad (5)$$

in which $\bar{k}_i$ is the effective spring constant, given by

$$\bar{k}_i = (k_i - mgL) + \frac{6mL^2}{\Delta t^2} + \frac{3c}{\Delta t} \qquad (6)$$

and $\Delta\bar{M}_i$ is the effective incremental moment force, expressed by

$$\Delta\bar{M}_i = \Delta M_i + mL^2\left\{\frac{6}{\Delta t}\dot{\theta}_i + 3\ddot{\theta}_i\right\} + c\left\{3\dot{\theta}_i + \frac{\Delta t}{2}\ddot{\theta}_i\right\} \qquad (7)$$

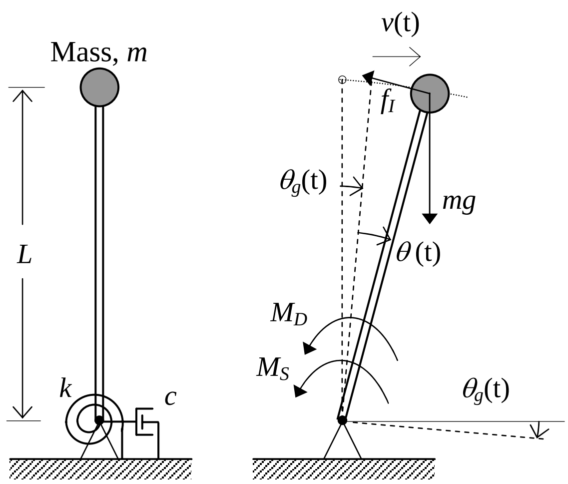

Fig.2 Mechanical model and its free-body diagram due to ground-induced motion

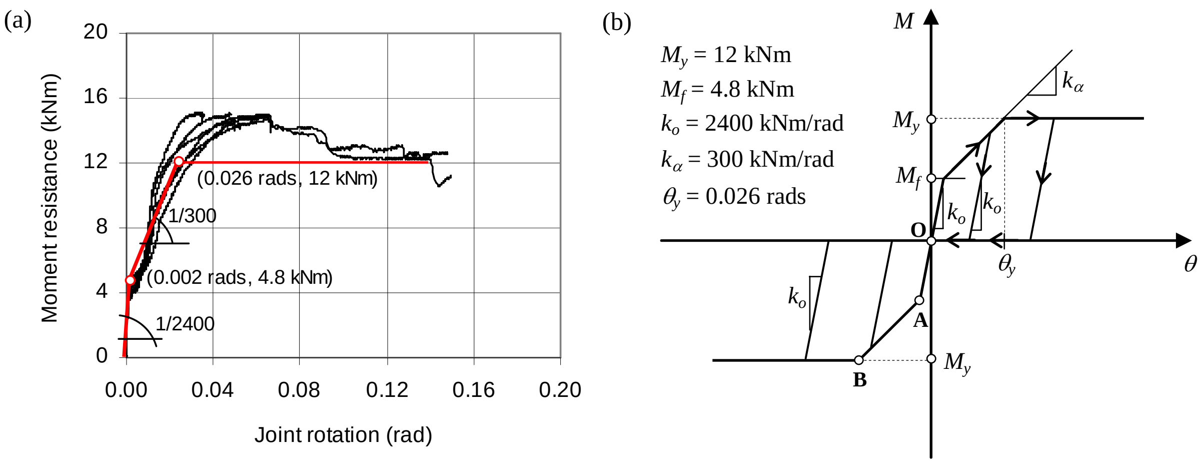

Fig.3 Moment rotation curve of pre-tensioned joint: (a) Experimental curve (ref. 3); and (b) Hysteretic model

The Eq. 5 is solved using step-by-step integration method where m is the mass of the concentrated weight plus a half of the column mass, c is viscous damping coefficient, and L shows the length of the test column. Viscous damping coefficient (c) is given by

$$c = 2\zeta\omega_n mL^2 \quad (8)$$

where ζ is equivalent viscous damping ratio and ω_n is natural circular frequency of the oscillator model. In the analysis, nonlinear responses of the test column are assumed to be determined only by rotational stiffness coefficient (k) and viscous damping coefficient (c), which is another element affecting the responses and is kept constant through the analysis.

