Flow-force relationship for two staggered circular cylinders with low angle of incidence

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The flow around two cylinders shows more complex phenomena than that for a single cylinder. Depending on the arrangement of the cylinders, the responses due to the flow behavior present various aspects. In this study, numerical simulations are performed to investigate the flow around two circular cylinders of equal diameter in staggered arrangements at a subcritical Reynolds number. The center-to-center distance ratio of the cylinders is set to 2 and the angle of incidence is varied as 5, 10 and 15 degrees. The incidence-angle dependencies on the flow pattern, the aerodynamic forces and the Strouhal number are investigated. In particular, a bistable flow pattern is obtained as reported in several experimental studies and its characteristics on the flow field and the forces are discussed.

Key Words: bistable flow, circular cylinder, LES, staggered arrangement

1. Introduction

Industrial and engineering constructions nowadays make use of various shape and combinations of shapes, among which circular cylinders in staggered arrangement can be named. The flow around multiple cylinders presents higher complexity phenomena, than for a single cylinder case. In general, for two circular cylinders, the shear layer separated from the upstream cylinder reattaches onto the downstream cylinder, or interferes with the stream formed around this (Fig.1). The shear layer interaction also causes a vibration of the cylinders and eventual structural damages; detailed examinations and understandings of the phenomena are then important.

Numerous researches using experiments and computational investigations for two circular cylinders in various arrangements have been made. For example, Sakamoto *et al.*¹⁾, Sumner *et al.*^{2), 3)}, Kiya *et al.*⁴⁾, Gu and Sun.⁵⁾, Zdravkovich^{6), 7)} and Mittal *et al.*⁸⁾ have revealed considerable complexity depending on the center-to-center distance ratio, L/D (*L*: center-to-center distance between the cylinders, *D*: cylinder section diameter), and the incidence angle, α , for the cylinders in staggered arrangement. They have classified the flow patterns or have investigated the aerodynamic force coefficients and the Strouhal number. Zdravkovich^{7), 9)} explored to classify the flow around two staggered circular cylinder and mentioned three main interference regions : "Proximity Interference", "Proximity and Wake Interference" and "Wake Interference". As a particular case of the last two, "Wake Displacement Regime" and "Gap

Flow Regime" were also mentioned. Additionally, Gu and Sun⁵⁾ showed that at angles of more than 20° the shear layers from the upstream cylinder do not interact with those of the downstream cylinder. Based on water channel experiments for flow visualization, Sumner et al.²⁾ identified no less than nine flow patterns for various center-to-center distances and the incident angles. While these previous achievements were obtained mainly with experiments, the numerical investigation started to be applied for the staggered circular cylinders in recent years. For example, Akbari and Price¹⁰⁾ numerically investigated the flow behavior, such as vortex formation, for two cylinders in staggered arrangements (L/D = 1.1 - 3.5, $\alpha = 30^{\circ} - 70^{\circ}$) and at a low Reynolds number, Re = 800. Five flow patterns were identified, and those are called "Base-Bleed", "Shear-Layer Reattachment", "Vortex Pairing and Enveloping", "Vortex Impingement" and "Complete Vortex Shedding".

On the other hand, Fig.2 shows a schematic of the time history of the lift acting on the downstream cylinder for L/D=2 and $\alpha = 10^{\circ}$, which was compiled from the experimental result by Sakamoto *et al.*¹⁾ Illustrated in this figure is a shift of the mean lift of the downstream cylinder, corresponding to a bistable flow (a successive alternation of two flow patterns). Based on the experimental results, they intensively discussed on the characteristics of the bistable flow observed around $\alpha = 10^{\circ}$ and at L/D = 2. The flow, when the lift force of the downstream cylinder had a lower mean value, was called "Mode 1" and that when the lift shifted to a higher mean value was "Mode 2". Only Mode 1 appeared for the incident angles smaller than 10°, and

Mode 2 singularly appeared for the angles larger than 10°.

In present study, we conduct numerical simulations to investigate the nature of the bistable flow. As was shown in Fig.2, the bistable flow is a nonstationary phenomenon. The numerical investigation is advantageous for assessing such a nonstationary flow field, because it allows us to observe the flow behaviors and the physical properties simultaneously. Following the above-mentioned geometry in the experiment by Sakamoto *et al.*¹⁾, the parameters on the position of two cylinders are set as L/D = 2 and $\alpha = 5^{\circ}$, 10° and 15°. The Reynolds number is set to 22,000, considering the consistency with the author's previous study¹¹⁾ on the numerical investigation for the flow around two circular cylinders in tandem arrangements.



Fig. 1 Geometry configuration for two staggered circular cylinders



Fig. 2 Schematic of time history of lift acting on downstream cylinders at L/D = 2 and $\alpha = 10^{\circ}$ (compiled from the experimental results by Sakamoto *et al.*¹⁾)

2. Computational Method

2.1 Algorithm

The large eddy simulation (LES) with the Smagorinsky subgrid-scale model was employed. The three dimensional incompressible Navier-Stokes equation and the equation of continuity in non-dimensional form are

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + 2 \frac{\partial}{\partial x_i} \left(\frac{1}{\text{Re}} + v_{sgs}\right) D_{ij}$$
(1)

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2}$$

where u_i is the velocity component of grid-scale, and *P* is the sum of the grid-scale pressure and the residual stress. $D_{i,j}$ in Eq. (1) is the strain-rate tensor on the grid-scale velocity components:

$$D_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$
(3)

The subgrid-scale eddy-viscosity, v_{sgs} , in Eq. (1) is expressed as

$$v_{sgs} = (C_s \bigtriangleup)^2 \sqrt{2D_{ij}D_{ij}} \tag{4}$$

In Eq. (4), Δ is the filter width and was given as the cubic-root of grid volume; the Smagorinsky constant, C_s , was set to 0.1 in this study. Additionally, near the cylinder surface, the van Driest function was considered as:

$$f_s = 1 - \exp \frac{-y^+}{A^+} \left(y^+ = \frac{y u_\tau}{v} , A^+ = 25 \right)$$
 (5)

which was multiplied to C_s in Eq. (4).

Equations (1) and (2) were transformed into a computational coordinates system of ξ^k (k = 1, 2, 3), and the contravariant component of the velocity

$$U^{k} = \frac{\partial \xi^{k}}{\partial x_{i}} u_{i} \tag{6}$$

was applied to the advection term in Eq. (1) and to Eq. (2). These equations yield to

$$\frac{\partial u_i}{\partial t} + \frac{1}{J} (JU^k) \frac{\partial u_i}{\partial \xi^k} = -\frac{\partial \xi^k}{\partial x_i} \frac{\partial P}{\partial \xi^k} + \frac{1}{J} \left(\frac{1}{\operatorname{Re}} + v_{sgs}\right) \frac{\partial}{\partial \xi^k} \left(\gamma^{kl} \frac{\partial u_i}{\partial \xi^l}\right)$$
(7)

$$\frac{1}{J}\frac{\partial(JU^{k})}{\partial\xi^{k}} = 0 \tag{8}$$

where J is the Jacobian and

$$\gamma^{kl} = J \frac{\partial \xi^k}{\partial x_m} \frac{\partial \xi^l}{\partial x_m} \tag{9}$$

Equations (7) and (8) were discretized with the finite difference method (FDM) in the collocated grid system (Rhie and Chow¹²⁾, Kajishima *et al.*¹³⁾), and were solved by the simplified maker and cell (SMAC) method. Following expressions are used for FDM discretization hereafter:

$$\delta_i f = -f_{i-\frac{1}{2}, j,k} + f_{i+\frac{1}{2}, j,k}$$
(10)

$$\delta_i' f = \frac{-f_{i-1,j,k} + f_{i+1,j,k}}{2} \tag{11}$$

$$\overline{f}^{i} = \frac{f_{i-\frac{1}{2},j,k} + f_{i+\frac{1}{2},j,k}}{2}$$
(12)

The second - order Adams - Bashforth method and the Crank -Nicolson method were applied to the advection term and the diffusion term of the incompressible Navier-Stokes equation, respectively. The predicted velocity u_i^P in the SMAC method is obtained by solving

$$u_{i}^{p} - \frac{\angle t}{2J} \left(\frac{1}{\operatorname{Re}} + v_{sgs} \right) \delta_{k} \left(\overline{\gamma^{k}}^{k} \delta_{i} \overline{u_{i}^{p}}^{k} \right) = u_{i}^{n} + \frac{\angle t}{2J} \left(\frac{1}{\operatorname{Re}} + v_{sgs} \right) \delta_{k} \left(\overline{\gamma^{k}}^{k} \delta_{i} \overline{u_{i}^{n}} \right) \\ - \angle t \frac{\partial \xi^{k}}{\partial x_{i}} \overline{\delta_{k}} \overline{P^{n}}^{k} + \angle t \frac{3A_{i}^{n} - A_{i}^{n-1}}{2}$$

$$(13)$$

where Δt is the non-dimensional time step and was set to 0.002, and where A_i denotes the advection term on which third-order upstream scheme was employed :

$$A_{i} = \frac{1}{J} (JU^{k}) \frac{\partial u_{i}}{\partial \xi^{k}} = \frac{1}{J} \Big[B_{i} + \alpha (\varDelta \xi^{k})^{3} \Big| JU^{k} \Big| \delta_{k}^{\prime 4} u_{i} \Big]$$
(14)