To obtain an accurate prediction of dynamic response caused by earthquake loadings, any hysteretic model of timber joints with dowel-type fastener must incorporate the rational load-slip characteristics, which are observed experimentally or evaluated analytically[14-16]. These characteristics are stiffness reduction due to reversely loaded multiple times, and pinching or narrowing the hysteretic curve at small range of joint rotation because of irrecoverable embedment of wood beneath fastenings. For the pre-tensioned joints evaluated in this study, their experimental moment rotation curves under quasi-static monotonic and cyclic loadings were previously examined[3].

Fig.3 shows the moment rotation curve of the pre-tensioned joints obtained from experiment and the hysteretic model implemented in this study. This hysteretic model is composed by a tri-linear skeleton that describes a compound behavior of interlayer slip, bearing action, and apparent yielding. When the applied moment exceeds the upper limit of interlayer slip, unloading follows the line whose stiffness is equal to the initial stiffness. As a result, plastic rotation will occur and this corresponds to the plastic embedment of wood under fastenings. Pinching or low stiffness at small range of joint rotation due to plastic embedment of wood beneath fastenings is modeled with a line of zero stiffness. Comparing with the hysteretic model that was used for the non-pretensioned joints[8)], the differences are only in the magnitude of resistance at upper limit of interlayer slip and tangential slope of the first line, which are essentially affected by fastener pre-tensioning. In the non-pretensioned joints, small resistance at interlayer slip might exist due to nature randomness of fastener clearance where some fasteners directly embed into the wood member. The apparent yielding resistance and the stiffness of the bearing action of the hysteretic model are found to be the same for the timber joints with and without pre-tension force.

4. Results and Discussions

4.1 Damping ratio and natural frequency

Shaking table previously generates the sine swept input excitation at non-detrimental peak ground acceleration (0.5g) for two different joint conditions: non-pretensioned and pre-tensioned joints. The observed joint rotation due to this swept excitation as presented in Fig. 4 can be used to evaluate the natural frequency of the joints in which the joint rotation will be a maximum at resonant frequency. From Fig. 4, it was found that the average natural frequency of the pre-tensioned joints and the non-pretensioned joints was 9.69 Hz and 6.15 Hz, respectively. Higher natural

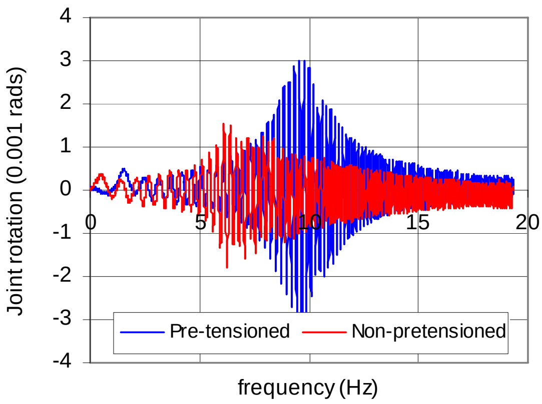

Fig.4 Time-history joint rotation due to a sine swept excitation

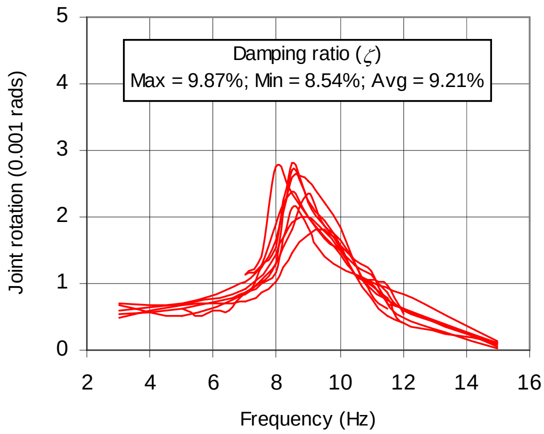

Fig.5 Frequency-response curve of the pre-tensioned joints

frequency of the pre-tensioned joints in comparison with the natural frequency of the non-pretensioned joints definitely reflects the increase of rotational stiffness due to pre-tension application. For a single-of-degree-freedom oscillator model shown in Fig. 2, its natural frequency is given by ref. 8 and is almost linear with the square root of its rotational stiffness. As a result, increase of natural frequency from 6.15 Hz to 9.69 Hz informs that the rotational stiffness of the pre-tensioned joints of this study would be around 2.48 times the stiffness of the non-pretensioned joints.