In Eq. (14), B_i is the fourth-order central FDM discretization on the advection term, and in accordance with the suggestion of Morinishi *et al.*¹⁴⁾, the following formulation was used in this study.

$$B_{i} = \frac{9}{8} \overline{(JU)^{k} \delta_{1k} u_{i}}^{1k} - \frac{1}{8} \overline{(JU)^{k} \delta_{3k} u_{i}}^{3k}$$
(15)

The second term in Eq. (14) is the numerical viscosity where α was set to 1 in this study, and this corresponds to utilizing the uniformly third-order polynomial interpolation algorithm (UTOPIA). Equation (13) was solved with the successive over relaxation (SOR) method. The predicted velocity u^{P} was transformed to the contravariant component and was interpolated at the staggered position after multiplication by J:

$$(JU^{j})^{P} = \overline{J \frac{\partial \xi^{j}}{\partial x_{i}} u_{i}^{P}}^{j}$$
(16)

Using $(JU^i)^p$ of Eq. (16), the Poisson equation on the potential ϕ is expressed as

$$\frac{\delta_j (JU^j)^P}{\Delta t} = -\delta_j \left(\frac{\gamma^{jk}}{\gamma^{jk}} \delta_k \phi \right)$$
(17)

This equation is derived such that JU^{i} at the next step

$$(JU^{j})^{n+1} = (JU^{j})^{P} - \Delta t \overline{\gamma^{jk}}^{j} \delta_{k} \phi \qquad (18)$$

satisfy the continuity on JU^i (Eq. (8)). Equation (17) was also solved by the SOR method in this study. Using ϕ obtained from Eq. (17), the velocity at the next step was corrected as

$$u_i^{n+1} = u_i^P + \frac{\partial \xi^k}{\partial x_i} \overline{\delta_k \phi}^k$$
(19)

and P at the next step was estimated by computing

$$P^{n+1} = P^n - \frac{\phi}{\angle t} + \frac{1}{2J} \left(\frac{1}{\text{Re}} + v_{sgs} \right) \delta_k \left(\frac{\phi}{\gamma^{kl}} \delta_l \phi \right)$$
(20)

2.2 Grid system and boundary condition

The flow around two circular cylinders in staggered arrangement was simulated in an elliptic-column space. This physical space was discretised with an O-type grid system. Figure 3 shows the schematic of the section of the grid system. The O-type grid system had the major axis of 60D, the minor axis of 30D and the thickness of 1D. In the Cartesian coordinates in the physical space, the origin was located at the center between the two cylinder-sections and on a side plane of the grid system; $x_1 - x_2$ - and x_3 - axes were along stream-wise direction, transverse to the stream-wise direction and along the cylinder-span, respectively. The computational coordinates were assigned in terms of body-conformed coordinates ξ^k . This threedimensional grid system was made as follow. A twodimensional grid system was made by solving the Poisson equation in the $x_1 - x_2$ plane. The three-dimensional grid system was organized by just aligning the two-dimensional grids along the x_3 axis. The number of grids on the circumference of each circular cylinder was 200 and that in cylinder span-wise direction was 26. Figure 4 shows the close-up of the x_3 - slice of the three-dimensional grid system in the case for L/D = 2 and α $= 15^{\circ}$.



Fig. 3 Physical space around two circular cylinders



Fig. 4 x_3 - slice of grid system for $\alpha = 15^{\circ}$

The non-slip boundary condition was specified on the surfaces of the cylinders, and the in-flow boundary condition was set to $u_1 = 1$, $u_2 = u_3 = 0$. On the out-flow boundary, an

advection-viscous condition (Miyauchi et al.¹⁵) was applied:

$$\frac{\partial u_i}{\partial t} + u_m \frac{\partial u_i}{\partial x_j} = \frac{1}{\text{Re}} \frac{\partial^2 u_i}{\partial x_i^2}$$
(21)

where u_m was given to 1. Between the $x_1 - x_2$ plane at $x_3 = 0$ and that at $x_3 = D$, the periodic boundary condition was used. The computation was impulsively started, i.e., the initial condition of flow was set to $u_1 = 1$, $u_2 = u_3 = 0$ and P = 0.