An attempt to determine the equivalent viscous damping ratio of the joint is carried out using Half-Power method based on the time history joint rotation given in Fig. 4. Two forcing frequencies on either side of the resonant frequency at which the joint rotation is about 0.707 times the joint rotation at the resonant frequency are seek. When the test column is assumed to be lightly damped, the damping ratio can finally be obtained by dividing the difference of these two forcing frequencies with the sum of them. This method gives damping ratio of 9.40% and 6.59%, respectively, for the pre-tensioned and non-pretensioned joints. The friction between joint members and increased friction between wood fibers, both of which are associated with transverse compression of the joint due to pre-stressing application, contribute to the increase of vibration energy absorption or damping ratio. This damping ratio increase due to bolt pre-tensioning has significant influence on dynamic performances of timber joints or overall timber structures; for instance, the response-amplitude caused by seismic forces would diminish rapidly.

Besides using swept function, the damping ratio of the pre-tensioned joint is also determined by using several sinusoidal excitation functions that were applied at different frequency magnitudes. For each forcing frequency value, the maximum joint rotation at steady state response is plotted as shown in Fig. 5. Although all the specimens are shaken at the same amplitude (0.25*g*), the resonant frequency of each specimen is not so close to each other but it scatters around 8 to 10 Hz. This might be caused by the non-uniformity of wood properties or pre-stressing level of each fastener. The Half-Power method yields an average damping ratio of 9.21% (min= 8.54%; max= 9.87%), though smaller interval of forcing frequency between 8 to 10 Hz is required for a better damping estimation. Average damping ratio of the non-prestressed joint was previously reported by ref. 8 and it was about 7.33%. These average damping ratios are almost the same as those that obtained by using the sine swept function. Since the magnitude of peak ground acceleration of the sinusoidal function used in this damping measurement is relatively small, the obtained damping mechanism mostly arises from friction between joint members and/or internal friction between molecules in the wood member[17-18]. The damping ratio attained in this study for the non-pretensioned joints is possibly on the high side because a damping ratio value of 5% is commonly assumed for small amplitude dynamic vibrations or elastic zone.

4.2 Seismic responses due to ground-induced motion

In the dynamic analysis, time increment of 0.001s is used to accurately capture the stiffness changes expressed in the hysteretic model and this time increment is also far less than one tenth of natural period of the system (0.1/9.69 = 0.01s). Time-histories of seismic response due to the ten-second ramped excitation are presented in Figs. 6 and 7. Even though the acceleration of input excitation is linearly increasing as shown in Figs. 6, the measured acceleration at

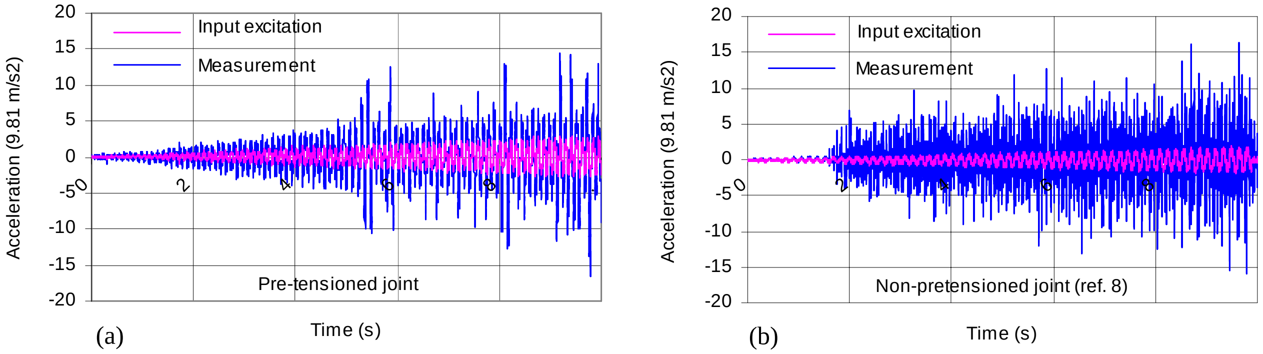

Fig.6 Time-histories acceleration at the top of the test columns: (a) Pre-tensioned joint; and (b) Non-pretensioned joint (ref. 8)