3. Numerical Results and Interpretation

3.1 Vorticity distribution

Figures 5 (a)-(c) show instantaneous x_3 -vorticity distributions at the middle of the cylinder span. The upper and lower shear layers separated from the upstream cylinder are notated as SL1up and SL1down respectively, while the upper and lower shear layers separated from the downstream cylinder are abbreviated by SL2up and SL2down. The points where shear layers reattach on, or detach from the surface of the downstream cylinder are notated with capital letters, A, B, C and D.

For $\alpha = 5^{\circ}$ (Fig. 5(a)), the upper shear layer coming from the upstream cylinder (SL1up) reattached on the downstream cylinder (point A). The reattaching SL1up was divided into two, and one of them flowed onward and combined with SL2up, till point D where this separated from the surface; vortices were formed by SL2up. The other part of the divided SL1up traveled along the inner surface (gap side) of the same cylinder till point B. The friction due to SL1up sliding on AB portion drove SL2down to be generated from the downstream-cylinder surface. After detaching at point B, these were deformed by a small eddy formed from SL1down. The small eddy merged with deformed SL1up and SL2down, and convected in downstream direction. These eddies impinged on the lower surface of the downstream cylinder (around point C) intermittently. Due to merging of the eddies and the layers as well as due to the reaction of their impingement on the downstream cylinder, the flow field underneath this cylinder became unsteady. The flow pattern registered for $\alpha = 5^{\circ}$ can be classified as being in the "Proximity and Wake Interference", in conformity with the classification stipulated by Zdravkovich⁹.

An example of the flow field for $\alpha = 10^{\circ}$ is shown in Fig. 5(b), and this is similar to the flow in the case for $\alpha = 5^{\circ}$. In comparison to the case for $\alpha = 5^{\circ}$, a difference found from the vorticity-movie observation was that, occasionally, SL1up discontinued on the upper part of the downstream cylinder (on AD portion) and was entirely redirected through the gap. After some sequences, this irregularity ceased and the flow field returned to the pattern as shown in Fig.5 (b). This can be a bistable characteristic of the flow pattern, which was observed in experiments by Sakamoto *et al.*¹, and this will be explained in detail in Section 3.4.

For $\alpha = 15^{\circ}$ (Fig.5 (c)), SL1up reattached onto the

downstream cylinder at point A, close to the cylinder front middle point, and this induced the formation of SL2down similarly to the cases of $\alpha = 5$ and 10°. Successively, SL1up streamed along with SL2down. SL2down slid on the cylinder surface, till a point B and detached. The parallel layers of SL1up and SL2down formed eddies of opposite sign underneath the downstream cylinder, which intersected SL1down. SL1down enveloped the pair of eddies and carried it in downstream direction; only sometimes the eddy formed by SL2down hit the surface of the downstream cylinder at point C. On the upper side of the downstream cylinder, distinct vortices were constantly formed by the SL2up. This flow pattern belongs to "Gap Flow Regime" (Zdravkovich⁹).

Additionally, it should be mentioned that, throughout Figs.5 (a)–(c), the points A, B, C and D did not have a constant position on the circumference of the downstream cylinder, but varied slightly.



Fig.5 Instantaneous vorticity distributions for (a) $\alpha = 5^{\circ}$, (b) $\alpha = 10^{\circ}$ and (c) $\alpha = 15^{\circ}$

3.2 Aerodynamic forces and Strouhal number

Based on the interpretation of the flow patterns above, the characteristics of the aerodynamic forces is discussed herewith. Figs.6 (a)-(c) show the time histories of the lift and drag coefficients, C_L and C_D , for $\alpha = 5^\circ$, 10° and 15°. The dotted line and the solid line represent the coefficients of the upstream and downstream cylinder respectively. In all the cases, the fluctuations of C_L of the downstream cylinder were much larger than those of the upstream cylinder, in which the upstream-cylinder C_L slightly oscillated around the value of 0. Similarly, the fluctuations of the downstream-cylinder C_D were stronger than for the upstream cylinder. This implies that, the

flow incoming from the upstream cylinder has an important influence upon any variations of forces acting on the downstream cylinder.