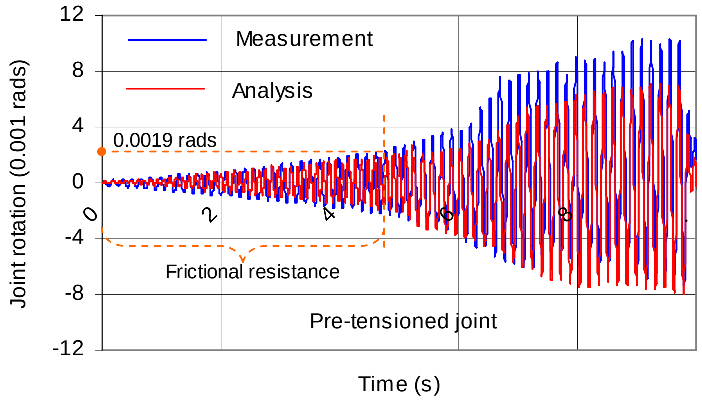

Fig.7 Time-history joint rotation of the pre-tensioned joint

the top of the test column shows a unique pattern. A few seconds after the initial excitation, the acceleration of the test column is not increasing linearly. But it exhibits sudden increase irregularly which is caused by stiffness degradation of the joint after part of wood beneath the steel fastenings are crushed. This sudden increase of acceleration is more substantial for the non-pretensioned joints. This can be expected since the degraded stiffness beyond the frictional resistance of the non-pretensioned joints is lower than that of the pre-tensioned joints.

The pre-stressing application keeps the pre-tensioned joint to response linearly up to joint rotation of 0.0019 radians as shown in Fig. 7. This upper limit of linear response is about same as the joint rotation that corresponds to friction resistance of the hysteretic model shown in Fig. 3a (0.002 radians). In this range of joint rotation, interlayer slip (slip between wood member and steel side plate) was resisted by the frictional force, which is developed due to bolt pre-tensioning, so that the joint members become a unity and respond consistently to the ground excitation. Good agreement was found between the measurement and analysis within this range of joint rotation indicates the appropriateness of assumptions proposed in the hysteretic model (see Fig. 7). However, the measured joint rotation shown in Fig. 7 shows unsymmetrical time-history of joint rotation right after the applied seismic force exceeds the frictional resistance. This is a typical response of timber connections with multiple fasteners and is primarily caused by unequal lead-hole clearance among their fasteners.

When the applied moment exceeds the upper limit of interlayer slip, the joint would experience pinching due to the presence of clearance around fastener. As the plastic embedment of wood under the steel fastenings progresses, the pinching region or range of joint rotation with relatively low stiffness increases. After reaching the upper limit of interlayer slip, however, the joint rotation does not grow rapidly till the end of ground excitation as shown in Fig. 7. Additional damping is potentially contributed during this pinching mechanism because less joint rotation than that of the prediction was measured. The clearance around fastener, which is caused by the difference diameter of pre-drilled hole and fastener or in-recoverable embedment of wood member beneath the steel fastenings, slightly reduced or damped out the ground excitation given by the shaking table. Dynamic responses of moment-resisting timber joints assembled with non-pretensioned fasteners were previously

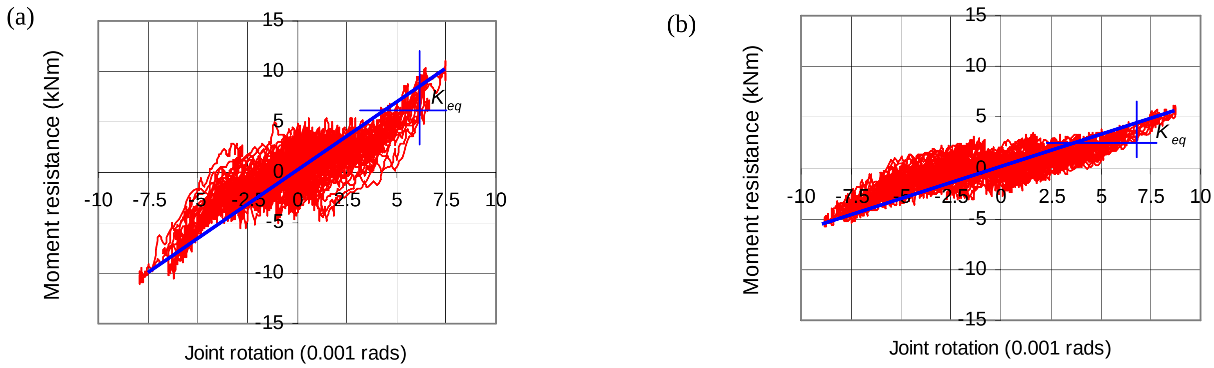

Fig.8 Predicted time-history moment rotation: (a) Pre-tensioned joint; and (b) Non-pretensioned joint (ref. 8)

examined both experimentally and analytically by the authors[8]. It was pointed out that the presence of clearance around the fastener provided additional damping, though it greatly reduced the joint stiffness. This additional damping mechanism therefore is also observed in the pre-tensioned joints as previously described.

Relationship between moment and rotation as shown in Fig. 8 indicates that less joint rotation is found in the pre-tensioned joints than that in the non-pretensioned joints for the same magnitude of moment resistance. If this moment rotation relationship is linearly represented by a dynamic stiffness or equivalent rotational stiffness (K_{eq}) as shown in Fig. 8, the dynamic response of two different joint conditions could be roughly compared to each other. From the experiment, this equivalent rotational stiffness of the non-pretensioned and pre-tensioned joints is around 627 kNm/rad and 1,333 kNm/rad, respectively. This result shows that the pre-tensioned joints of this study are about two times stiffer and would perform better than the non-pretensioned joints during earthquake or seismic forces. Pre-stressing application therefore improves the structural performances of timber joints not only their static-cyclic responses, but also their seismic responses. The magnitude of dynamic stiffness obtained from this study was about the same as the average value of frictional stiffness and bearing stiffness of the hysteretic model shown in Fig. 3a. And this dynamic stiffness is much stiffer than static stiffness of a line that is connecting the initial/zero rotation point to the yield rotation point in Fig. 3a.

Increase of both static-cyclic stiffness and dynamic stiffness of the joint greatly limits the total deformation of timber structures since large portion of their deformation is due to displacement of their connections. The enhancement of joint stiffness is also useful especially in serviceability limit state design where displacement or deformation is being the most fundamental restriction. As the joint stiffness is increased, moreover, vibration characteristics of the whole structure would improve accordingly. Since the pre-stress may dissipate due to creep behavior of wood member, a regular re-stressing program is therefore required to be carried out so that a certain pre-stress level will remain throughout the service life of the structure.

4.3 Stress-relaxation problem

For design values, the tangential slope or rotational stiffness and magnitude of moment resistance at interlayer slip of hysteretic model should also consider the rheology mechanism of wood properties. As time-dependent material wood shows relaxation when it is subjected to a constant deformation. Therefore, the bearing stress of wood member beneath the steel plate due to fastener pre-tensioning will relax or diminish. Loss of pre-stress as a consequence of bearing stress relaxation will occur after the initial pre-stressing and the amount of this stress loss is greatly influenced by moisture content change and the initial pre-stress level[19-20)]. Stress relaxation measurement for one year after the initial pre-tensioning is under way in which the specimen is exposed to temperature and relative humidity changes of in-door environment condition. The complete result of this stress relaxation measurement, which would be useful for establishment of re-stressing program, will appear in a future paper.

5. Conclusions

Attempts to investigate the seismic performances of moment-resisting timber joints with pre-tensioned bolts are presented. The joints are placed on shaking table and excited with ten-second ramped excitation at a forcing frequency close to their natural frequencies. The seismic responses are evaluated based on a single-degree-of-freedom oscillator model that has constant viscous damping coefficient. The relation between stiffness and rotation of the joints was modeled by a tri-linear skeleton, which characterizes a compound action of interlayer slip between joint members, bearing action, and apparent yielding. The effect of axial pre-tension applied to the bolt is considered in the hysteretic modeling, which is indicated by very high initial stiffness and moment resistance at upper limit of the interlayer slip.

Based on the presented results, several important findings can be summarized as follows: 1) The enhancement of frictional resistance between joint members because of pre-stressing application contributes to the increase of natural frequency and vibration energy absorption or damping ratio; 2) By applying a pre-stress level of 1,600 kPa, the natural frequency of the joint increases from 6.15 Hz to 9.69 Hz, while the damping ratio changes from 6.59% to 9.40%; 3) Pre-stressing increases the dynamic stiffness of the joint so that the pre-tensioned joints would be stiffer and perform better than the non-pretensioned joints under seismic loadings; And 4) the presence of clearance around the fastener due to initial clearance or in-recoverable embedment of wood beneath steel fastenings provides additional damping, though it reduces the joint stiffness.

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