For $\alpha = 5^{\circ}$ (Fig.6 (a)), large magnitudes were observed in the C_L fluctuation of the downstream cylinder on certain period, e.g., 40 < t < 60 and 120 < t < 240. These strong C_L -fluctuations are possibly caused by the intermittent impingement of the small scale eddy (merging with SL1up and SL2down) onto point C (Fig.5 (a)). Other fluctuations of C_L and C_D of the downstream cylinder are caused by the constant reattachment at point A and the separation at points D and B along with the formation of the vortices from SL2up and the convection of the small eddies merging with SL1up and SL2down.

For $\alpha = 10^{\circ}$ (Fig.6 (b)), the magnitude of the C_L fluctuation of the downstream cylinder was influenced by the shear layer reattachment at points A and the separation at points B and D, as well as by the vortices generated from SL2up and the small eddies from SL1down merging with parallel two layers (Fig.5 (b)). However, a particularity in this case was the shift of the mean in the C_L time history of the downstream cylinder. For example, the mean C_L of the downstream cylinder became low locally during 120 < t < 190. This shift of the mean C_L is associated with a bistable characteristic of the flow and will be explained in Section 3.4.

For $\alpha = 15^{\circ}$, as observed in Fig.5(c), SL1up reattached to the downstream cylinder near its middle front point without dividing. This results in the drag of the downstream cylinder being stronger relatively to the other α cases. Indeed, the mean C_D of the downstream cylinder (Fig.6(c) bottom) was larger than that for the other cases. On the other hand, the fluctuations of C_L and C_D became large in the range of 160 < t < 240, and this is due to variation of the reattachment and separation points. However, even in this range, the mean value of the downstream-cylinder C_L remains almost the same. This implies that no bistable flow appeared in this case.

Figure 7 shows the mean values of C_L and C_D . The mean lift coefficients for the upstream cylinder, $C_{L,1}$, was almost zero for all incidence angles (Fig.7 (a)), and the drag coefficient for the upstream cylinder, $C_{D,1}$, had almost same value of 0.8 for all cases (Fig.7 (b)). For the downstream cylinder, the mean drag coefficient, $C_{D,2}$, became negative only for $\alpha = 5^{\circ}$. This is because, at this angle, the downstream cylinder is almost immersed in the wake of the upstream cylinder. The reattachment point A of SL1up locates on the upper side of the downstream cylinder (Fig.5 (a)), and SL1up just "wipes" the downstream-cylinder surface and does not fully hit it. Hence, the drag of the downstream cylinder is more influenced by the flow along AB portion and the separation at point B (Fig.5 (a)), which induce a negative drag force, oriented to the inside of the gap. For the other two cases of $\alpha = 10^{\circ}$ and 15° (Figs.5 (b) and (c)), the reattachment point A is close to front middle point of the downstream cylinder, and SL1up sliding around AB portion pushes away the downstream cylinder in the downstream direction. Then, as shown in Fig.7 (b), the mean drag coefficient becomes positive and increases with the incidence angle α . The mean lift coefficient, $\overline{C_{L,2}}$, had similar values of - 0.25 for $\alpha =$ 5° and $\alpha = 15^\circ$, but for 10° had a lower value of -0.4. The lift of the downstream cylinder was negative for all the angles. This



Fig.6 Time histories of C_L and C_D for (a) $\alpha = 5^\circ$, (b) $\alpha = 10^\circ$ and (c) $\alpha = 15^\circ$; — : upstream cylinder, — : downstream cylinder



can be explained from Fig.5: the flow attached around AD portion of the cylinder surface pushes the cylinder down, and the shear layers around BC portion pull down the cylinder. While the above results were totally found to be in agreement with experimental results obtained by Summer *et al.*³⁾ for Re = 32,000, a discrepancy exists in $\overline{C_{L,2}}$ for $\alpha = 10^{\circ}$. The differences in the Reynolds number and the aspect ratio of the cylinder between the experiment and the present simulation might be the causes.

The Strouhal number, St, was obtained from power spectra of the C_L time histories of the downstream cylinder. Figure 8 shows St versus α . The value of St decreased slightly with the increase of incidence angle. For $\alpha = 15^{\circ}$, because two peaks were found in the power spectrum of the downstream-cylinder C_L , both reduced frequencies were plotted in the figure. The experimental result for $\alpha = 16^{\circ}$ by Sumner *et al.*²⁾ also indicated two peaks in the power spectrum and the corresponding Sts are plotted in Fig. 8: the value of the first St was around 0.12 and the second one was around 0.33. While the value of the second St in the present study were larger than that by Sumner *et al.*, both results are in support of the existence of two vortex shedding frequencies around this incidence angle. It is conjectured that the occurrence of the second St is related to the pair of two eddies



Fig.8 Strouhal number versus α

underneath the downstream cylinder (Fig. 5(c)), however further investigations are needed to clarify the exact cause. Except for the case of $\alpha = 15^{\circ}$, the values of St in the present study were in a good agreement with previous experimental results.

3.3 Pressure distribution on circular cylinders

Fig.9 shows the mean pressure coefficient, \overline{C}_p , and the r.m.s pressure coefficient, C'_p , on the cylinder surface. The symbol θ for the horizontal axis denotes the clockwise angle from the upstream side of the cylinder, and the dotted and solid lines represent the values for the upstream and downstream cylinders, respectively. Hereafter, we discuss on the characteristics of the pressure on the downstream cylinder.



Fig.9 Distributions of \overline{C}_p for (a) $\alpha = 5^\circ$, (c) $\alpha = 10^\circ$, (e) $\alpha = 15^\circ$ and C'_p for (b) $\alpha = 5^\circ$, (d) $\alpha = 10^\circ$, (f) $\alpha = 15^\circ$; ----- : upstream cylinder, — : downstream cylinder

In case of a single body, the \overline{C}_{p} - θ curve has a positive peak indicating the stagnation point. From similar discussion, a positive peak in the \overline{C}_{p} distribution of the downstream cylinder indicates the reattachment point of the shear layer from the upstream cylinder. Thus, the reattachment point (point A in Figs.5 (a)-(c)), where SL1up reattached onto the downstream cylinder, is reconfirmed by the presence of the sharp peaks in Figs.9 (a), (c) and (e). In the case for $\alpha = 5^{\circ}$ (Fig.9 (a)), the reattachment point located around $\theta = 60^{\circ}$, and those for $\alpha = 10^{\circ}$ and 15° positioned around $\theta = 40^{\circ}$ and 20° respectively. Then, consistently with the flow observation in Figs.5 (a)-(c), the reattachment angle decreases with increase in α .

It is known that the separation point corresponds to the inflection point in the \overline{C}_p - θ curve. In Fig. 9, the separation point of SL2up for $\alpha = 5^{\circ}$ (point D in Fig.5 (a)) approximately corresponded to $\theta = 100^{\circ}$, and those for $\alpha = 10^{\circ}$ and 15° had almost the same angle of $\theta = 90^{\circ}$. For the separation point of SL2down (point B in Fig.5), it is difficult to identify the angle from Fig.9, but they were approximately around $\theta = 10^{\circ}$ for both $\alpha = 5^{\circ}$ and 10° (Figs.9 (a) and (c)) and around $\theta = 320^{\circ}$ for $\alpha = 15^{\circ}$ (Fig.9 (e)).

In Figs.9 (b) and (d), the C'_{P} distribution of the downstream cylinder had a peak near the reattachment angle of SL1up ; the angles of the peak were of $\theta = 50^{\circ}$ for $\alpha = 5^{\circ}$ (Fig.9 (b)) and $\theta =$ 20° for $\alpha = 10^{\circ}$ (Fig.9 (d)). It is believed that this peak is due to unstable fluctuations of SL1up and SL2down around AB portion (Figs.5(a) and (b)). For $\alpha = 15^{\circ}$ (Fig.9 (f)), the C'_{p} peak due to these shear-layers fluctuation located at $\theta = 350^{\circ}$, and the level of the peak was small relatively to those in the other α cases. This implies that the flow near AB portion (Fig.5 (c)) is rather stable because whole SL1up flows into the gap between the cylinders without being divided. Additionally, in Fig.9 (b) again, a mild peak existed around $\theta = 270^{\circ}$. Judging from the discussion on the flow field in Section 3.1, it is thought that this peak in C'_{P} is resulted from the small eddies (Fig.5 (a)) passing underneath the downstream cylinder; the angle of $\theta =$ 270° in Fig.9 (b) corresponds to the position of point C, approximately. In the cases for $\alpha = 10^{\circ}$ and 15° (Figs.9 (d) and (f)), this mild peak in C'_{P} was also found around $\theta = 270^{\circ}$. The level of this C'_{p} peak became low with increase in α , gradually. This is because the distance between the convection path of the small eddy merging with SL1up+SL2down and the lower surface of the downstream cylinder become wide with increase in α .

3.4 Bistable flow pattern at $\alpha = 10^{\circ}$

Sakamoto *et al.*¹⁾ observed a bistable flow in their experiments using two staggered circular cylinders at Re = 55,000. This bistable flow was characterized as an alternate occurrence of two flow patterns called "Mode 1" and "Mode 2", and synchronizing with the flow pattern change, the mean lift of the downstream cylinder varied discontinuously (Fig. 2). In this section, comparing with the experimental results by Sakamoto *et al.*¹⁾ for L/D = 2 and $\alpha = 10^\circ$, the characteristics of the bistable flow observed in the present simulation are explained in detail. Same notations as Sakamoto *et al.*¹⁾ of Mode 1 and Mode 2 are preserved in this study.

Figure 10 shows instantaneous x_3 -vorticity distributions simulated for L/D = 2 and $\alpha = 10^\circ$, which represents the two flow patterns of the bistable flow. Schematics explaining about

the flow fields are also illustrated in Fig. 10. In Fig.10 (a), SL1up reattached to the downstream cylinder surface (point A) and flowed inside the gap between the cylinders. In the gap, SL1up traveled comparatively near the downstream-cylinder surface along with SL2down. This flow pattern is notated as Mode 1 hereafter. Meanwhile, the flow pattern in Fig.10 (b) is called Mode 2; SL1up was divided into two flows after reattaching the downstream cylinder. Note that comparing with Mode 1 in Fig.10 (a), the position of the SL1up reattachment point in Mode 2 (point A in Fig.10(b)) located at somewhat upper portion of the downstream cylinder. One of the divided SL1up slid along the upper side surface of the downstream cylinder, and the other passed downward through the gap between the cylinders. Thus, two main differences can be noticed between the two flow patterns as follows. In Mode 1, SL1up flows only through the gap between the cylinder and stays in the vicinity of the downstream cylinder together with SL2down. In Mode 2, SL1up flows in two directions after reattachment : one through the gap and one over the upper side of the cylinder. These differences are signaled with dashed-line circles in the schematics in Fig. 10.



Fig .10 Bistable flow pattern for $\alpha = 10^{\circ}$ (a) Mode 1 (b) Mode 2



Fig .11 Relationship between flow pattern and time history of aerodynamic force coefficient of the downstream cylinder in case for $\alpha = 10^{\circ}$ (a) C_L time history (b) C_D time history

In Fig.11(a), the time histories of C_L and C_D for the downstream cylinder are shown, which are the same as the ones in Fig.6 (b). In Fig. 11(a), the C_L time history showed shifts in its mean value : the local \overline{C}_{L} calculated from the data in the range of 120 < t < 190 had a value of -0.386 while that in the range of 220 < t < 290 was of -0.297. Along with the shift of \overline{C}_{t} , a slight switch of mean values in the C_D time history occurred (Fig.11(b)). This evolution of C_L and C_D time histories is associated with the bistable flow pattern. The flow patterns shown in Figs.10(a) and (b) represent the flows at t = 140 and 240 respectively; Mode 1 then occurred around 120 < t < 190while Mode 2 appeared in the range of 220 < t < 290. Additionally, because the values of the local \overline{C}_{i} in the other time ranges (e.g., 40 < t < 100 and t > 300) were almost the same as that in 220 < t < 290, the flow in those time range are categorized as Mode 2. Thus, Mode2 was dominantly induced in the present simulation. On the other hand, when Mode1 was induced, the downstream-cylinder C_L fluctuated with large magnitude around the mean value of -0.386 as seen in Fig 11(a). This strong fluctuation occurs probably due to unsteady flows near the lower part of the downstream cylinder.

Figure 12(a) shows the distributions of the mean pressure coefficients, \bar{C}_{ν} , on the downstream cylinder surface. The solid line represents the values of \overline{C}_{P} in Mode 1, which was obtained from the pressure coefficient, C_p , in the range of 120 < t< 190 (see Fig.11). The dotted line is for the \overline{C}_{p} values in Mode 2, which was from the C_p data in the range of 220 < t <290. In order to compare the present results with the experimental data in detail later, those distributions are presented with two figures, where the left figure in Fig.12 represents the values of \overline{C}_{P} in the range of $0^{\circ} \le \theta \le 180^{\circ}$ and the right one is for those in $180^{\circ} \le \theta \le 360^{\circ}$. The \overline{C}_{P} distribution had a peak representing the reattachment point of SL1up. In the left figure in Fig. 12 (a), this peak in Mode 1 located around $\theta = 30^{\circ}$ and that in Mode 2 positioned around $\theta = 40^{\circ}$. In addition to the difference of the peak position, the \overline{C}_{p} values for Mode 1 around $0^{\circ} < \theta \le 40^{\circ}$ (upstream and upper side of the cylinder) were much larger than those for Mode 2. Furthermore, in the ranges of $90^{\circ} < \theta \le 270^{\circ}$ (downstream side of the cylinder): $90^{\circ} < \theta \le 180^{\circ}$ in the left figure in Fig.12 (a) and $90^{\circ} < \theta$ $\leq 270^{\circ}$ in the right one, the \overline{C}_{p} values in Mode 1 were lower than those in Mode2. These characteristics result in the drag in Mode 1 being larger than that in Mode2. Indeed, in Fig. 11(b), the drag at which Mode 1 appeared (120 < t < 190) became slightly larger than that in Mode 2. Special attention is paid to the \overline{C}_{P} values on the lower side surface of the downstream cylinder. In the range of $180^{\circ} < \theta < 360^{\circ}$ (right figure in Fig.12 (a)), the \overline{C}_{P} values in Mode 1 were entirely lower than those in Mode 2. The reason for this is that in Mode 1, as observed in Fig. 10(a), SL1up and SL2down travel close to the lower side surface of the downstream cylinder and a stronger suction is driven on the surface. This works so as to pull the cylinder

downward. Additionally, as mentioned above, the \overline{C}_p values in Mode 1 around $0^\circ < \theta \le 40^\circ$ were much larger and the positive pressures push the cylinder downward. These two characteristics mainly contribute to induce a stronger negative lift in Mode 1(Fig. 11(a)).

Figure 12(b) indicates the \overline{C}_{p} distributions for Mode 1 and Mode 2 experimentally obtained by Sakamoto *et al.*¹, where L/D = 2, $\alpha = 10^{\circ}$ and Re = 55,000. In the range of $0^{\circ} \le \theta \le 180^{\circ}$ (left figure in Fig. 12 (b)), the \overline{C}_{p} values were almost the same between Mode 1 and Mode 2. Comparing with the present result in the left figure in Fig.12(a), the values around $0^{\circ} \le \theta \le 30^{\circ}$ and $60^{\circ} \le \theta \le 180^{\circ}$ were large. For $240^{\circ} \le \theta$ \leq 360° in the right figure in Fig. 12(b), the difference between Mode 1 and 2 can be noticed distinctly: the \overline{C}_{p} values for Mode 1 was much lower than those in Mode 2. Additionally, their \overline{C}_{p} values in this θ -range for Mode 1 were much lower than those of the present study. In the experiment by Sakamoto et al.¹⁾, the mean C_L for Mode 1 was the value of -0.8 approximately, while for Mode 2 that was the value around -0.4. The values of the mean C_L for Mode 1 and 2 in their study were much larger than the present ones (Mode 1: -0.386, Mode 2: -0.297). This large discrepancy of the mean C_L for both modes is resulted from the difference in \overline{C}_{p} mainly on the lower side surface of the downstream cylinder. Although it has not been clarified the reason for this difference in \overline{C}_{p} , possible causes are the differences in the surface roughness (surface boundary condition) and aspect ratio of the cylinder as well as in Re.



Fig. 12 Comparison of \overline{C}_p distribution on the downstream cylinder surface between Mode 1 and Mode 2 (a) present study (b) data compiled from experimental result by Sakamoto *et al.*¹⁾

4. Conclusions

Numerical investigations of cross-flow past a pair of circular cylinders in staggered arrangement for angles $\alpha = 5^{\circ}$, 10° and 15° at Re = 22,000, having the center to center distance ratio of

L/D = 2.0, were performed. The relationship between the flow patterns and the aerodynamic forces induced by fluid upon cylinders were studied in detail. The values of the Strouhal number and the aerodynamic coefficients were almost in agreement with those in previous experimental researches. The pressure distributions on the downstream cylinder showed clear correlations with the flow patterns. A bistable flow pattern, accompanied by a shift of the mean values of lift and drag coefficients of the downstream cylinder, was simulated at the incidence angle of 10°. However, for the two flow modes involved in the bistable flow, the lifts and the pressure distributions of the downstream cylinder differed slightly from experimental results.

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